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INFERENTIAL STATISTICS ASSESSMENT OF URBAN RAINFALL-RUNOFF MODELS

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ABSTRACT

Thorough understanding of the rainfall-runoff processes that influence watershed hydrological response is important and can be incorporated into the planning and management of water resources. This study assessed rainfall-runoff models through inferential statistics and benchmarked their runoff predictive accuracies against a proposed new runoff model. Linear regression model has been in use to model urban rainfall-runoff. However, the model was found to be statistically in-significant in this study. Hydrological implications from the regression model became in-consistent and obsolete. The 1954 simplified SCS runoff model was also statistical in-significant under two Null hypotheses rejection and paved way for the regional model calibration study. A new rainfall-runoff model was developed with calibration according to regional hydrological conditions. It out-performed simplified SCS runoff model and reduced RSS by 54%.

Keywords: urban runoff model, linear regression model, SCS, inferential statistics.

INTRODUCTION

In Malaysia, about 97% of the total water demands for irrigation, domestic and industrial uses come from surface runoff (DID, 2000). Rain that falls on watershed surfaces will be transformed into surface runoff or become interflow after being infiltrated and percolated into soil. The flows will leave the watershed as discharge into receiving water and eventually into the sea to complete the hydrological cycle. As a result of rapidly growing human activities, stormflow volume and peakflow have increased significantly due to expansion of impervious land area and the decreased availability of depression storages (Adams and Papa, 2000). These increased flows are conveyed to natural watercourses and eventually discharged through the watershed outlet. Unfortunately, the natural receiving watercourses in downstream areas are often not sufficient to cope with the larger and more frequent runoff events. The resulting effects are the increased frequency of flooding in the downstream of urban watersheds. Impacts of disturbance at local watersheds tend to aggravate or vary as the watershed scale becomes larger, affecting people living downstream. Therefore, from a management viewpoint, a thorough understanding of the rainfall-runoff processes that influence watershed hydrological response is important and can be incorporated into the planning and management of water resources (Chan, 2005).

Linear Regression Model

One dimensional linear regression model has been in use to model urban rainfall-runoff. The slope of the regression model could be implied as hydrological reduction factor (Harremoës and Arnbjerg-Nielsen, 1996) or as the percentage of impervious area (Abustan and Ball, 2000). The interception on x-axis is regard as the estimation of initial loss, local depression or the depression loss (Huber and Dickinson, 1988), (Abustan *et al.* 2008, 2008b) of a watershed. The base form of this model is:

$$Q = \mathrm{m}P + \mathrm{c} \tag{1}$$

Q = Runoff amount (mm)
P = Rainfall depth (mm)
m = gradient (slope)
c = constant

SCS Runoff Model

In 1954, the United States Department of Agriculture (USDA), then Soil Conservation Services (SCS) proposed a rainfall runoff prediction model. Since its inception, the model was incorporated into many official hydro design manuals and even led to the derivation and development of curve number (CN) methodology but many researchers around the world reported inconsistent results using the model (Hawkins *et al.* 2009), (Schneider and McCuen, 2005), (Hawkins, 2014), (Ling and Yusop, 2013). The base model was proposed as:

$$Q = \frac{(P - I_a)^2}{P - I_a + S} \tag{2}$$

Q = Runoff amount (mm)
P = Rainfall depth (mm)
I_a = the initial abstraction (mm)
S = maximum potential water retention of a watershed (mm)

The initial abstraction is also known as the event rainfall required for the initiation of runoff. SCS also hypothesized that $I_a = \lambda\, S = 0.20S$. The value of 0.20 was referred to as the initial abstraction coefficient ratio (λ), a correlation parameter between I_a and S. The value of 0.20 was presented as a constant (λ value falls within 0 to 1 only). The substitution of I_a = 0.20S simplifies equation (2) into a common simplified SCS runoff prediction model:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \qquad (3)$$

Equation (3) is subjected to a constraint that $P > 0.2S$, else Q = 0. However, there were increasing evidential study results leaning against the prediction accuracy of equation (3) and the hypothesis that I_a = 0.20S. The literature review of fifty-one worldwide studies showed inconsistent runoff results using equation (3), many researchers urged to perform regional hydrological conditions calibration instead of blindly adopting it as proposed by SCS (Hawkins *et al.* 2009), (Ling and Yusop, 2013). This study was inspired by a developed methodology (Hawkins *et al.* 2009) and utilised numerical analysis algorithm guided by inferential statistics to derive a new rainfall runoff model based on equation (2). New model was calibrated according to regional hydrological conditions as pertain to the given dataset in Melana watershed.

DATA AND METHODOLOGY

Study Site

This study adopted the rainfall-runoff dataset from a different research which was carried out in Melana Watershed. It is located in Johor, Malaysia between 1° 30' N to 1° 35' N and 103° 35' E to 103° 39' E (Figure-1). Drained by Melana River which starts in the hilly area of Gunung Pulai in the north, the watershed covers an area of 21.12 km^2.

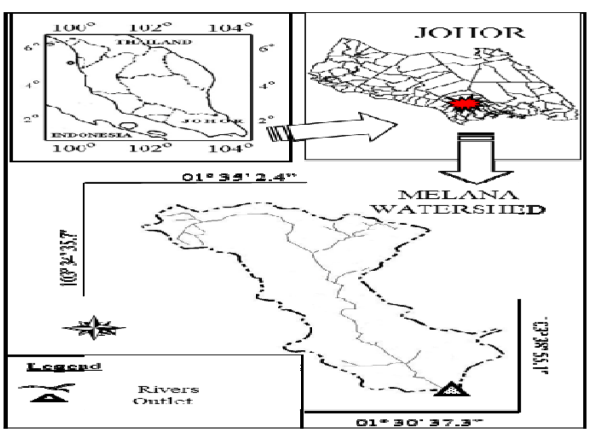

Figure-1. Melana watershed in Johor (Chan, 2005).

In 1993, only about 20% of the area in Melana Watershed was covered by urbanised area, by 2010, more than 60% of the area was developed mainly for residential area (MPJBT, 2001).

METHODOLOGY

27 rainfall-runoff datapairs were recorded between July and October of 2004 at this site. Linear regression model was first fitted with all events and analysed with descriptive statistics using IBM PASW version 18. Non-parametric Bootstrapping technique, Bias corrected and accelerated (BCa) procedure (2000 samples) was conducted to double check the statistics results at 95% confidence level. Bootstrapping BCa statistics was selected for robustness and the inferential ability through its confidence interval. The slope and the constant from the linear regression model are the main focus under the assessment as these parameters have significant hydrological implications, and therefore it is crucial for both parameters to be statistically significant at least at alpha = 0.05 level in order for linear regression model to be considered as an acceptable rainfall-runoff model in this study.

To the best of our knowledge, no attempt was made to validate previous research findings by performing regional hydrological characteristics calibration on SCS base runoff prediction model equation (2) in Malaysia until now. We are also unaware of any research which calibrate and apply SCS model in urban runoff study. Inferential non-parametric statistics was employed for two claim assessments set forth by the 1954 SCS proposal with two Null hypotheses (Rochoxicz, 2011), (Howell, 2007), (Wright, 1997):

Null Hypothesis 1 (H_{01}): I_a = 0.20S globally.

Null Hypothesis 2 (H_{02}): The value of 0.20 is a constant in H_{01}.

The rainfall (P) and runoff (Q) data pairs from Melana site were used to derive I_a in order to calculate S and λ using a developed methodology by US researchers (Hawkins *et al.* 2009), (Schneider and McCuen, 2005), (Hawkins, 2014). The difference of rainfall depth (P) and initial abstraction (I_a) is the effective rainfall depth (P_e) to initiate runoff (Q) thus $P - I_a = P_e$. Substitute this relationship into equation (2), the model can be re-arranged in order to calculate S and λ for each P-Q data pair. Bootstrapping, Bias corrected and accelerated (BCa) procedure was used to aid numerical optimisation technique in the selection of the optimum λ value and to assess both hypotheses. Rejection of H_{01} implies that equation (3) is invalid and not applicable for Melana dataset, while H_{02} rejection indicates that λ is not a constant as initialy proposed by SCS in 1954 but a variable. Rejection of both hypotheses will pave way to derive new λ value. The selection of the optimum λ and S value will formulate a new calibrated runoff prediction model of Melana watershed.

STATISTICS AND HYPOTHESES ASSESSMENT

Linear Regression Model

Based upon linear-intercept basic form equation (1), PASW identified a best fitted linear regression model for the given rainfall-runoff dataset as:

$$Q = 0.364P - 1.613 \qquad (4)$$

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Equation (5) has adjusted R square = 0.748 and standard error = 3.751, the statistics is tabulated in Table-1.

Table-1. 95% confidence interval.

Model	Coeff.	*p* value	BCa *p* value
Constant	-1.613	0.112	0.334
Gradient	0.364	0.000	0.275

The constant coefficient is not significant under 95% confidence interval and BCa test with *p* value >0.05. BCa test discounted entire linear regression model because both fitting coefficients of the model are not significant. When the constant becomes in-significant (x-intercept = 0), another alternate linear regression fitting form is regression through the origin (RTO) with gradient as the sole fitting coefficient. PASW re-analysed Melana dataset with RTO and identified the best fitted RTO as:

$$Q = 0.318P \tag{5}$$

The statistics is tabulated in Table-2.

Table-2. (RTO) 95% confidence interval.

Model	Coeff.	*p* value	BCa *p* value
Gradient	0.318	0.000	0.194

Although RTO model survived the 95% confidence interval with *p* value < 0.01but stringent BCa test rejected the significancy of the only fitting coefficient of the model. Therefore, every possible linear regression fitting model was rejected by BCa test.

Simplified SCS Runoff Model

BCa results provided confidence interval span for λ at Melana watershed (Table-3). BCa results consist of confidence intervals for λ, which can also be used to assess Null hypotheses. The span of λ confidence interval will be used to asses H_{01} while H_{02} will be based on the standard deviation of the derived λ dataset (Ling and Yusop, 2014, 2014b). Neither the mean nor the median's confidence interval span includes λ value of 0.2 while the standard deviation of λ dataset is not equal to zero. The assessment of H_{01} and H_{02} will base on these results. Both hypotheses must not be rejected in order to apply equation (3) for this study.

Calibrated SCS Runoff Model

Twenty-seven λ values were derived from Melana dataset. The study will identify a best collective representation of λ value for the dataset in order to formulate a new runoff prediction model and benchmark against the empirical model equation (3) where λ was assumed to be 0.2 by SCS. The descriptive statistics of the data distribution of λ values was tabulated in Table-3. Bootstrapping technique, Bias corrected and accelerated (BCa) procedure (2000 samples) was conducted at a stringent 99% confidence level on the λ dataset to include confidence intervals and aid the selection of an optimum λ value (Rochoxicz, 2011), (Howell, 2007), (Wright, 1997).

Table-3. BCa results of Melana dataset.

Melana λ dataset	Statistics	99% BCa Lower	Upper
Mean	0.059	0.009	0.154
Median	0.009	0.004	0.015
Skewness	4.677		
Kurtosis	22.778		
Std. Deviation	0.188		

λ optimization study was conducted via numerical analyses approach base on equation (2). The least square fitting algorithm was set to identify an optimum λ value by minimizing the residual sum of squares (*RSS*) between final runoff model's predicted *Q* and its observed values. The optimization study was based on λ variation within the median confidence interval due to the skewed λ dataset. The optimization study via numerical analysis identified the optimum λ value to be 0.015 and the best collective representation *S* value to be 81.804 mm for Melana watershed. Since $I_a = \lambda\ S$ the substitution of λ and *S* value yields I_a = 1.248 mm. With the substitution of I_a and *S* back to equation (2), the calibrated rainfall runoff prediction model was formulated as:

$$Q_{0.015} = \frac{(P-1.248)^2}{P+80.555} \tag{6}$$

Equation (6) is subjected to a constraint that P>1.248 mm, else $Q_{0.015}$ = 0. The formulation of the calibrated SCS runoff prediction model equation (6) using the optimum λ value will have the same inherent significant level (at alpha = 0.01).

RESIDUAL MODELLING

Calibrated new runoff model (with λ=0.015) equation (6) was benchmarked against non-calibrated SCS runoff model equation (3), linear-intercept model equation (4) and RTO equation (5). Model's prediction efficiency index (*E*), *RSS* and predictive model *BIAS* were calculated in order to draw further comparison. Model runoff prediction comparison results were tabulated in Table-4 with following formulas:

$$RSS = \sum_{i=1}^{n}\left(Q_{predicted} - Q_{observed}\right)^2 \tag{7}$$

$$E = 1 - \frac{RSS}{\sum_{i=1}^{n}\left(Q_{predicted} - Q_{mean}\right)^2} \tag{8}$$

$$BIAS = \frac{\sum_{i=1}^{n}\left(Q_{predicted} - Q_{observed}\right)}{n} \quad (9)$$

Q = Runoff amount (mm)
n = Total number of datapairs

Table-4. Runoff predictive models comparison.

Model	New Calibrated model (6)	Non-calibrated SCS model (3)	Eq. (4)	Eq. (5)
E	0.87	0.71	0.76	0.73
RSS	193.2	423.4	351.8	390
BIAS	0.068	1.869	0.009	0.88

RSS value indicates the residual spread from a model. Lower *RSS* indicates a better predictive model. Model efficiency index (*E*) ranges from minus to 1.0 where index value = 1.0 indicates a perfect predictive model. When E<0, the predictive model peforms worse than using the average to predict the dataset. Predictive model *BIAS* shows the overall model prediction error calculated by the summation of predictive model's residual to indicate the overall model prediction pattern. Zero value indicates a perfect overall model prediction with no error, the negative value indicates the overall model tendency of under-prediction and vice versa. It is noteworthy to mention that equation (3), (4) and (5) are not statistically significant even at alpha = 0.05 level.

Equation (6) and (3) are derivative from (2) with same mathematical framework, and therefore it is possible to perform residual modelling between them. A statistical significant model can correct and narrow the runoff prediction gap between two models. An effective residual model can transform equation (3) into proximate predictive model as equation (6) with goals to increase *E* index, reduce *RSS* and improve overall model *BIAS* of equation (3). The residual between two models was calculated by subtracting runoff predictions of equation (3) to equation (6) in order to quantify runoff prediction error of the simplified SCS runoff model against the new calibrated runoff predictive model.

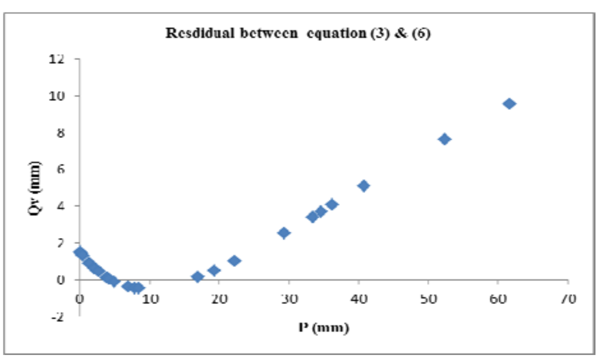

Figure-2. Runoff prediction equation (3) – equation (6).

As shown in Figure-2, the residual distribution between equation (3) and (6) shows a clear pattern of trend. A statistical significant residual model will adjust the runoff prediction difference and correct runoff predictions into proximate results as produced by equation (6). Runoff prediction difference (residuals) between equation (3) and (6) was mapped with several non-linear regression models in IBM PASW. The best correlation was modelled with the following equation. (Adjusted R square = 0.991, Standard error = 0.231, p< 0.000):

$$Q_v = 1.3E-4P^3 + 0.015P^2 - 0.282P + 1.194 \quad (10)$$

Q_v = Runoff difference between models (mm)
P = Rainfall depth (mm)

Equation (10) is the runoff predictive correction of the simplified SCS runoff prediction model. It can be amended to equation (3) to improve its overall model predictive accuracy. The simplified SCS runoff prediction model correction was proposed as:

$$Q = \frac{(P-0.2S)^2}{P+0.8S} - equation(10) \quad (11)$$

RESULTS AND DISCUSSION

This study explored the possibility of the application of linear regression model for urban runoff study in Melana catchment. Linear-intercept regression model had an in-significant fitting constant term. According to previous studies, the constant term is vital for the solution of x-intercept which implies the estimation of initial loss, local depression or the depression loss of a watershed. In-significant fitting constant infers that there is no initial or depression loss (local depression = 0) at Melana watershed according to its *P-Q* dataset. The hydrological interpretation onward is that the watershed is fully saturated with 100% runoff from any rainfall amount. The only reasonable runoff model will have to be $Q = P$ but the RTO regression results stated otherwise. Although equation (5) is significant (under 95% confident interval), the gradient's coefficient which was proposed by previous researchers to estimate the total impervious area was barely 32% instead of 100% as expected (with 68% of pervious area, 100% runoff is impossible). If RTO regression results is valid (32% of impervious area within Melana watershed), depression loss must still exists but linear-intercept regression model discounted its existence with an in-significant fitting constant term. Contrary, stringent BCa test consistently discounted both linear-intercept and RTO regression models (Table-1 and 2). Same interpretation conflict was cited in Sungai Kerayong and Sungai Kayu Ara catchments in Kuala Lumpur (Abustan *et al.* 2008, 2008b).

Researchers across the world concluded that SCS runoff prediction model had to be calibrated according to regional specific characteristics and the conventional simplified SCS runoff prediction form of equation (3) could not be blindly adopted for study use. As such, we assessed two hypotheses as well as model efficiencies of non-calibrated (simplified) SCS model equation (3) and the new calibrated runoff prediction model equation (6).

The initial SCS hypothesis of the λ value of 0.2 (the value was used to simplify SCS base runoff model) as a constant was rejected at alpha = 0.01 level because the 99% BCa confidence interval span did not include the value of 0.2 (Table-3) which deduced that equation (3) was invalid and not applicable to the Melana dataset. H_{02} was also rejected (at alpha = 0.01 level) because the BCa results showed a non-zero standard deviation of λ (Table-3) which indicated its fluctuation nature thus λ could not be a constant as proposed by SCS in 1954 but a variable for Melana dataset. The rejection of both Null hypotheses in this study paves the way for model calibration.

This study used numerical analysis approach guided by non-parametric inferential statistics to identify the best collective representation of λ and S value from the dataset for the formulation of a better runoff predictive model at Melana watershed. The common pitfall in the least square fitting algorithm is to wrongly identify local minima or maxima as optimum solution thus producing inconsistent results. The initial guess point for least square fitting algorithm to commence an optimization search often played an influential role to end results. Researchers often started the initial guess point with a wild guess which could lead to a wrong conclusion. Inferential statistics can be an effective guide to narrow the search and identify a statistical significant optimum solution in swift and precise manner. Inferential statistics narrowed the optimum search band while optimization study pin pointed an optimum value within the BCa confidence interval range; both methods supplemented each other in this regard. The optimum λ value was identified as 0.015 and S = 81.804 mm to model runoff in this study at alpha = 0.01. Therefore, the optimum λ and S value are statistical significant, best collective representation of the dataset. The formulation of the new calibrated runoff prediction model equation (6) using these optimum values will have the same inherent significant level (at alpha = 0.01).

The rejection of both hypotheses concluded that equation (3) is invalid and not statistical significant for this study. Therefore, it is imminent to model the runoff difference between equation (3) and (6) to produce a correction equation and adjust the runoff predictability of equation (3) in order for SCS practitioners or its software users to perform runoff results adjustment. The correction equation (11) improved the runoff prediction results through a site specific characteristics calibration protocol which corrected *RSS* of equation (3) by almost 55% and achieved proximate runoff prediction results as equation (6).

In the benchmark assessment against the new calibrated runoff prediction model represented by equation (6), (un-calibrated) simplified SCS model equation (3) over-predicted runoff depth amount by 1.801 mm on average in this study (in non-linear format). When compared to equation (6), equation (3) showed runoff over-predictions at rainfall depths below 5 mm, under-predicted runoff between 5 – 9 mm and returned to runoff over-predictions thereafter (Figure-2). Using the (un-calibrated) simplified SCS runoff prediction model equation (3) results will incur different design risks and commit a type II error.

In comparison to the new calibrated runoff model, un-calibrated SCS equation (3) over-predicted 38,039 m^3 (on average in non-linear format) under different rainfall scenarios from Melana catchment area in this study. The over-prediction risk was significant and further magnified under higher rainfall depths.

CONCLUSIONS

Linear regression model is not applicable to model urban runoff in this study because both linear-intercept and RTO regression models were not statistical significant. Linear regression model produced in-consistent results and left the hydrological interpretation in question thus it was discarded to model the runoff conditions at Melana watershed.

Inferential statistics assessments rejected both H_{01} and H_{02} at alpha = 0.01 level. Therefore, simplified SCS runoff model equation (3) became obsolete and not applicable in this study. It paved way for the development of regional hydrological conditions calibration methodology to formulate a new calibrated runoff predictive model on SCS runoff model framework. Inferential statistics guided numerical optimization algorithm to search for best collective representation values within 99% confidence interval span. Therefore, new calibrated runoff predictive model has the inherent statistical significancy (at alpha = 0.01). New calibrated runoff predictive model was the only model that was statistically significant, it also out-performed its counterpart models with high model efficiency (E) and low *BIAS*. A runoff correction equation was devised to restore simplified SCS predictive model accuracy. This study proved that SCS base runoff predictive model can be calibrated to predict urban runoff.

Equation (10) and (11) are only applicable for this particular study and do not apply to any rainfall depths larger than 62 mm. However, design engineers and users of the conventional SCS runoff prediction model are encouraged to conduct regional specific calibration for this model and formulate appropriate adjustment equation(s) as proposed.

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REFERENCES

[1] Abustan, I. and Ball, J.E. (2000). Modelling the Export of Phosphorous from Urban Catchments. Australian Journal of Water Resources, 41.

[2] Abustan I., Sulaiman, A.H., Wahid, N.A. and Baharudin, F. (2008). Determination of Rainfall-Runoff Characteristics in an urban area: Sungai Kerayong Catchment, Kuala Lumpur. 11th International Conference on Urban Drainage, Edinburgh, Scotland, UK. http://web.sbe.hw.ac.uk/staffprofiles/bdgsa/11th_International_Conference_on_Urban_Drainage_CD/ICUD08/pdfs/706.pdf

[3] Abustan I., Sulaiman, A.H., Wahid, N.A. (2008b). Urban Rainfall-Runoff Study to Validate the Design Chart in the Malaysian Urban Stormwater Management Manual (MSMA). 11th International Conference on Urban Drainage, Edinburgh, Scotland, UK. http://eprints.usm.my/13596/1/urban_rainfall-runoff.pdf

[4] Adams, B.J. and Papa, F. (2000). Urban Stormwater Management Planning with Analytical Probabilistic Models. New York: John Wiley & Son.

[5] Chan, C.H. master thesis (2005). Modelling Watershed Cumulative Effects On Runoff Based On Major Land Uses.

[6] DID (2000). Urban Stormwater Management Manual for Malaysia. Kuala Lumpur, Malaysia: Department of Irrigation and Drainage.

[7] Harremoës, P. and Arnbjerg-Nielsen, K. (1996). Prediction of Hydrological Reduction Factor and Initial Loss in Urban Surface Runoff from Small Ungauged Catchments. Elsevier: Atmospheric Research, pp.137--147.

[8] Hawkins, R.H., Ward, T., Woodward, D. E. and Van Mullem J. (2009): Curve Number Hydrology: State of the Practice. Reston, Virginia, ASCE.

[9] Hawkins, R.H. (2014 e-mail communication).

[10] Howell, D.C. (2007). Statistical methods for psychology (6th Ed.). CA: Thomson Wadsworth, Belmont.

[11] Huber, W.C. and Dickinson, R.E. (1988). Stormwater Management Model, Version 4, Users Manual, Athens, Georgia, USA: Environmental Research Laboratory, US Environmental Protection Agency.

[12] Ling, L. and Yusop, Z. (2013). A micro focus with macro impact: Exploration of initial abstraction coefficient ratio (λ) in Soil Conservation Curve Number (CN) methodology. IOP Conf. Ser.: Earth Environ. Sci. 18 012121 doi:10.1088/1755-1315/18/1/012121, issue 1.

[13] Ling, L. and Yusop, Z. (2014). Inferential Statistics of Claim assessment. AIP Conference Proceedings: ISBN: 978-0-7354-1274-3; http://dx.doi.org/10.1063/1.4903675.805

[14] Ling, L. and Yusop, Z. (2014b). Inferential Statistics Modelling and Claim re-assessment. ICCEMS Conference Proceedings: ISBN: 978-967-11414-7-2; http://www.iccems.com/2014/ICCEMSProcAll.pdf. 835-840.

[15] MPJBT (2001). Rancangan Tempatan Skudai (1993-2010). Johor Bahru, Malaysia: Pihak Berkuasa Perancangan Tempatan Daerah Jahor Bahru.

[16] Rochoxicz, John A. Jr. (2011). Bootstrapping Analysis, Inferential Statistics and EXCEL. Spreadsheets in Education (eJSiE). 4(3).

[17] Schneider, L. and McCuen, R.H. (2005). Statistical Guidelines for Curve Number Generation. J. Irrig. Drain. Eng.,131, pp.282--290.

[18] Wright, D.B. (1997). Understanding statistics: an introduction for the social sciences. London, Sage.