Blocking Probability of f-Cast Optical Banyan Networks on Vertical Stacking

Lisheng Ma
Chuzhou University
Chuzhou, P. R. China 239000
Email:mls@chzu.edu.cn

Xiaohong Jiang
Future University Hakodate
Hokkaido, Japan 041-8655
Email: jiang@fun.ac.jp

Bin Wu
Tianjin University
Tianjin, P. R. China 300072
Email: binwu.tju@gmail.com

Achille Pattavina
Politecnico di Milano
Milano, Italy 20133
Email: pattavina@elet.polimi.it

***Abstract*—Vertical stacking of banyan networks has been an attractive architecture to construct optical switching networks due to its small depth, absolute signal loss uniformity and good fault tolerance property. Recently, F.K.Hwang extended the study of banyan-based networks to the general f-cast case, which covers the unicast ($f = 1$) and multicast ($f = N$) as special cases. In this paper, we study the blocking probability of f-cast optical banyan networks under crosstalk-free constraint. It is expected that the proposed probability model can be used to dimension such an f-cast network and achieve a graceful tradeoff between hardware cost and blocking probability.**

***Index Terms*—Optical switching network, f-cast, blocking probability, banyan networks.**

I. Introduction

As the result of the development of communication technology, there is an increasing demand for the high bandwidth and high capacity communication systems. Optical networks hold a great promise to meet this demand and serve as the backbone network of next generation Internet, because each fiber can support hundreds of wavelengths with each wavelength operating at rates above 10 Gbit/s [1]. Optical switching network is a key element in an optical network, which can be built by combining the directional coupler (DC) technology [2], [3] and banyan network architecture [4], [5]. This is because banyan network has simple switch setting (self-routing), a smaller and exact same number of switching elements (SEs) along any path between an input-output pair, an absolutely loss uniformity and smaller attenuation of optical signals.

It is notable, however, that DC-based optical switching networks suffers from an intrinsic crosstalk problem [6], in which a small portion of optical power in one waveguide will be coupled into the other unintended waveguide when two optical signals pass through a DC at same time. This undesirable coupling is called the *first-order crosstalk*. Crosstalk elimination is an important issue for improving the signal-to-noise ratio of the optical flow transmission, which can meet the stringent bit-error rate requirement of fiber optics. A cost-effective solution to the crosstalk problem is to guarantee that only one signal passes through a DC at a time, which can eliminate the first-order crosstalk. Due to Banyan topology only a unique path can be found from each network input to each network output, the network is degraded as a blocking one. To deal with this situation, horizontal expansion or vertical stacking, or a combination of them are usually applied to construct banyan-based nonblocking optical switching networks [7], [8].

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In this paper, we focus on vertical stacking of optical banyan networks that are free of first-order crosstalk constraint in every SE in the networks (we refer to this constraint as crosstalk-free constraint hereafter). In recent years, vertical stacking crosstalk-free optical banyan networks have attracted extensive research activities [7]-[18]. These works can be roughly divided into two categories, nonblocking condition study and blocking probability analysis. In this paper, our interest is on blocking probability analysis. In [7], [8], [9], the nonblocking conditions of crosstalk-free banyan-based optical switches have been explored. F.K.Hwang extended the study of banyan-based networks to the general f-cast case [10], [11], which covers the unicast ($f = 1$) and multicast ($f = N$) as special cases. In [12], the strictly nonblocking condition under various crosstalk constraints has been determined for f-cast banyan-based photonic networks. In [13], the rearrangeable nonblocking condition for f-cast banyan-based photonic networks was studied under both node-blocking scenario and link-blocking scenario. Previous works on the blocking probability analysis mainly focus on the one-to-one request (unicast) [14], [15], [16], [17], [18], in which each input can request only one output. To the best of our knowledge, however, no study has been reported for blocking probability analysis of f-cast vertical stacking optical banyan networks.

This paper conducts a comprehensive study for the blocking probability of f-cast vertical stacking optical banyan networks with the consideration of crosstalk-free constraint. We extend the probabilistic methods used for the analysis of vertical stacking optical banyan networks with only unicast to analyze the blocking probability of f-cast vertical stacking optical banyan networks. We propose an upper bound on blocking probability of f-cast crosstalk-free vertical stacking optical banyan networks, which can help us to explore the inherent relationship between the blocking probability and network

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hardware cost (number of vertical stacking planes).

The rest of the paper is organized as follows: Section II provides preliminaries that will facilitate our discussions. Section III presents the upper bound on blocking probability. Comparison result is provided in section IV. Section V concludes the paper.

II. Preliminaries

In general, a typical $N \times N$ banyan network consists of $log_2 N$ stages, each of which contains $N/2$ 2×2 switches. The link connection between each pair of adjacent stages is arranged in such a way that a butterfly interconnection pattern is recursively applied as shown in Fig. 1. We use VSOB(N, m) to denote an $N \times N$ VSOB network that has m stacked copies (planes) of an $N \times N$ banyan network. In this paper, we consider VSOB(N, m) that can support f-cast communication, where an input can simultaneously request up to f $(1 \leq f \leq N)$ distinct outputs. The blocking probability is defined as the probability that a feasible connection request is blocked, where a feasible connection request is a connection request between an idle input port and an idle output port of the network.

Due to topological symmetry of banyan networks, all paths in a banyan network have the same property in terms of blocking. To study the blocking probability, we will select the path between the first input and the first output, which is termed as the *tagged path* in the following section. All the SEs on the tagged path are called *tagged SEs*. The stages of SEs are numbered by 1 to n from left to right. For the tagged path, an input intersecting set $I_i = \{2^{i-1}, 2^{i-1}+1, ..., 2^i - 1\}$ at stage i is defined as the set of all inputs that intersect a tagged SE at stage i. Likewise, an output intersecting set $O_i = \{2^{i-1}, 2^{i-1}+1, ..., 2^i - 1\}$ associated with stage i is defined as the set of all outputs that intersect a tagged SE at stage $log_2 N - i + 1$, $1 \leq i \leq log_2 N$.

The following notations will be used in the remainder of this paper.

N: Network size

n: $n = log_2 N$ is number of network stages

f: Maximum fanout of a request, $1 \leq f \leq N$

m: Number of network planes

r_I: The occupancy probability of an input link (input workload)

r_O: The occupancy probability of an output link (output workload)

ξ_i: Number of connections from input port i, where $0 \leq \xi_i \leq f$

β_i: Number of connections from input set I_i, where $0 \leq \beta_i \leq |I_i| \cdot f = 2^{i-1} \cdot f$

In general, when the number of both input and output ports involved in multicast in a banyan network is considered at any time, we can find that the fewer input ports than output ports are involved in multicast. Then we use $\overline{f}$ denote the average fanout of an input, which is given by:

$$\overline{f} = \sum_{i=1}^{f} i \cdot \frac{1}{f} = \frac{1}{2}(f+1)$$

under f-cast communication, r_I can be determined as:

$$r_I = \frac{r_O}{\overline{f}} = \frac{2}{f+1} \cdot r_O$$

We assume that the correlation among connection requests arriving at input (output) ports is neglected, so the status (busy or idle) of an input (output) port is independent from that of others.

III. Upper Bound on Blocking Probability

In this section, we will develop the upper bound on blocking probability of f-cast VSOB(N, m) for the cases that $\lceil n - log_2 f \rceil$ is even and odd, respectively.

A. Blocking Probability When $\lceil n - log_2 f \rceil$ is Odd

For this case, from the result in [12], we can determine a unique integer j_1 under the constraint of crosstalk-free, such that $2^{n-2j_1-1} \leq f \leq 2^{n-2j_1}$ or $|O_{n-j_1}| \leq |I_{j_1+1}| \cdot f \leq |O_{n-j_1+1}|$ We can prove that the unique j_1 above is determined as: $j_1 = (\lceil n - log_2 f \rceil - 1)/2$

By defining J_1 as j_1, i.e., $J_1 = j_1 = (\lceil n - log_2 f \rceil - 1)/2$

then, we can get that the maximum number of blocked planes of the tagged path is given by

$$\sum_{i=1}^{J_1} |I_i| \cdot f + \sum_{i=1}^{n-J_1} |O_i| = (2^{J_1} - 1) \cdot f + 2^{n-J_1} - 1$$

which is determined by both the connections originated from set $\cup_{i=1}^{J_1} I_i$, and the connections destined for set $\cup_{i=1}^{n-J_1} O_i$. Let c_I denote the connections originated from set $I_{odd} = \cup_{i=1}^{J_1+1} I_i$ and c_O denote the connections destined for set $O_{odd} = \cup_{i=1}^{n-J_1} O_i$ that block the tagged path, among which we use $c_{I,O}$ to denote the connections from I_{odd} to O_{odd}. If we use $Pr(a)$ to denote the probability that event a happens, and use $Pr^+(a)$ to denote the upper bound of $Pr(a)$, then we have the following result.

Theorem 1: For f-cast VSOB(N, m), where $\lceil n - log_2 f \rceil$ is odd,

$$Pr^+(blocking) = 1 - \sum_{k_I=0}^{min\{m-1, |I_{odd}| \cdot f\}} \sum_{k_O=0}^{min\{m-1, |O_{odd}|\}} \sum_{k_{I,O}=max\{0, k_I+k_O-m+1\}}^{min\{k_I, k_O\}} \frac{Pr(c_I = k_I, c_{I,O} = k_{I,O}) \cdot Pr(c_O = k_O, c_{I,O} = k_{I,O})}{Pr(c_{I,O} = k_{I,O})} \quad (1)$$

where

$$|I_{odd}| = \sqrt{2^{\lceil n - log_2 f \rceil + 1}} - 1, |O_{odd}| = \sqrt{2^{\lfloor n + log_2 f \rfloor + 1}} - 1$$

Proof: (see [19] for details)

1) Calculation of $Pr(c_{I,O} = k_{I,O})$

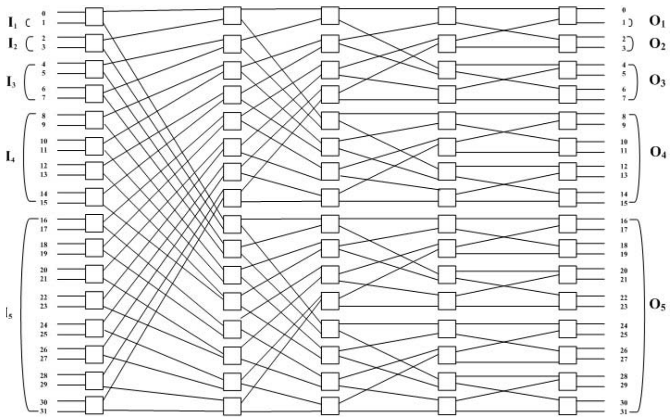

Fig. 1. 32×32 banyan network (odd number of stages)

Let η_f denote the number of connections from I_{odd}, where

$$0 \le \eta_f = \sum_{i=1}^{J_1+1} \beta_i = \sum_{i=1}^{|I_{odd}|} \xi_i \le |I_{odd}| \cdot f$$

Then, the probability $Pr(c_{I,O} = k_{I,O})$ can be evaluated as

$$Pr(c_{I,O} = k_{I,O}) = \sum_{g=k_{I,O}}^{|I_{odd}|\cdot f} Pr(c_{I,O} = k_{I,O}|\eta_f = g) \times Pr(\eta_f = g)$$

where the evaluation of $Pr(\eta_f = g)$ is provided in the following section, and $Pr(c_{I,O} = k_{I,O}|\eta_f = g)$ is given by:

$$Pr(c_{I,O} = k_{I,O}|\eta_f = g) = \begin{pmatrix} g \\ k_{I,O} \end{pmatrix} \cdot \left(\frac{|O_{odd}|}{N-1}\right)^{k_{I,O}} \cdot \left(1 - \frac{|O_{odd}|}{N-1}\right)^{g-k_{I,O}}$$

2) Calculation of $Pr(\eta_f = g)$

The exact distribution of $Pr(\eta_f = g)$ can be evaluated based on the technique of mother function. About the calculation of probability $Pr(\eta_f = g)$, we have the following Lemma.

Lemma 1:$Pr(\eta_f = g)$ is given by:

$$Pr(\eta_f = g) = \sum_{\substack{b_0, b_1, ..., b_f \ge 0 \\ b_0 + b_1 + \cdots + b_f = |I_{odd}| \\ b_1 + 2b_2 + \cdots + fb_f = g}} \frac{|I_{odd}|!}{b_0! b_1! \cdots b_f!} \times (1 - r_I)^{b_0} \cdot \left(\frac{r_I}{f}\right)^{b_1 + \cdots + b_f} \tag{2}$$

Here, we use the notation $b_i, 0 \le i \le f$, to represent the number of connection requests originated from set I_{odd} in which each connection request has fanout i.

Proof: Given ξ_i $(0 \le \xi_i \le f)$, η_f and r_I, we have $Pr(\xi_i = 0) = 1 - r_I$ and $Pr(\xi_i = t) = r_I \cdot (1/f)$, $t = 1, ..., f$. Then the mother function of ξ_i is given by:

$$\psi_{\xi_i}(s) = \sum_{t=0}^{f} Pr(\xi_i = t) \cdot s^t = (1 - r_I) + \frac{r_I}{f}(s + \cdots + s^f)$$

To calculate the probability $Pr(\eta_f = g)$, we first express it as:

$$Pr(\eta_f = g) = Pr(\sum_{i=1}^{|I_{odd}|} \xi_i = g)$$

Since ξ_i are i.i.d, and $\eta_f = \sum_{i=1}^{|I_{odd}|} \xi_i$, so the mother function of η_f is given by:

$$\psi_{\eta_f}(s) = \prod_{i=1}^{|I_{odd}|} \psi_{\xi_i}(s) = [(1 - r_I) + \frac{r_I}{f}(s + \cdots + s^f)]^{|I_{odd}|}$$

Thus,

$$Pr(\eta_f = g) = \frac{1}{g!} \psi_{\eta_f}^{(g)}(0) \tag{3}$$

Since it is very difficult to directly use (4) for the calculation of $Pr(\eta_f = g)$, so we first express $\psi_{\eta_f}(s)$ as the following format based on the multinomial series formula:

$$\begin{aligned} \psi_{\eta_f}(s) &= \sum_{\substack{b_0, b_1, \cdots, b_f \ge 0 \\ b_0 + b_1 + \cdots + b_f = |I_{odd}|}} \frac{|I_{odd}|!}{b_0! b_1! \cdots b_f!} \times (1 - r_I)^{b_0} \cdot \prod_{i=1}^{f} \left(\frac{r_I}{f} s^i\right)^{b_i} \\ &= \sum_{\substack{b_0, b_1, \cdots, b_f \ge 0 \\ b_0 + b_1 + \cdots + b_f = |I_{odd}|}} \frac{|I_{odd}|!}{b_0! b_1! \cdots b_f!} \times (1 - r_I)^{b_0} \cdot \left(\frac{r_I}{f}\right)^{b_1 + \cdots + b_f} s^{b_1 + 2b_2 + \cdots + fb_f} \end{aligned} \tag{4}$$

Since in general the polynomial $\psi_{\eta_f}(s)$ with the order $|I_{odd}|f$ can be expressed as:

$$\psi_{\eta_f}(s) = \sum_{g=0}^{|I_{odd}|f} c(g) \cdot s^g \tag{5}$$

By comparing equations (4) and (5), we can easily see that the coefficient $c(g)$ in (5) for term s^g is given by

$$c(g) = \sum_{\substack{b_0, b_1, \cdots, b_f \ge 0 \\ b_0 + b_1 + \cdots + b_f = |I_{odd}| \\ b_1 + 2b_2 + \cdots + fb_f = g}} \frac{|I_{odd}|!}{b_0! b_1! \cdots b_f!} \times (1 - r_I)^{b_0} \cdot \left(\frac{r_I}{f}\right)^{b_1 + \cdots + b_f} \tag{6}$$

Summarizing the equation (3), (5) and (6), we can see that the probability $Pr(\eta_f = g)$ is determined by (2).

3) Calculation of $Pr(c_I = k_I, c_{I,O} = k_{I,O})$

To calculate the probability $Pr(c_I = k_I, c_{I,O} = k_{I,O})$, we first express it as:

$$Pr(c_I = k_I, c_{I,O} = k_{I,O}) = \sum_{\substack{y_1 + \cdots + y_{J_1+1} \geq k_I \\ 0 \leq y_i \leq |I_i| f, i=1,\cdots J_1+1}} Pr\left(c_I = k_I, c_{I,O} = k_{I,O} | \bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right) \times Pr\left(\bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right)$$

where the probability $Pr\left(\bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right)$ is provided in the following section, and $Pr\left(c_I = k_I, c_{I,O} = k_{I,O} | \bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right)$ is given by:

$$Pr\left(c_I = k_I, c_{I,O} = k_{I,O} | \bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right) = \sum_{\substack{L_1 + \cdots + L_{J_1+1} = k_{I,O} \\ 0 \leq L_i \leq y_i, i=1,\cdots J_1+1}} \sum_{\substack{T_1 + \cdots + T_{J_1} = k_I - k_{I,O} \\ 0 \leq T_i \leq y_i - L_i, i=1,\cdots,J_1}} \left(\prod_{i=1}^{J_1} \binom{y_i}{L_i} \times \binom{y_i - L_i}{T_i} \lambda_I^{L_i} [\mu_I(i)]^{T_i} \times [1 - \lambda_I - \mu_I(i)]^{y_i - L_i - T_i}\right) \cdot Pr_1^*$$

$$Pr_1^* = \binom{y_{J_1+1}}{L_{J_1+1}} \lambda_I^{L_{J_1+1}} (1-\lambda_I)^{y_{J_1+1} - L_{J_1+1}}$$

$$\lambda_I = \frac{|O_{odd}|}{N-1} = \frac{2^{n-J_1} - 1}{N-1}, \mu_I(i) = \frac{\sum_{l=1}^{n-i+1} |O_l| - |O_{odd}|}{N-1} = \frac{2^{n-i+1} - 2^{n-J_1}}{N-1}, i = 1, \cdots, J_1.$$

Here, λ_I is the probability that a connection from I_{odd} to O_{odd} blocks the tagged path, and $\mu_I(i)$ is the probability that a connection from I_i blocks the tagged path but is not destined for O_{odd}. L_i and T_i are the numbers of connections from I_i that block the tagged path but are destined for and not destined for O_{odd}, respectively. These connections originated from I_{J_1+1} are treated separately which brought the extra probability factor Pr_1^*.

4) Calculation of $Pr(\bigcap_{i=1}^{J_1+1} (\beta_i = y_i))$, $(0 \leq \beta_i \leq |I_i| \cdot f = 2^{i-1} \cdot f)$

To calculate the probability $Pr(\bigcap_{i=1}^{J_1+1} (\beta_i = y_i))$, we further express it as:

$$Pr\left(\bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right) = \prod_{i=1}^{J_1+1} Pr(\beta_i = y_i)$$

Then we only need to evaluate $Pr(\beta_i = y_i)$ to get $Pr\left(\bigcap_{i=1}^{J_1+1} (\beta_i = y_i)\right)$. Since $\beta_i = \sum_{j=2^{i-1}}^{2^i - 1} \xi_j$, thus the mother function of β_i is given by:

$$\psi_{\beta_i}(s) = \left[(1 - r_I) + \frac{r_I}{f} \cdot (s + \cdots + s^f)\right]^{|I_i|}$$

Then,

$$Pr(\beta_i = y_i) = \frac{1}{y_i!} \psi_{\beta_i}^{(y_i)}(0) = \sum_{\substack{b_0, b_1, \cdots, b_f \geq 0 \\ b_0 + b_1 + \cdots + b_f = |I_i| \\ b_1 + 2b_2 + \cdots + f b_f = y_i}} \frac{|I_i|!}{b_0! b_1! \cdots b_f!} (1 - r_I)^{b_0} \cdot \left(\frac{r_I}{f}\right)^{b_1 + \cdots + b_f}$$

5) Calculation of $Pr(c_O = k_O, c_{I,O} = k_{I,O})$

The probability $Pr(c_O = k_O, c_{I,O} = k_{I,O})$ can be evaluated as:

$$Pr(c_O = k_O, c_{I,O} = k_{I,O}) = \sum_{\substack{L_1 + \cdots + L_{n-J_1} = k_{I,O} \\ 0 \leq L_i \leq |O_i|, i=1,\cdots n-J_1}} \sum_{\substack{T_1 + \cdots + T_{n-J_1-1} = k_O - k_{I,O} \\ 0 \leq T_i \leq |O_i| - L_i, i=1,\cdots n-J_1-1}} \left(\prod_{i=1}^{n-J_1-1} \binom{|O_i|}{L_i} \binom{|O_i| - L_i}{T_i} \lambda_O^{L_i} [\mu_O(i)]^{T_i} \times [1 - \lambda_O - \mu_O(i)]^{|O_i| - L_i - T_i}\right) \cdot Pr_2^*$$

$$Pr_2^* = \binom{|O_{n-J_1}|}{L_{n-J_1}} \lambda_O^{L_{n-J_1}} \cdot (1 - \lambda_O)^{|O_{n-J_1}| - L_{n-J_1}}$$

where

$$\lambda_O = \frac{|I_{odd}|}{N-1} \cdot r_O = \frac{2^{J_1+1} - 1}{N-1} \cdot r_O, \mu_O(i) = \frac{\sum_{l=1}^{n-i+1} |I_l| - |I_{odd}|}{N-1} \cdot r_O = \frac{2^{n-i+1} - 2^{J_1+1}}{N-1} \cdot r_O, \quad i = 1, \cdots, n - J_1 - 1.$$

Here, λ_O is the probability that a connection destined for O_{odd} blocks the tagged path but is originated from I_{odd}, and $\mu_O(i)$ is the probability that a connection destined for O_i blocks the tagged path but is not originated from I_{odd}. L_i and T_i are the numbers of connections destined for O_i that block the tagged path but are originated from and not originated from

I_{odd}, respectively. These connections destined for O_{n-J_1} are treated separately which brought the extra probability factor Pr_2^*.

B. Blocking Probability When $\lceil n - log_2 f\rceil$ is Even

Similar to the odd case analysis, for the case when $\lceil n - log_2 f\rceil$ is even, we can determine a unique integer j_2 such that $2^{n-2j_2-2} \le f \le 2^{n-2j_2-1}$ or $|O_{n-(j_2+1)}| \le |I_{j_2+1}| \cdot f \le |O_{n-j_2}|$ We can prove that the unique j_2 above is determined as:

$$j_2 = (\lceil n - log_2 f\rceil)/2 - 1.$$

By defining J_2 as $j_2 + 1$, i.e.,

$$J_2 = j_2 + 1 = (\lceil n - log_2 f\rceil)/2$$

then, we can get that the maximum number of blocked planes of the tagged path is given by:

$$\sum_{i=1}^{J_2} |I_i| \cdot f + \sum_{i=1}^{n-J_2} |O_i| = (2^{J_2} - 1) \cdot f + 2^{n-J_2} - 1$$

which is determined by both the connections originated from set $\cup_{i=1}^{J_2} I_i$, and the connections destined for set $\cup_{i=1}^{n-J_2} O_i$. Let c_I denote the connections originated from set $I_{even} = \cup_{i=1}^{J_2} I_i$ and c_O denote the connections destined for set $O_{even} = \cup_{i=1}^{n-J_2} O_i$ that block the tagged path, among which we use $c_{I,O}$ to denote the connections from I_{even} to O_{even}. Similar to the odd case, we give the following theorem.

Theorem 2: For f-cast VSOB(N,m), where $\lceil n - log_2 f\rceil$ is even,

$$Pr^+(blocking) = 1 - \sum_{k_I=0}^{min\{m-1,|I_{even}|\cdot f\}} \sum_{k_O=0}^{min\{m-1,|O_{even}|\}} \sum_{k_{I,O}=max\{0,k_I+k_O-m+1\}}^{min\{k_I,k_O\}} \frac{Pr(c_I = k_I, c_{I,O} = k_{I,O}) \cdot Pr(c_O = k_O, c_{I,O} = k_{I,O})}{Pr(c_{I,O} = k_{I,O})}$$

where

$$|I_{even}| = \sqrt{2^{\lceil n - log_2 f\rceil}} - 1, |O_{even}| = \sqrt{2^{\lfloor n + log_2 f\rfloor}} - 1$$

Proof: The theorem can be proved in a similar way as that of the theorem 1.

Here, the probabilities $Pr(c_{I,O} = k_{I,O})$, $Pr(c_I = k_I, c_{I,O} = k_{I,O})$ and $Pr(c_O = k_O, c_{I,O} = k_{I,O})$ can be evaluated in a similar way as that of the odd case.

IV. Experimental Results

An extensive experimental study has been performed to verify our upper bound on blocking probability of an f-cast VSOB(N,m) network. We developed a network simulator that consists of two major modules: the request pattern generator and the request router. The request pattern generator randomly generates a connection request pattern consisting a list of the feasible connection requests for an f-cast VSOB$(N,m$ $)$ network while the occupancy probability of an output link r_O, an fanout f and the number of input/output ports N are considered. The request router attempts to route the connection requests of the request pattern through the network using routing strategy. Here the packing strategy is considered to verify our upper bound on blocking probability. In the packing strategy, a connection is realized on a path found by trying the most used plane of the network first and the least used plane last. If no plane can satisfy the request of tagged path using the packing strategy, the connection request pattern is recorded as a blocked connection pattern corresponding to the routing strategy. The blocking probability of a routing strategy is then estimated by the ratio of the number of blocked connection patterns to the total number of connection patterns generated.

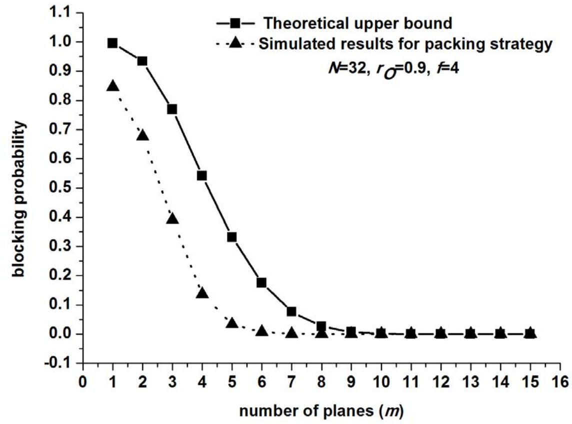

Fig. 2. Blocking probability of 32×32 VSOB network

A. Theoretical Versus Simulated Bounds on Blocking Probability

We have examined an f-cast VSOB$(32,m)$ network. The blocking probabilities were generated by both theoretical bounds and simulator for r_O=0.9 and $f = 4$. The corresponding comparison results are summarized in Figs. 2. The results in Figs. 2 show that our theoretical bounds are correct in estimating the upper bound blocking probabilities of f-cast VSOB(N,m) networks. The upper bound can guide the network designers to estimate the maximum blocking probability of an f-cast VSOB(N,m) network in which any routing strategy might be applied. Figs. 2 also show clearly that it is possible for us to reduce the hardware cost (number of planes) by allowing a small and predictable blocking probability. So our bounds can be used to display the tradeoffs between hardware cost and blocking probability.

B. Hardware Cost Versus Blocking Probability

For the f-cast VSOB(N,m) networks of different sizes, when output workload $r_O = 100\%$ and fanout $f = \{2,4,8\}$, the minimum numbers of planes estimated by our upper bound for different requirements of blocking probability (also denoted by BP hereafter) are summarized in table I. From table I we can observe that the minimum number of planes required can be reduced by allowing an almost negligible blocking probability. For example, as shown in table I, for the setting of $N = 128$ and $f = 8$, the minimum number of planes achieved by the condition of strictly nonblocking is $m = (\sqrt{2^{\lceil n-log_2 f\rceil}}-1)f+\sqrt{2^{\lfloor n+log_2 f\rfloor}}=56$, whereas the minimum number of planes achieved by our upper bound is only 18 for $BP < 0.1\%$, so $(56-18)/56 \cong 67.9\%$ of the hardware required can be saved.

TABLE I
MINIMUM NUMBERS OF PLANES FOR DIFFERENT REQUIREMENTS OF $r_O = 100\%$, $BP = \{0.0, 0.001, 0.01, 0.05\}$ AND $f = \{2, 4, 8\}$

		N=16	N=32	N=64	N=128
$BP = 0.0$	$f = 2$	10	14	22	30
	$f = 4$	12	20	28	44
	$f = 8$	16	24	40	56
$BP < 0.1\%$	$f = 2$	8	10	12	13
	$f = 4$	9	11	13	15
	$f = 8$	9	14	15	18
$BP < 1\%$	$f = 2$	7	9	10	12
	$f = 4$	8	10	12	13
	$f = 8$	8	12	13	15
$BP < 5\%$	$f = 2$	6	8	9	10
	$f = 4$	7	8	10	11
	$f = 8$	6	10	11	13

V. CONCLUSIONS

In this paper, we presented the upper bound on blocking probability of f-cast VSOB(N, m) networks. The bound can nicely describe the blocking behaviors of f-cast VSOB(N, m) networks, and it enables a network developer to find a desirable tradeoff between blocking probability and hardware cost in the design of such networks. We conclude that the network hardware cost in an f-cast VSOB(N, m) network can be dramatically reduced if a small and predictable blocking probability is allowed.

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