Optimal delegated portfolio management with background risk

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Abstract

Most investors delegate the management of a fraction of their wealth to portfolio managers who are given the task of beating a benchmark. However, in an influential paper [Roll, R., 1992. A mean/variance analysis of tracking error. Journal of Portfolio Management 18, 13–22] shows that the objective functions commonly used by these managers lead to the selection of portfolios that are *suboptimal* from the perspective of investors. In this paper, we provide an explanation for the use of these objective functions based on the effect of *background risk* on investors' optimal portfolios. Our main contribution is to provide conditions under which investors can *optimally* delegate the management of their wealth to portfolio managers.
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1. Introduction

Most investors delegate the management of a fraction of their wealth to portfolio managers who are given the task of beating a benchmark. These managers commonly use objective functions that lead to the selection of portfolios with minimum tracking error variance (TEV) for a given expected gain over the benchmark (hereafter, 'mean-TEV objective functions').[1] However, in an influential paper, Roll (1992) shows that these portfolios are typically *suboptimal* from the perspective of investors.[2] In particular, managers tend to select portfolios that are overly risky for investors (see Jorion, 2002, 2003).

It is important to emphasize that Roll's suboptimality result is obtained under the assumption that investors face solely portfolio risk. In practice, however, investors often face additional sources of risk such as those arising from labor income and real estate that might not be insurable in financial markets. These sources of risk are commonly referred to as *background risk* (see, e.g., Gollier, 2001).

As Cornell and Roll (2005) point out, the literature has yet to present an explanation for the use of mean-TEV objective functions by portfolio managers. In this paper, we fill this gap in the literature by exploring the effect of background risk on the optimality of these objective functions from the perspective of investors. Our main contribution is

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[1] A portfolio's *tracking error* is the difference between the returns on: (i) the portfolio and (ii) the benchmark.

[2] There is an extensive literature recognizing that portfolio managers might have incentives to take actions that are not optimal from the perspective of investors. First, these incentives can be induced *explicitly* by compensation contracts that are based on the managers' performance relative to a benchmark; see, e.g., Davanzo and Nesbitt (1987), Kritzman (1987), Record and Tynan (1987), Starks (1987), Grinblatt and Titman (1989), Bailey (1990), Admati and Pfleiderer (1997) and Elton et al. (2003). Second, the aforementioned incentives can be induced *implicitly* by the relationship between fund inflows and performance; see, e.g., Chevalier and Ellison (1997) and Sirri and Tufano (1998).

to provide conditions under which investors can *optimally* delegate the management of their wealth to portfolio managers with mean-TEV objective functions.

The idea of our result is simple. In the presence of background risk, the optimal portfolio of an investor is mean–variance inefficient under fairly general conditions.[3] As Roll (1992) shows, managers with mean-TEV objective functions select portfolios that are also mean–variance inefficient. Accordingly, there exist conditions under which investors can optimally delegate the management of their wealth to portfolio managers with mean-TEV objective functions.

The case when the investor seeks to optimally delegate his or her wealth to a single portfolio manager is examined first. We provide a necessary and sufficient condition for an investor to be able to optimally delegate his or her wealth to a single manager. This condition involves: (i) the composition of the benchmark, (ii) the degree of risk aversion of both the investor and the manager, and (iii) the joint distribution of asset returns and background risk.

The assumption that the investor can only delegate the management of his or her wealth to a single manager is restrictive. Furthermore, when the aforementioned condition does not hold, the investor may still be able to optimally delegate his or her wealth to an 'appropriate' set of managers. Accordingly, the case when the investor seeks to optimally delegate his or her wealth to more than a single portfolio manager is examined next. We provide a necessary and sufficient condition for an investor to be able to optimally delegate his or her wealth by choosing: (i) an 'appropriate' set of managers and (ii) an 'appropriate' allocation of wealth among them. Similar to the case when a single manager is considered, the condition involves: (i) the composition of the benchmarks, (ii) the degree of risk aversion of the investor and managers, and (iii) the joint distribution of asset returns and background risk.

Previous papers explore the portfolio selection implications arising from managers using mean-TEV objective functions and propose several methods to mitigate the problem that these functions lead to the selection of suboptimal portfolios from the perspective of investors who do not face background risk.[4] Roll (1992) characterizes the

set of portfolios with minimum TEV for various levels of expected gain over the benchmark. Under certain conditions, he shows that adding a beta constraint to a manager's portfolio selection problem benefits investors. Jorion (2003) shows that the set of portfolios that maximize the expected gain over the benchmark subject to variance and TEV constraints are represented by an ellipse in mean–variance space. Due to the flat shape of this ellipse, he shows that adding a variance constraint to a manager's portfolio selection problem benefits investors.[5] More recently, Alexander and Baptista (forthcoming) show that adding a value-at-risk constraint to the portfolio selection problem of a manager with a mean-TEV objective function is beneficial from the perspective of investors.[6] Our work differs from these papers in that we provide conditions under which the use of a mean-TEV objective function by managers is optimal from the perspective of investors who face background risk.

There is an extensive literature recognizing the effects of background risk on the willingness of individuals to bear other risks (see Gollier, 2001 for a survey). Specifically, the literature provides conditions on utility functions under which the presence of background risk makes individuals less willing to bear other risks (see, e.g., Pratt and Zeckhauser, 1987; Kimball, 1993; Gollier and Pratt, 1996; Eeckhoudt et al., 1996). Our paper differs from this literature in that we investigate conditions under which an investor can optimally delegate the management of his or her wealth to managers with mean-TEV objective functions.

Our paper is organized as follows. Section 2 characterizes the optimal portfolio for an investor who faces background risk. Section 3 characterizes the optimal portfolio for a portfolio manager with a mean-TEV objective function. Section 4 provides a necessary and sufficient condition for an investor to be able to optimally delegate the management of his or her wealth to a single portfolio manager with a mean-TEV objective function. Section 5 shows that when this condition does not hold, there exist conditions under which an investor can optimally delegate the management of his or her wealth by choosing: (i) an 'appropriate' set of portfolio managers and (ii) an 'appropriate' allocation of wealth among them. Section 6 provides an example that illustrates our main results. Section 7 concludes.[7]

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[3] Flavin and Yamashita (2002) show that when an investor holds real estate and is subject to a housing constraint involving the ratio of house value to net worth, he or she ends up selecting a portfolio of financial assets that is mean–variance inefficient. Our paper differs from theirs in several respects. First, while we do not specify the particular source of background risk in our paper, they focus on housing as the source of background risk. Second, while we provide a closed-form solution to the portfolio selection problem of an investor who faces background risk, they use numerical methods to find the optimal portfolio of financial assets in the presence of the aforementioned housing constraint. Third, while we seek conditions under which investors can optimally delegate their wealth to managers with mean-TEV objective functions, they focus on effect of housing on an investor's optimal portfolio of financial assets.

[4] Equilibrium implications arising from delegated portfolio management have also been examined in the absence of background risk; see Brennan (1993), Gómez and Zapatero (2003) and Cornell and Roll (2005).

[5] In related work, Leibowitz et al. (1992) find that the set of portfolios that meet a surplus return constraint lie on an 'egg-shaped' curve in mean–standard deviation space. They define *surplus* as the difference between the current value of assets and liabilities. The ratio of the change in surplus to the initial value of liabilities is referred to as the *surplus return*.

[6] In contrast, Alexander and Baptista (2006) find that adding a maximum drawdown constraint to a manager's portfolio selection problem may not be beneficial from the perspective of investors.

[7] An Appendix containing the proofs of the theoretical results in our paper can be downloaded at: http://home.gwu.edu/~alexbapt/JBF3Appendix.pdf.