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A Model for Project Scheduling with Fuzzy Precedence Links

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Abstract In the real world for many projects we have to use human judgment for estimating the duration of activities. A way to deal with this imprecise data is to employ the concept of fuzziness, where the vague activity times can be represented by fuzzy sets. This paper presents a new method based on fuzzy theory for solving fuzzy project scheduling in fuzzy environment. The duration of activities is triangular fuzzy numbers (TFN). Also for the first time in the literature, we consider the relationship between activities is not crisp and assume it is a triangular fuzzy number. The proposed solution method is based on linear programming (LP) model that gives us several parameters of project such as earliest time, latest time, and slack times in term of TFN.

Key words: Fuzzy CPM, fuzzy PERT, project scheduling, linear programming

INTRODUCTION

In today's highly competitive business environment, project management's ability to schedule activities and control progress within cost, time, and quality is becoming important to obtain competitive priorities such as on-time delivery and customization. When the activity times in the project are deterministic and known, critical path method (CPM) has been demonstrated to be a useful tool in managing projects in an efficient manner to meet this challenge. However, there are many cases where the activity times may not be presented in a precise manner. To solve these problems with imprecise data researchers have presented the program evaluation and review technique (PERT) and Monte Carlo simulation (Kenzo, 2002; Ragsdale, 1986), which are based on the probability theory. However, there are some critiques of PERT. In PERT there are some assumptions for simplifying the model, for example use of the beta distribution. Also to provide an appropriate distribution for activity times we need historic data. But in the real world some activities have never proceeded and we have to use human judgment instead of stochastic assumptions to determine activity times. The detailed critiques of PERT can be found in the paper of Shipley (Shipley *et al.,* 1997). An alternative way to deal with imprecise is to use of fuzzy theory instead of probability theory. In this case activity times are represented by fuzzy numbers.

In this paper we propose a mathematical model to deal with project scheduling problem in uncertain environment. We have supposed duration of activities are imprecise and showed in the form of triangular fuzzy numbers. Also in this model we have supposed for first time the relationship between activities are not crisp, and are supposed fuzzy numbers. For example in the figure 1 activity 2-3 cannot start before finishing activity 1-2, but in the real world relationship is not an assured relationship, and there is an imprecise in relationship, and some days after of finishing 1-2 activity 2-3 can be started. For example in figure 1 degree of relationship between activities 1-2,2-3 equal $\tilde{S}_{123} = (0.45, 0.55, 0.65)$

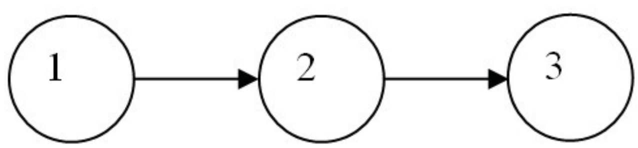

Fig. 1: The organization of this paper is as follows; in the next section ranking fuzzy numbers is presented. In the second section the structure of fuzzy project network is described. In the

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Literature Review:

In many papers fuzzy project scheduling have discussed. Most of them, by using the available algorithms in CPM method, have computed the project parameters such as earliest time, in which the deterministic activity times are replaced by fuzzy numbers. For more details refer to(Yao, and Lin, 2000; Nasution, 1994; McCahon, 1993; Kamburowsai, 1983; Yasuhiro et al., 1995; Gazdik, 1983; Chen and Sue, 2007). considering fuzzy numbers for activity times computed the criticality of network paths. Then by summing fuzzy numbers on the paths with high criticality, they computed the finished time of project. (Shin, 2007). via concept of α_cut level and using of various α and (Yager, 1981). method have computed the criticality degree of activities and paths and presented the completion time of project via fuzzy number. (Shih and Yi, 2007). proposed a simple approach to the critical path problem with fuzzy activity duration times. Considering CPM problem as the opposite of the shortest path problem, they formulated the problem as an LP. (Didier, 2003). et.al has proposed several heuristics models for computing the latest start time and float times of activities. (Hua and Baoding, 2007). have assumed that times of activities are mixture of stochastic and fuzzy numbers. Activities times were assumed as a normal distribution with known and deterministic variance and with fuzzy mean. Then they have solved three fuzzy stochastic models. But their methods do not support backward pass calculation, because subtraction of two fuzzy numbers might lead to a negative number for some parameters of project such as latest times and slack times. (Pawel, 2005). have announced that the backward recursions for computing the latest start time and floats of activities is not a suitable method and he have proposed a new algorithm for computing the latest start times and float of activities in projects with fuzzy durations time. As it could be seen in the literature, almost of all of the fuzzy CPM papers have been supposed that the project linkages are crisp. (Iranmanesh *et al.,* 2008). have introduced a network with flexible links (NFL) and extended tradition CPM calculation for these networks. This paper extends this concept by using fuzzy theory.

Ranking Fuzzy Numbers:

This section is about ranking two fuzzy numbers. For two real numbers there is no problem in ordering them from smallest to largest. However in the fuzzy case there is no universally accepted way to do this. There are probably more than 40 methods proposed in the literature of defining $\tilde{M} \le \tilde{N}$ for two fuzzy numbers $\tilde{M}$ and . A few key references on this topic are(Bortolon and Degani, 1985; Wang and Kerre, 2001; Wang and Kerre, 2001; Yager, 1981). proposed a procedure for ordering fuzzy sets, in which a ranking index $I(\tilde{t})$ is calculated for the convex fuzzy numb $\tilde{t}$ r from its $\alpha - cut\ \alpha_i = [t_\alpha^l, t_\alpha^u]$ according to the following formula:

$$I(\tilde{t}) = \int_0^1 \frac{1}{2}\left(t_\alpha^l + t_\alpha^u\right) d\alpha$$

Which is the center of the mean value of $\tilde{t}$ Considering two fuzzy numbers $\tilde{M}$ and $\tilde{N}$ he case of $I(\tilde{M}) \ge I(\tilde{N})$ shows that $\tilde{M} \ge \tilde{N}$ and then max $\{\tilde{M}, \tilde{N}\} = \tilde{M}$

Fuzzy Project Network:

A network $G = \langle N, A, T \rangle$, being a project model, is defined where N is a set of nodes (events) and $A \subset V \times V$ is a set of arcs (activities). The network G is a directed, compact, acyclic graph. The set $V = \{1, 2, \ldots, n\}$ is labeled in such a way that the following condition holds:. $(i, j) \in A \Rightarrow i < j$ By means of function T, $T : A \to R^+$ the activity times in the network are determined, d_{ij} is a duration of activity $(i, j) \in A$

For a fuzzy network $G = \langle N, A, \tilde{T} \rangle$ all assumptions are the same as in the deterministic case except for function $\tilde{T}$ that is defined now in the following way: $\tilde{T} : A \to P(R^+)$, where $P(R^+)$ is the set of fuzzy numbers with non-negative parts. And $\tilde{d}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ is a fuzzy triangular number for duration of activity (i,j).

The Solution Procedure:

In this section we extend the traditional critical path method (CPM) to our case in which activity durations and relationship between them are fuzzy numbers.

Fuzzy Forward Calculation:

In this phase, we calculate the earliest start and finish time of activities, also the completion time of the project. The modified formulas based on our assumptions are as follow:

$$\tilde{E}_j = \left(e_j^a, e_j^b, e_j^c\right) = \begin{cases} \underset{i \in P(j)}{M\tilde{a}x} \left\{ \tilde{E}_i \oplus \tilde{d}_{ij} \otimes \underset{k \in S(j)}{m\tilde{a}x} \left\{ \tilde{S}_{ijk} \right\} \right\} & if\ P(j) \neq \phi \\ \tilde{T}_S = \left(t_s^a, t_s^b, t_s^c\right) & if\ P(j) = \phi \end{cases} \tag{1}$$

$$E\tilde{S}_{ij} = \tilde{E}_i = \left(es_{ij}^a, es_{ij}^b, es_{ij}^c\right) \tag{2}$$

$$E\tilde{F}_{ij} = E\tilde{S}_{ij} \oplus \tilde{d}_{ij} \tag{3}$$

$$T\tilde{F} = \left(tf^a, tf^b, tf^c\right) = \underset{i \in N}{M\tilde{a}x}\, \tilde{E}_i \tag{4}$$

In the above $\tilde{E}_j$ is the fuzzy earliest occurrence time of event j, $\tilde{S}_{ijk}$ is a triangular fuzzy number that shows the amount of dependence between activity (i,j) and activity (j,k), $E\tilde{S}_{ij}$, $E\tilde{F}_{ij}$ re the fuzzy earliest start and finish time of activity (i,j) respectively, and $T\tilde{F}$ is the fuzzy completion time of the project. In the Eqs (1) $M\tilde{a}x$ operator is calculated by (Yager, 1981).`s ranking method.

Fuzzy Modified Backward Pass (MBP) Calculation:

Backward pass calculation is used to calculate the latest times of activities in the project network.
Ahmad and Rasoul (Soltani and Haji, 2007). presented a backward pass named (MBP) for fuzzy project network as follow:

$$\tilde{L}_i = \left(l_i^a, l_i^b, l_i^c\right)$$

$$l_i^c = \max\left(0, \min_{j \in S(i)}\left(l_j^c - d_{ij}^c\right)\right) \tag{6}$$

$$l_i^b = \max\left(0, \min\left(l_i^c, \min_{j \in S(i)}\left(l_j^b - d_{ij}^b\right)\right)\right)$$

$$ax\left(0, \min\left(l_i^b, \min_{j \in S(i)}\left(l_j^a - d_{ij}^a\right)\right)\right)$$

In this paper we have a new assumption that the relationship between activities is triangular fuzzy number. Thus, the fuzzy latest occurrence time of event i is rewritten as follow:

$$\tilde{L}_i = \begin{cases} \underset{j \in S(i)}{M\tilde{i}n} \left\{ \tilde{L}_j - \tilde{d}_{ij} \otimes \underset{k \in S(j)}{m\tilde{a}x} \left\{ \tilde{S}_{ijk} \right\} \right\} & if\ S(i) \neq \phi \\ T\tilde{F} & if\ S(i) = \phi \end{cases} \tag{7}$$

In the above equation operators $M\tilde{a}x$ and $M\tilde{i}n$ are calculated by (Yager, 1981)'s method.
Since the addition and the subtracting are not inverse relations in the fuzzy environment, in backward pass calculation we might achieve a negative number for some parameters of project. For example, lets

$\tilde{L}_2 = (8,10,14)$ and $\tilde{d}_{12} = (9,11,14)$ $\tilde{L}_1 = (-6,-1,5)$ we will have and it is a triangular fuzzy number with

negative part and it is possible that the latest time of event 1 occurs in negative time. To avoid this problem, we follow the approach proposed by Ahmad and Rasoul (Wang and Kerre, 2001).

Now we can calculate the fuzzy latest occurrence time of event i by the following equation.

$$
\begin{aligned}
&\tilde{L}_i = \left(l_i^a, l_i^b, l_i^c\right)\\
&l_i^c = \max\left(0, \min_{j \in S(i)}\left(l_j^c - d_{ij}^c * \left\{S_{ijk}^c \,\middle|\, S_{ijk}^c \text{ is the third part of } \underset{k \in S(j)}{\tilde{\max}}\left\{\tilde{S}_{ijk}\right\}\right\}\right)\right)\\
&l_i^b = \max\left(0, \min\left(l_i^c, \min_{j \in S(i)}\left(l_j^b - d_{ij}^b * \left\{S_{ijk}^b \,\middle|\, S_{ijk}^b \text{ is the second part of } \underset{k \in S(j)}{\tilde{\max}}\left\{\tilde{S}_{ijk}\right\}\right\}\right)\right)\right)\\
&l_i^a = \max\left(0, \min\left(l_i^b, \min_{j \in S(i)}\left(l_j^a - d_{ij}^a * \left\{S_{ijk}^a \,\middle|\, S_{ijk}^a \text{ is the first part of } \underset{k \in S(j)}{\tilde{\max}}\left\{\tilde{S}_{ijk}\right\}\right\}\right)\right)\right)
\end{aligned} \tag{8}
$$

The fuzzy latest finishing time of activity (i,j) is calculated by :

$$
L\tilde{F}_{ij} = \tilde{L}_j = \left(lf_{ij}^a, lf_{ij}^b, lf_{ij}^c\right) \tag{9}
$$

If we use this equation for calculating $L\tilde{F}_{ij}$ $E\tilde{F}_{ij}$ possibly would be bigger than. $L\tilde{F}_{ij}$ For example using Equation (9) for numerical example in Section 5 leads to $L\tilde{F}_{12} = (13.6, 25.15, 31.3)$ and $E\tilde{F}_{12} = (25, 28, 32)$

To deal with this problem we developed equation (10) for calculating $L\tilde{F}_{ij}$

$$
L\tilde{F}_{ij} = \tilde{L}_j - \tilde{d}_{ij} \otimes \underset{k \in S_j}{\tilde{\max}} \tilde{S}_{ijk} \oplus \tilde{d}_{ij} \tag{10}
$$

Since there is a subtraction operator in this formula, as we described above, the equation (10) can be rewritten as.

$$
\begin{aligned}
&L\tilde{F}_{ij} = \left(lf_{ij}^a, lf_{ij}^b, lf_{ij}^c\right)\\
&lf_{ij}^c = \max\left(0, \min\left(tf^c, \left(l_j^c - d_{ij}^c * \left\{S_{ijk}^c \,\middle|\, S_{ijk}^c \text{ third part of the } \max_{k \in S(j)}\left\{\tilde{S}_{ijk}\right\}\right\} + d_{ij}^c\right)\right)\right)\\
&lf_{ij}^b = \max\left(0, \min\left(tf^b, lf_{ij}^c, \left(l_j^b - d_{ij}^b * \left\{S_{ijk}^b \,\middle|\, S_{ijk}^b \text{ second part of the } \max_{k \in S(j)}\left\{\tilde{S}_{ijk}\right\}\right\} + d_{ij}^b\right)\right)\right)\\
&lf_{ij}^a = \max\left(0, \min\left(tf^a, lf_{ij}^b, \left(l_j^a - d_{ij}^a * \left\{S_{ijk}^a \,\middle|\, S_{ijk}^a \text{ first part of the } \max_{k \in S(j)}\left\{\tilde{S}_{ijk}\right\}\right\} + d_{ij}^a\right)\right)\right)
\end{aligned} \tag{11}
$$

For calculation of the fuzzy latest start time of activity (i,j) we have

$$
L\tilde{S}_{ij} = L\tilde{F}_{ij} - \tilde{d}_{ij} \tag{12}
$$

For the same reason as we had for $\tilde{L}_j$ and $L\tilde{F}_{ij}$, we use

$$
\begin{aligned}
&L\tilde{S}_{ij} = \left(ls_{ij}^a, ls_{ij}^b, ls_{ij}^c\right)\\
&ls_{ij}^c = \max\left(0, \left(lf_{ij}^c - d_{ij}^c\right)\right)\\
&ls_{ij}^b = \max\left(0, \min\left(ls_{ij}^c, \left(lf_{ij}^b - d_{ij}^b\right)\right)\right)\\
&ls_{ij}^a = \max\left(0, \min\left(ls_{ij}^b, \left(lf_{ij}^a - d_{ij}^a\right)\right)\right)
\end{aligned} \tag{13}
$$

Fuzzy slack times:

For identifying critical path we calculate fuzzy total slack. $\left(T\tilde{F}_{ij}\right)$ If classical relations of CPM are applied for the calculation of this parameter in the fuzzy environment, we can write the following relation:

$$
T\tilde{F}_{ij} = L\tilde{F}_{ij} - E\tilde{F}_{ij} \tag{14}
$$

As it is possible that the slack times be a negative numbers, similar to the calculation of $\tilde{L}_j$ in modified backward pass, we can calculate these parameters from bellow equations.

$$
\begin{aligned}
&T\tilde{F}_{ij} = \left(tf_{ij}^{a}, tf_{ij}^{b}, tf_{ij}^{c}\right)\\
&tf_{ij}^{c} = \max\left(0, \left(lf_{ij}^{c} - ef_{ij}^{c}\right)\right)\\
&tf_{ij}^{b} = \max\left(0, \min\left(tf_{ij}^{c}, \left(lf_{ij}^{b} - ef_{ij}^{b}\right)\right)\right)\\
&tf_{ij}^{a} = \max\left(0, \min\left(tf_{ij}^{b}, \left(lf_{ij}^{a} - ef_{ij}^{a}\right)\right)\right)
\end{aligned} \quad (15)
$$

Numerical Example:

The network representing a structure of project is given in figure.2.

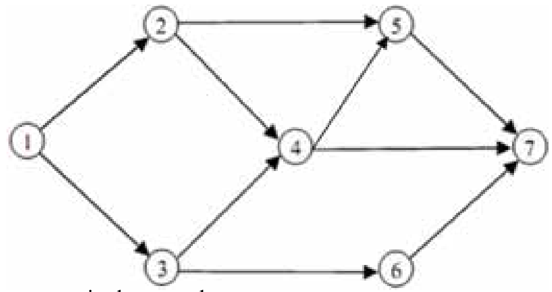

Fig. 2: network of project in numerical example

The duration of activities are given in the table 1. and the start time of project is (0,0,0).

Table 1: $\tilde{d}_{ij}$ and $\tilde{S}_{ijk}$ of project in numerical example.

Activity (i,j)	$\tilde{S}_{ijk}$	Duration ($\tilde{d}_{ij}$)
(1,2)	S_{125}=(0.3,0.4,0.5)	(25,28,32)
(1,3)	S_{124}=(0.4,0.5,0.6)	(40,55,65)
(2,4)	S_{134}=(0.6,0.7,0.8)	(32,37,43)
(3,4)	S_{136}=(0.45,0.55,0.7)	(20,25,35)
(2,5)	S_{257}=(0.8,0.9,1)	(35,38,42)
(3,6)	S_{245}=(0.2,0.3,0.5)	(42,45,55)
(4,5)	S_{247}=(0.7,0.8,0.9)	(20,25,28)
(4,7)	S_{345}=(0.4,0.5,0.6)	(60,65,75)
(5,7)	S_{347}=(0.6,0.65,0.7)	(65,75,85)
(6,7)	S_{367}=(0.3,0.4,0.6)	(15,18,22)
No	S_{456}=(0.5,0.7,0.8)	No
No	S_{457}=(0.5,0.6,0.65)	No

Table 2 shows the earliest and latest occurrence of events that were calculated by equations (1), (8).

Table 2: the $\tilde{E}_i$ and $\tilde{L}_i$ of project in numerical example

Event (I)	$\tilde{E}_i$	$\tilde{L}_i$
1	(0,0,0)	(0,0,0)
2	(10,14,19.2	)(13.6,25.15,31.3)
3	(24,38.5,45.5)	(24,38.5,45.5)
4	(36,54.75,70)	(36,54.75,70)
5	(46,69.75,88.2)	(46,69.75,88.2)
6	(36.6,56.5,78.5)	(96,126.75,151.2)
7	(111,144.75,173.2)	(111,144.75,173.2)

Table 3 shows the other parameters such as fuzzy earliest and latest start and finish time of activities and fuzzy slack times of activities by using eqs (2,3,11,13,15) of project in numerical example. So the critical path is.1→3 →4 →5 →7.

Conclusion:

In this paper a new method has been presented for solving the project scheduling problem in the fuzzy environment. In this paper we considered the duration of activities in a project is positive triangular fuzzy numbers and for first time it is assumed that the relationship between activities are fuzzy numbers. Previous researches have proposed some models for solving project scheduling problem in the fuzzy environment. Their proposed methods do not support the backward pass calculations. Our proposed method calculates all parameters of project such as earliest and latest start and finish time and slack times.

Table.3: the $\tilde{ES}_{ij}, \tilde{EF}_{ij}, \tilde{LS}_{ij}, \tilde{LF}_{ij}, \tilde{TF}_{ij}$ of project in numerical example.

(i,j)	$\tilde{ES}_{ij}$	$\tilde{EF}_{ij}$	$\tilde{LS}_{ij}$	$\tilde{LF}_{ij}$	$\tilde{TF}_{ij}$
(1,2)	(0,0,0)	(25,28,32)	(3.6,11.15,12.1)	(28.6,39.15,44.1)	(3.6,11.15,12.1)
(1,3)	(0,0,0)	(40,55,65)	(0,0,0)	(40,55,65)	(0,0,0)
(2,4)	(10,14,19.2)	(42,51,62.2)	(13.6,25.15,31.3)	(45.6,62.15,74.3)	(3.6,11.15,12.1)
(3,4)	(24,38.5,45.5)	(44,63.5,80.5)	(24,38.5,45.5)	(44,63.580.5)	(0,0,0)
(2,5)	(10,14,19.2)	(45,52,61.2)	(18,35.55,46.2)	(53,73.55,88.2)	(8,21.55,27)
(3,6)	(24,38.5,45.5)	(66,83.5,100.5)	(69,99.75,118.2)	(111,144.75,173.2)	(45,61.25,72.7)
(4,5)	(36,54.75,70)	(56,79.75,98)	(36,54.75,70)	(56,79.75,98)	(0,0,0)
(4,7)	(36,54.75,70)	(96,119.75,145)	(51,79.75,98.2)	(111,144.75,173.2)	(15,25,28.2)
(5,7)	(46,69.75,88.2)	(111,144.75,173.2)	(46,69.75,88.2)	(111,144.75,173.2)	(0,0,0)
(6,7)	(36.6,56.5,78.5)	(51.6,74.5,100.5)	(96,126.75,151.2)	(111,144.75,173.2)	(59.4,70.25,72.7)

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