# Curviness as a Parameter for Route Determination 

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#### Abstract

The determination of optimized routes requires the stipulation of costs for various road segments. These costs can then be used in, e.g., the Dijkstra algorithm to determine optimal routes. The functions for stipulating costs vary with the goal of the optimization. Typical goals are the determination of the shortest, fastest, or simplest route. The paper discusses a parameter that has not yet received much attention: curviness of the road. Special user groups like motorbike riders or truck drivers have specific requirements. Motorbike riders try to avoid long, straight roads whereas truck drivers have to avoid sharp bends. Different approaches to model these goals and results for a specific route are presented in the paper.


## 1 Introduction

The determination of optimal routes is a standard problem both in transportation of goods and individual navigation. Standard solutions like the Dijkstra or the A*-algorithm (DIJKSTRA 1959, HART et al. 1968) search for optimal routes in graphs based on costs assigned to the edges in the graph. Typical costs are based on geometrical distance or travel time and the solutions are then the shortest respectively fastest route. Extensions of this concept led to solutions like the simplest path (Duckham 2003), the least risk path (Grum 2005), or the minimum work path (Shirabe 2008). Finally, legal aspects like one way streets or turn restrictions requires further adaptation of the model (GEROE et al. 2011).
Curviness is a parameter that has not yet been addressed extensively in navigation literature. In analogue trip planning the decision for a specific route is made by a human based on an analogue map. This process is typically quite accurate in the choice of a suitable route. Selecting an interesting route using topographic maps is a simple task for an experienced map user. The same task, however, is difficult for navigational devices. The goal of the paper is the development of strategies to find optimal routes for two extremes: (i) avoiding sharp turns and (ii) avoiding straight roads. The first case is suitable for truck drivers. Hensher and Sullivan showed that truck drivers prefer straight roads over curvy ones (Hensher \& Sullivan 2003). Reasons for this behaviour are the significant speed reduction and the increased travel time, fuel consumption, mechanical wearing of the truck, and accident risk. The second case is suitable for motorbike riders. They prefer winding roads because these roads are more fun to drive whereas straight roads are boring and lead to unfavourable wear of the tires (LEDER 2011).

Section two provides a brief survey of route determination problems and cost definition. In the following section, the requirements for the two specific types of vehicles, trucks and motorcycles, are discussed. These requirements are then modelled in section 4. Section 5 presents the results of a test based on the OpenStreetMap dataset of a part of Austria. Some conclusions and further research questions finish the paper.

## 2 Route Determination and Cost Definition

Routing algorithms work on networks, which consist of nodes and edges. Edges are the road themselves and nodes mark intersections of roads or road ends. The edges have specific costs attached to them and the standard routing algorithms (e.g., Dijkstra or A*) search the path with the least total costs given a start and an end point. Route determination thus requires a suitable definition of costs. The costs must be non-negative but otherwise are completely unconfined. The costs are attached to the edges assuming that the costs at the nodes are insignificant in comparison to the costs of the edges. In some cases this assumption may be violated, e.g., left turns on major roads may cost more than the continuation in any other direction. In such cases the costs caused at the nodes are typically added to the costs of the edges (compare Winter 2002).

Different kinds of costs functions have been successfully used:

- Geometrical distance: In this case the known length of the road segment is used to determine the route. The result is the geometrically shortest route. Problems with this solution are that this is not necessarily the fastest solution and may include low level roads.
- Driving time: The assessment of a reasonable driving speed together with the geometrical length leads to driving time as a cost function. The assessment can be based on the legal speed limit, experience, or even floating car data. The result is unless there is an unforeseen event like an accident - the fastest possible route.
- Natural beauty, personal safety, etc.: Sometimes time is not the most important factor. Personal safety may be a crucial factor for route selection, e.g., when walking home in the night (e.g., HÄUSLER et al. 2011). Other factors like variety or beauty of the surrounding may influence the subjective perception of travel time. Several solutions for such cost functions have been proposed already (e.g., Hochmair \& Navratil 2008). The contradiction in this kind of approaches is that they shall maximize the benefits, e.g., for an interesting tourist route, and at the same time minimize the costs. This may lead to difficulties in assessing the parameters to balance the contradicting goals. It is easier if a property shall be minimized, e.g., the exposure to dangerous locations or bad weather (compare LITZINGER et al. to appear).
- Energy consumption: In times where energy conservation is a constant topic, energyefficient paths are feasible. The routes have to include the costs of acceleration after sharp turns and on ascending roads. Numerical solutions for this problem have been already developed (SHIRABE 2008).


## 3 Requirements for Trucks and Motorcycles

The selection of routes for specific user groups is based on the specific requirements of this user group. The requirements can be of legal or physical nature. Legal requirements may include the absence of driving prohibition for the used vehicles or cargo. Other requirements express physical limits, like maximum height, length, or mass of the vehicle. Physical requirements also include the limitation of inclinations or declination. Inclinations slow down the vehicle and steep declinations require deceleration to keep a constant speed. A standard method for deceleration is the use of an exhaust break because standard wheel
breaks may overheat the break discs leading to break force reduction. The possible driving speed is thus reduced and longer routes avoiding inclinations or declinations may even be faster than the shorter routes including these obstacles.

Trucks and motorcyclists have specific requirements for selected routes. Typically, routing solutions assume that all means of transportation have the same properties. This leads to the well known situations where trucks get stuck in narrow, winding roads, routes for pedestrians incorporate highways, or driving restrictions are ignored. Some prerequisites are of legal nature, others depend on geometrical properties. Paths must be wide enough, for example, to let the vehicle pass. Curviness is only one of a number of parameters that can and must be addressed for reasonable routes.

Trucks are vehicles of considerable mass and size. Acceleration of such a mass causes significant fuel consumption. Thus the search for fuel-saving routes is a reasonable goal. Such routes should avoid acceleration as far as possible. Size and mass limit possible curve speed and thus sharp curves require speed reduction and subsequent acceleration. Routes for trucks should therefore avoid sharp curves.

Motorcyclists have the opposite demands. Driving a motorbike on a straight road is boring. Thus motorcyclists try to select twisting routes. Varying radiuses demand constant action from the motorcyclists and thus keep the concentration high. In addition, motorcycle tyres have a rounded cross section to allow slanting positions of the motorcycle. Driving causes abrasion and driving straight roads only restricts the abrasion to the centre-line of the tyre. This causes edges in the cross section of the tyre and results in balance problems at curve entries and exits. Such situations are dangerous and thus motorcyclists try to avoid this type of wear by avoiding long straight roads like highways.

## 4 Modelling the Costs

Finding curvy roads is a problem similar to finding beautiful roads (compare Hochmair \& NaVRatil 2008). In both cases a parameter shall be maximized (the curviness or the beauty) while the algorithms used to determine the route minimize costs. Optimal routes on curvy roads will be longer than the shortest route and travelling will take more time than on the fastest route but how much may the costs increase to still produce suitable results? This question has no universal answer because the acceptance of elongation may depend on personal preferences and the reasons for the trip. For example, the route to relatives for a visit could be longer than the daily route to work. The deviation from the shortest or fastest route can be influenced by changing the balance between the costs for the geometrical length or travel time and the assessed cost for curviness.

A simplistic way to determine the curviness of a road is to compute the quotient between Euclidean distance of the end points of the road and the actual length of the road. A completely straight road would produce a length quotient of one and a value of zero is only possible for roads that form closed loops or roads with infinite length. The second case is impossible but the first case may happen, e.g., in case of roundabouts. However, roundabouts are typically small and thus the effect on travel time or distance should be small, too. Thus the zero value should have an insignificant impact on the determined route.

$$
\text { Length quotient: } p_{i}=\frac{s_{E u k l}}{s_{\text {true }}} \in[0,1] \text {. }
$$

A length quotient of 0.5 describes a road where the actual length is twice the Euclidean distance. Unfortunately it does not describe the shape of the road (compare figure 1).


Fig. 1:
Schematic representation of different shapes of roads that may be described by the same length quotient

The radius of the curves along a road segment may be important to assess the suitability of a road. This could be assessed by

- maximum absolute value of the curvature,
- average absolute value of the curvature,
- frequency of sign changes in the curvature, or
- a combination of above parameters.

A simple method to include curviness is the determination of an average curvature. Assuming that each line segment is a polygon consisting of at least 3 points, this can be approximated by a simple process:

- Take the first three points,
- determine the radius of a circle through these points,
- determine the length of the arc (or the part of the polygon),
- invert the radius to get the curvature, and
- shift the window by dropping the first polygon point and adding the next one until the end of the line segment is reached.

This approach assumes that the road network is not heavily generalized and thus curves in reality are represented by a polygon and not a simple corner. The result is a list of curvatures combined with corresponding lengths. These values are then combined by a weighted average:

$$
\bar{c}=\frac{\sum_{i} c_{i} l_{i}}{\sum_{i} l_{i}}
$$

where $c_{i}$ is the curvature and $l_{i}$ is a function of the length of the arc. Alternatively, also the median could be used instead of the average. The resulting average curvature is normalized and must be multiplied by the length of the road segment to provide a balance between segments with similar curvature but different length.

A final approach models the avoidance of sharp turns. It is currently restricted to sharp curves in road segments. Turns between road segments may also provide a problem for truck drivers but are ignored in this paper because the problem then differs significantly from the motorcyclist problem. The mathematical approach for avoiding sharp turns is similar to the previous one: The radiuses of arcs through the polygon points are determined and the smallest radius is used for the cost function. This again assumes that curves are modelled by several points and not a single corner. The cost function itself is the curvature, which is the inverse of the radius. However, it must be considered that an absolutely straight road would have an infinite radius and thus a costs factor of zero. This must be compensated, e.g., by adding a value of 1 to the cost factor. The cost factor then is again multiplied by the geometrical length of the road segment.

## 5 Tests



Fig. 2: Test area used for the paper. Vienna is located in the north-western corner of the map (Map data © OpenStreetMap contributors, CC-BY-SA)

The Austrian road network from OpenStreetMap was used for testing (figure 2). The test area is located in the south-west of Vienna and comprises some valleys in east-west direction, which are perpendicular to the determined routes. This should provide sufficiently different routing results for the different approaches. In order to increase the
number of possible solutions, all available paths were used including tracks and bicycle paths. It is obvious that the resulting routes are then not feasible in practice. However, the goal of the paper is to show the determination of different solutions based on existing road networks. The selection of feasible subsets of the road network is a question that is ignored here because some of the necessary data (e.g., driving restrictions, weight restrictions, etc.) are not part of the data set and some information like the actual weight and type of the vehicle is only known in real situations.

The first result (figure 3) is the geometrically shortest past from the starting point in the south to the end point in the north. It is evident that this is not the fastest route because this would capitalize on the higher deriving speed permitted by the highways. The linear distance between start and end point is 55.9 km and the resulting route has a length of 86 km .


Fig. 3: Shortest route between the defined points (Map data © OpenStreetMap contributors, CC-BY-SA)

The next solution applies the length quotient strategy (figure 4a). It deviates strongly from the shortest route and thus the length of the route increases to 117.8 km . However, the route consists of segments with a higher curvature as visible in the comparison of a detail in figure 4 b .

(a)

(b)

Fig. 4: (a) Cost function based on the length quotient strategy.
(b) Detail from the solution using the length quotient strategy. The dark gray line is the same as in figure 4 a , the black line in the south-eastern area is a part of the shortest path as shown in figure 3 (Map data © OpenStreetMap contributors, CC-BY-SA).

The solution in figure 5a is based on average curvature. The routing then avoids any kind of straight route as shown in figure 5 b. The deviations from the straight route also increase the total length of the solution, which is now 141.0 km .

(a)

(b)

Fig. 5: (a) Cost function based on average curvature.
(b) Detail of the route determined by a average curvature cost function (Map data (C) OpenStreetMap contributors, CC-BY-SA).

The final solution is shown in figure 6. It aims at avoiding tight curves. Therefore the costs of a segment are based on the maximum curvature. The resulting route tends to follow straight lines. Since the basis of the cost changes was the shortest route and not the fastest route, the highways are still not used. The length of the route is now 106.5 km .


Fig. 6: Cost function based on maximum curvature (Map data © OpenStreetMap contributors, CC-BY-SA)

## 6 Conclusions and Future Work

The paper shows how one parameter for route selection, curviness, can be modelled based on the geometry of the line segments. Obviously any deviation from the fastest route will extend the travel time and any deviation from the shortest route elongates the route. Table 1 summarizes the elongation of the different cost functions. Avoiding sharp curves for trucks elongates the given route by $24 \%$. The length quotient strategy also provided usable results. The elongation of $37 \%$ is still reasonable when assuming that motorcyclists tend to enjoy driving and may see curvy routes as an added benefit. However, in case of time pressure even motorcyclists would probably use the fastest route and this is easily possible by adding driving restrictions and speed limits for motorcycles and adapting the average speed. Figure 7 shows the different solutions in their geographical context.

Table 1: Comparison of the route length of the four different solutions

| Method | Length in [km] | Elongation in [\%] |
| :--- | :---: | :---: |
| Shortest | 86.0 | 0 |
| Quotient of length | 117.8 | 37 |
| Average curvature | 141.0 | 64 |
| Maximum curvature | 106.5 | 24 |



Fig. 7: Comparison of the three solutions (from left to right): Average curvature, quotient of length, shortest, maximum curvature (Map data © OpenStreetMap contributors, CC-BY-SA).

However, curvature is only one of a few parameters, which needs to be addressed. LEDER determined preferences from interviews and was able to separate six parameters, which are important for motorcyclists during route determination (LEDER 2011):

- Curviness of the route,
- existence of inclinations and declination on the route,
- landscape around the route,
- grip level,
- legal speed limit, and
- truck traffic.

Only the first one has been addressed in this paper. The problem of inclinations and declinations is similar to the curviness problem. Legal speed limit and grip level are easy to include if the data are available but it is difficult to assess the emotional costs for the driver. Driving at low speed or on slippery ground may be boring or dangerous for motorbike riders and may be a reason to avoid the respective routes even if they are fast. The same is true for the amount of truck traffic along the route. The most difficult problem is probably assessing the beauty of the landscape but some ideas have already been presented. The last open question is the operator used for combining the effects and the balance between the different types of costs. This requires additional user testing.

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