Theory of Ferromagnetic Superconductors with Spin-Triplet Electron Pairing[*]

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Abstract. A general quasi-phenomenological theory that describes phases and phase transitions of ferromagnetic superconductors with spin-triplet electron Cooper pairing is presented. The theory is based on extended Ginzburg-Landau expansion in powers of superconducting and ferromagnetic order parameters. A simple form for the dependence of theory parameters on the pressure ensures a correct theoretical outline of the temperature-pressure phase diagram where a stable phase of coexistence of p-wave superconductivity and itinerant ferromagnetism appears. This new theory is in an excellent agreement with the experimental data for intermetallic compounds, for example, UGe_2, URhGe, UCoGe, and UIr that are experimentally proven to be itinerant ferromagnets exhibiting spin-triplet superconductivity. The mechanism of appearance of superconductivity due to itinerant ferromagnetism (M-trigger effect) is established and demonstrated. On the basis of the same theory, basic features of quantum phase transitions in this type of ferromagnetic superconductors are explained in agreement with the experimental data. The theory allows for a classification of the spin-triplet ferromagnetic superconductors in two different types: type I and type II. The classification is based on quantitative criteria, i.e., on simple relations between theory parameters. Both theory and experiment indicate that the two types of p-wave ferromagnetic superconductors are well distinguished by essential differences in their physical properties.

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1 Introduction

There exists an intensive research on Cooper pairing of fermions in condensed matter. Both experimental and theoretical studies reveal fascinating phenomena of superconductivity, unconventional superfluidity and coexistence of superconductivity and ferromagnetism. The fermion Cooper pairs in s-wave state are referred to as conventional pairing and, hence, the respective superfluidity or superconductivity are labeled *conventional*. The unconventional (non-s-wave)

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pairing, for example, the p-wave pairing creates another type of similar phenomena – unconventional superfluidity and unconventional superconductivity of p-wave type. These phenomena can be described within the general scheme of spin-triplet states of fermion pairs. The phenomenon of unconventional (spin-triplet) Cooper pairing of fermions was firstly described for the case of superfluid Helium 3 [1]. Here we focus the attention to unconventional superconductivity of spin-triplet type which allows for electron pairing in p-wave states with spin equal to unity.

The spin-triplet superconducting phases are described in the framework of the general Ginzburg-Landau (GL) effective free energy functional with the help of the symmetry groups theory and detailed thermodynamic analysis [2, 3]; see also [4, 5]. This unconventional superconductivity has been discovered in itinerant ferromagnets – the intermetallic compounds UGe$_2$ [6], URhGe [7], UCoGe [8] and UIr [9].

At low temperature ($T \sim 1$ K) all these compounds exhibit coexistence of p-wave superconductivity and itinerant (f-band) electron ferromagnetism (in short, FS phase). In the mentioned compounds, the FS phase appears only in the ferromagnetic phase domain of the $T - P$ diagram. At given pressure P, the temperature $T_F(P)$ of the normal-to-ferromagnetic phase (or N-FM) transition is never lower than the temperature $T_{FS}(P)$ of the ferromagnetic-to-FS phase (or FM-FS) transition. This is consistent with the point of view that the superconductivity in these compounds is triggered by the spontaneous magnetization $\boldsymbol{M}$, as explained in details in [10]). The main system properties are affected by a term in the Ginzburg-Landau (GL) expansion of the form $\boldsymbol{M} \cdot (\boldsymbol{\psi} \times \boldsymbol{\psi})$, which represents the interaction of $\boldsymbol{M} = \{M_j; j = 1, 2, 3\}$ with the complex superconducting vector field $\boldsymbol{\psi} = \{\psi_j\}$. This term triggers $\boldsymbol{\psi} \neq 0$ for certain T and P values ($\boldsymbol{M}$-trigger effect [10]).

The mechanism of formation of pairs in these systems is under conjectures. The electron band theory of such compounds is quite complex and the density of states $D_n(E)$ at the Fermi surface E_F can hardly be evaluated in details. In this situation, the reliable theoretical interpretation of experiments is performed within the general phenomenological theory of GL type. In such theory, the Landau parameters, $\mu = (a, b, \dots)$ should depend on the shape of $D_n(E)$. While this dependence is unknown, one may reliably assume the dependence of $D_n(E)$ on temperature T and pressure P, and hence, the double functional dependence $\mu = \mu[D_n(T, P)]$. Although unknown, the latter justifies the function $\mu(T, P)$, i.e., the dependence of the Landau parameters on T and P. This dependence can be reliably proposed on the basis of heuristic arguments and experimental data. So, let us focus on the phenomenological approach.

The spin-triplet ferromagnetic superconductors are described by the quasi-phenomenological theory based on an extended GL in powers of the fields ψ and M (for details, see [5, 10–12]). In this Report, the theory and related topics

are briefly presented; for details, see [5, 10–12]. New aspects and outstanding problems are briefly indicated.

2 Theory

Here we hold the consideration in the lowest (mean-field) approximation. The latter is enough to elucidate the main system properties in a quite correct way. The free energy per unit volume, $F/V = f(\psi, M)$, can be written in the form

$$f(\psi, M) = a_s|\psi|^2 + \frac{b_s}{2}|\psi|^4 + \frac{u_s}{2}|\psi^2|^2 + \frac{v_s}{2}\sum_{j=1}^{3}|\psi_j|^4 \tag{1}$$

$$+a_f M^2 + \frac{b_f}{2}M^4 + i\gamma_0 M \cdot (\psi \times \psi^*) + \delta M^2|\psi|^2.$$

The material parameters satisfy $b_s > 0$, $b_f > 0$, $a_s = \alpha_s(T - T_s)$, and $a_f = \alpha_f[T - T_f(P)]$. The terms proportional to u_s and v_s describe, respectively, the anisotropy of the spin-triplet electron Cooper pairs and the crystal anisotropy. Next, $\gamma_0 \sim J$ (with $J > 0$ the ferromagnetic exchange constant) and $\delta > 0$ are parameters of the ψ-M interaction terms. Previous mean-field studies [10] have shown that the anisotropy represented by the u_s and v_s terms in Eq. (1) slightly perturb the size and shape of the stability domains of the phases, while similar effects can be achieved by varying the b_s factor in the $b_s|\psi|^4$ term. For these reasons, in the present analysis we ignore the anisotropy terms, setting $u_s = v_s = 0$, and consider $b_s \equiv b > 0$ as an effective parameter. Then, without loss of generality, we use the gauge $M = (0, 0, M)$.

A convenient dimensionless free energy can now be defined by $\widetilde{f} = f/(b_f M_0^4)$, where $M_0 = [\alpha_f T_{f0}/b_f]^{1/2} > 0$ is the value of M corresponding to the pure magnetic subsystem ($\psi \equiv 0$) at $T = P = 0$ and $T_{f0} = T_f(0)$. On scaling the order parameters as $m = M/M_0$ and $\varphi = \psi/[(b_f/b)^{1/4}M_0]$, we obtain

$$\widetilde{f} = r\phi^2 + \frac{\phi^4}{2} + tm^2 + \frac{m^4}{2} + 2\gamma m\phi_1\phi_2\sin\theta + \gamma_1 m^2\phi^2, \tag{2}$$

where $\phi_j = |\varphi_j|$, $\phi = |\varphi|$, and θ is the phase angle between the complex φ_2 and φ_1. The dimensionless constants are $t = [\widetilde{T} - \widetilde{T}_f(P)]$, $r = \kappa(\widetilde{T} - \widetilde{T}_s)$ with $\kappa = \alpha_s b_f^{1/2}/\alpha_f b^{1/2}$, $\gamma = \gamma_0/[\alpha_f T_{f0}b]^{1/2}$, and $\gamma_1 = \delta/(bb_f)^{1/2}$. The reduced temperatures are $\widetilde{T} = T/T_{f0}$, $\widetilde{T}_f(P) = T_f(P)/T_{f0}$, $\widetilde{T}_s(P) = T_s(P)/T_{f0}$.

3 Results

The analysis involves making simple assumptions for the P dependence of the t, r, γ, and γ_1 parameters in Eq. (2). Specifically, we assume that only T_f has
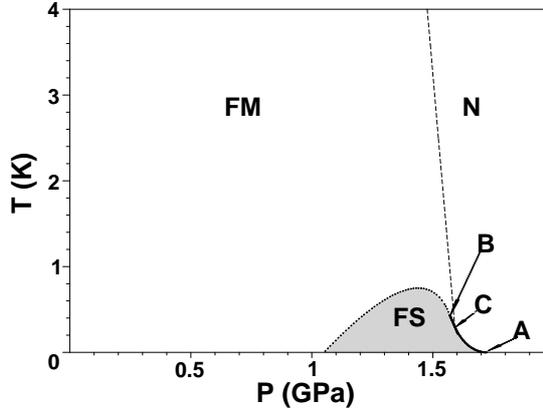
Figure 1. Low-temperature part of the $T - P$ phase diagram of UGe$_2$. The points A, B, C are located in the high-pressure part ($P \sim P_c \sim 1.6$ GPa). The FS phase domain is shaded. The solid lines AC and BC show the first-order transitions N-FS, and FM-FS, respectively. The dotted line shows a second-order FS-FM transition, the dashed line stands for the N-FM transition.

a significant P dependence, described by $\widetilde{T}_f(P) = (1 - \widetilde{P})$, where $\widetilde{P} = P/P_0$ and P_0 is a characteristic pressure deduced later; for UGe$_2$, $P_0 \sim P_c$ - the critical pressure above which both magnetic and supercionducting order vanish at $T \sim 0$ (see Figure 1); for some substances this is not the case, but then another dependence of T_f on P takes place. Within this simple variant, all other theory parameters are considered P-independent.

The simplified model in Eq. (2) is capable of describing the main thermodynamic properties of spin-triplet ferromagnetic superconductors [10]. For example, in Figure 1 the low temperature part of the $T - P$ diagrams of UGe$_2$ is shown. The result is in a remarkable agreement with the experimental shape of the same diagram [6]. The type of the phase transitions and the multicritical points (A, B, C) also correspond to the experimental observations. There are three stable phases: (i) the normal (N) phase, given by $\phi = m = 0$; (ii) the pure ferromagnetic (FM) phase, given by $m = (-t)^{1/2} > 0$, $\phi = 0$; and (iii) the FS phase, given by $\phi_1^2 = \phi_2^2 = (\gamma m - r - \gamma_1 m^2)/2$, $\phi_3 = 0$, where $\sin\theta = -1$ and m satisfies

$$(1 - \gamma_1^2)m^3 + \frac{3}{2}\gamma\gamma_1 m^2 + \left(t - \frac{\gamma^2}{2} - \gamma_1 r\right)m + \frac{\gamma r}{2} = 0. \qquad (3)$$

Although Eq. (3) is complicated, some analytical results follow, e.g., we find that the second order phase transition line $\widetilde{T}_{FS}(P)$ separating the FM and FS phases (the dotted lines in Figure 1) is the solution of

$$\widetilde{T}_{FS}(P) = \widetilde{T}_s + \frac{\gamma_1}{\kappa}t(T_{FS}) + \frac{\gamma}{\kappa}[-t(T_{FS})]^{1/2}. \qquad (4)$$

Under certain conditions, which are satisfied for UGe$_2$, the $T_{FS}(P)$ curve has a maximum at $\widetilde{T}_m = \widetilde{T}_s + (\gamma^2/4\kappa\gamma_1)$ with pressure P_m found by solving $t(T_m, P_m) = -(\gamma^2/4\gamma_1^2)$.

Under certain conditions, Eq. (2) describes pure superconducting phases, too. To date there are no experimental observations of such phase, which means that the respective conditions are not satisfied by the parameters of the currently known superconductors. Negative values of T_s are possible, and they describe a phase diagram topology in which the FM-FS transition line terminates at $T = 0$ for $P < P_c$. This might be of relevance for other compounds, e.g., URhGe.

As in experiment [6], the dashed line in Figure 1 slopes to $T = 0$ at another critical pressure P_{0c}. This is possible in UGe$_2$ because for this compound the condition $\gamma < \gamma_1$ is satisfied. Such systems are spin-triplet ferromagnetic superconductors of type I, whereas systems which fulfill the condition $\gamma > \gamma_1$ are of type II [10, 12]. The quantum phase transitions at P_0c and P_c have remarkable properties, as shown in [11]. Depending on the system properties, T_C – the temperature locating point C, can be either positive (when a direct N-FS first order transition is possible), zero, or negative (when the FM-FS and N-FM phase transition lines terminate at different zero-temperature phase transition points). The last two cases correspond to $T_s < 0$. All these cases are possible in spin-triplet ferromagnetic superconductors. The zero temperature transition at P_{c0} is found to be a quantum critical point, whereas the zero-temperature phase transition at P_c is of first order. As noted, the latter splits into two first order phase transitions. This classical picture may be changed through quantum fluctuations [10].

The quantum phase transitions at P_{0c} and P_c have remarkable properties. An investigation [11] performed by renormalization group methods revealed a fluctuation change in the order of the zero temperature first order phase transition at P_c to a continuous phase transition belonging to an entirely new class of universality. However, this option exists only for magnetically isotropic order (Heisenberg symmetry) and is unlikely to apply in the known spin-triplet ferromagnetic superconductors, which are magnetically anisotropic.

The application of the theory has been demonstrated on the example of UGe$_2$, but the same theory has ample volume of options to describe various real superconductors of the same type and is not restricted to a particular compound, or, to a particular group of such materials.

Even in its simplified form, this theory has been shown to be capable of accounting for a wide variety of experimental behavior. A natural extension to the theory is to add a M^6 term which provides a formalism to investigate possible metamagnetic phase transitions and extend some first order phase transition lines, as required by experimental data. Another modification of this theory, with regard to applications to other compounds, is to include a P dependence for some of the other GL parameters.

Among the outstanding problems are: local gauge effects on the vortex phase and the phase transitions, the outline of the upper critical magnetic field $H_{c2}(T, P)$, thermal and magnetic properties, and the description of $T - P$ diagrams with topologies, which are observed in experiments. Such studies require an extension of the theory by including M^6 term, the magnetic induction $B = H + 4\pi M$, and a more precise P-dependence of some theory parameters.

Acknowledgements

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