For solvable DAE systems, a general methodology was developed wherein the DAE system was modified by a dynamic state feedback compensator such that the resulting system was solvable and possessed a control invariant state space, thereby allowing the derivation of standard state-space realizations. For the feedback-modified system, a state-space realization was derived that can be used as the basis for controller synthesis. Extension of the proposed methodology for nonlinear DAE systems will be explored in future work.

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Adaptive Nonlinear Output-Feedback Schemes with Marino–Tomei Controller

Miroslav Krstić and Petar V. Kokotović

Abstract— **Three new adaptive nonlinear output-feedback schemes are presented. The first scheme employs the tuning functions design. The other two employ a novel estimation-based design consisting of a strengthened controller-observer pair and observer-based and swapping-based identifiers. They remove restrictive growth and matching conditions present in the previous output-feedback nonlinear estimation-based designs and allow a systematic improvement of transient performance.**

I. INTRODUCTION

In the last few years, adaptive control of nonlinear systems has emerged as an exciting research area. Early efforts focused on the state-feedback problem and resulted in a systematic design procedure called adaptive backstepping [5]. The more challenging output-feedback problem was then addressed for systems with nonlinearities which depend on the output only. This problem was first solved under restrictive structural and growth conditions on the nonlinearities [3], [4]. Subsequently, the growth restrictions were removed [6], but the structural restriction remained: the output nonlinearities were not allowed to precede the control input.

The removal of this structural restriction by Marino and Tomei in [15] was a breakthrough in adaptive nonlinear output-feedback control. This was achieved by merging the filtered transformations of [13] and [14] with the adaptive backstepping of [5] and using a novel compensation of the estimation error effects. An alternative approach for the same class of systems was presented in [7]. The nonlinear systems considered in [15] and [7] are still the largest class for which asymptotic tracking of arbitrary smooth reference signals can be achieved. A more general class of systems was considered in [16], but only for the set-point regulation problem.

In [15], the authors view their adaptive scheme as an existence result because of its complexity and high-dynamic order which are primarily due to the overparameterization inherited from the original adaptive backstepping procedure [5]. The overparameterization amounts to employing ρ different update laws for the same parameter vector, ρ being the relative degree of the plant. Another drawback of the scheme in [15] is that it is restricted to the unnormalized gradient update law. Furthermore, in the setting of [15], the passivity of the observer error system could not be exploited to design a simple observer-based identifier.

The three new adaptive schemes proposed in this paper remove these drawbacks. They achieve minimal parameterization in two different ways. The first scheme, presented in Section III, employs the "tuning functions" technique developed in [9]. In this scheme we modify the Marino–Tomei controller with terms which compensate the mismatch between the actual update law and the tuning functions. The other two adaptive schemes avoid overparameterization by using

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a novel estimation-based approach. They are motivated by our recent state-feedback results [10], [11]. The "observer-based" scheme in Section V has a simple identifier which exploits the passivity of the observer error system. This scheme is still restricted to an unnormalized gradient update law, as are [15] and the tuning functions scheme. The "swapping-based" scheme in Section VI removes this last restriction and can incorporate any standard update law: gradient, least-squares, normalized, or unnormalized.

Until recently, the estimation-based approach to adaptive nonlinear control [18] was unable to guarantee global boundedness without restrictive growth or matching conditions. This is now accomplished by strengthening the controller-observer pair with nonlinear damping including the κ-terms of [8], so that its boundedness properties are achieved independently of the identifier. This strengthening guarantees boundedness of all closed-loop states whenever $\dot{\tilde{\theta}}$ is bounded, $\tilde{\theta} \in \mathcal{L}_\infty$, and either $\dot{\tilde{\theta}} \in \mathcal{L}_\infty$ or $\dot{\tilde{\theta}} \in \mathcal{L}_2$. The identifiers, in turn, independently guarantee that $\tilde{\theta} \in \mathcal{L}_\infty$ and either $\dot{\tilde{\theta}} \in \mathcal{L}_\infty$ (swapping-based scheme) or $\dot{\tilde{\theta}} \in \mathcal{L}_2$ (observer-based scheme).

In Section VII we show analytically that the schemes given in this paper can be used for systematically improving transient performance.

Notation: $X_{(i)}$ and X_j denote the ith row and the jth column of matrix X, respectively.

II. Marino–Tomei Filtered Transformations and Observer

Problem Statement: As in [15], we consider SISO nonlinear systems transformable into the output-feedback canonical form

$$\begin{aligned} \dot{x} &= Ax + \phi(y) + \Phi(y)a + \begin{bmatrix} 0 \\ b \end{bmatrix} \sigma(y)u, \qquad x \in \mathbb{R}^n \\ y &= x_1 \end{aligned} \tag{1}$$

where $a = [a_1, \cdots, a_q]^{\mathrm{T}} \in \mathbb{R}^q, b = [b_m, \cdots, b_0]^{\mathrm{T}} \in \mathbb{R}^{m+1}$ are vectors of unknown constant parameters, and

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \cdots & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} \varphi_{0,1} \\ \vdots \\ \varphi_{0,n} \end{bmatrix}$$
$$\Phi = \begin{bmatrix} \varphi_{1,1} & \cdots & \varphi_{q,1} \\ \vdots & & \vdots \\ \varphi_{1,n} & \cdots & \varphi_{q,n} \end{bmatrix}. \tag{2}$$

It is assumed that $\varphi_{j,i}, 0 \le j \le q, 1 \le i \le n$, and σ are smooth. Geometric conditions which characterize the class of nonlinear systems that can be transformed into this form have been given in [15]. The class of systems which are globally stabilizable by output feedback is not much broader than (1). It was shown in [17] that the system $\dot{x}_1 = x_2, \dot{x}_2 = x_2^n + u, y = x_1$ cannot be globally stabilized by dynamic output feedback for $n > 2$.

Assumption 2.1: The polynomial $B(s) = b_m s^m + \cdots + b_1 s + b_0$ is Hurwitz, and $\sigma(y) \neq 0 \forall y \in \mathbb{R}$. The sign of b_m is known.

Assumption 2.2: The reference signal $y_r(t)$ and its first ρ derivatives are known and bounded, and $y_r^{(\rho)}(t)$ is piecewise continuous.

Filtered Transformations: The filtered transformations employ the input filters

$$v_i = \frac{1}{s+\lambda} v_{i+1} = \frac{1}{(s+\lambda)^{\rho - i}} \sigma(y) u, \qquad i = 1, \cdots, \rho - 1 \tag{3}$$

$$\mu_j = \frac{1}{s+\lambda} \mu_{j-1} = \frac{1}{(s+\lambda)^j} v_1, \qquad j = 1, \cdots, m \tag{4}$$

and the output filters

$$\dot{\xi} = A_l \xi + B_l \phi(y), \qquad \xi \in \mathbb{R}^{n-1} \tag{5}$$
$$\dot{\xi} = A_l \Xi + B_l \Phi(y), \qquad \Xi \in \mathbb{R}^{(n-1)\times q} \tag{6}$$

where A_l and B_l are given by

$$A_l = \begin{bmatrix} & I_{n-2} \\ -\bar{l} & \\ & 0 \cdots 0 \end{bmatrix}, \quad B_l = [A_l, e_{n-1}] \tag{7}$$

and the vectors $\bar{l}$ and l are defined via the coefficients of polynomial $(s+\lambda)^{n-1}$

$$l = \left[1, \binom{n-1}{1}\lambda, \cdots, \binom{n-1}{n-2}\lambda^{n-2}, \lambda^{n-1}\right]^{\mathrm{T}} \triangleq \begin{bmatrix} 1 \\ \bar{l} \end{bmatrix}. \tag{8}$$

For later use, (4) is rewritten in the compact form

$$\dot{\mu} = \Lambda \mu + e_1 v_1. \tag{9}$$

Lemma 2.1 [15]: For (1) and (3)–(6), there exist a vector $\beta(b) \in \mathbb{R}^m$ and a matrix $S(\theta) \in \mathbb{R}^{n \times [(q+2)(n-1)]}$ where

$$\theta = [b_m, \beta(b)^{\mathrm{T}}, a^{\mathrm{T}}]^{\mathrm{T}} \tag{10}$$

such that the parameter-dependent coordinate transformation

$$\chi = x - S(\theta)[v^{\mathrm{T}}, \mu^{\mathrm{T}}, \xi^{\mathrm{T}}, \mathrm{col}\,(\Xi)^{\mathrm{T}}]^{\mathrm{T}} \tag{11}$$

takes (1) into the "adaptive observer form"

$$\begin{aligned} \dot{\chi} &= A\chi + l(\omega_0 + \omega^{\mathrm{T}}\theta) \\ y &= \chi_1 \end{aligned} \tag{12}$$

where ω_0 and the "regressor" ω are given by

$$\omega_0 = \phi_1(y) + \xi_1 \tag{13}$$
$$\omega^{\mathrm{T}} = [v_1, \mu^{\mathrm{T}}, \Phi_{(1)}(y) + \Xi_{(1)}]. \tag{14}$$

Observer: Once they have brought (1) to the adaptive observer form, Marino and Tomei design the observer for the transformed state χ

$$\dot{\hat{\chi}} = A\hat{\chi} + K_o(y - \hat{\chi}_1) + l(\omega_0 + \omega^{\mathrm{T}}\hat{\theta}) \tag{15}$$

where K_o is chosen so that $A_o = A - K_o e_1^{\mathrm{T}}$ satisfies

$$\det(sI - A_o) = (s + c_o)(s+\lambda)^{n-1} \tag{16}$$

namely, $e_1^{\mathrm{T}}(sI - A_o)^{-1} l = 1/(s + c_o)$. The observer error $\varepsilon = \chi - \hat{\chi}$ is governed by

$$\dot{\varepsilon} = A_o \varepsilon + l\omega^{\mathrm{T}}\tilde{\theta}. \tag{17}$$

III. Tuning Functions Scheme

The tuning function's design [9] is applied to the system consisting of the states $y, v_1, \cdots, v_{\rho-1}$. The design steps are only briefly outlined.

Step 1: The first component of the state of the error system is the tracking error $z_1 = y - y_r$. The first stabilizing function is designed as

$$\alpha_1(X, t) = -\hat{\varsigma}(c+d)z_1 + \hat{\varsigma}\overline{\alpha}_1 \triangleq \hat{\varsigma}\alpha_\varsigma \tag{18}$$

where $X \triangleq (y, \hat{\chi}, \xi, \Xi, \mu, \hat{\theta}, \hat{\varsigma})$, the quantity $\hat{\varsigma}$ is an estimate of $\varsigma = 1/b_m$, and

$$\overline{\alpha}_1 = -\hat{\chi}_2 - \omega_0 - \overline{\omega}^{\mathrm{T}}\hat{\theta} + \dot{y}_r \tag{19}$$

where the truncated regressor is given by

$$\overline{\omega}^{\mathrm{T}} = [0, \mu^{\mathrm{T}}, \Phi_{(1)}(y) + \Xi_{(1)}]. \tag{20}$$

We design the first tuning function as

$$\tau_1 = \Gamma(\overline{\omega} z_1 + \gamma_\varepsilon \omega \varepsilon_1), \qquad \gamma_\varepsilon > 0. \tag{21}$$

Step 2: Introducing $z_2 = v_1 - \alpha_1$, the second stabilizing function

$$\alpha_2(X, v_1, t) = -\left[c + d\left(\frac{\partial \alpha_1}{\partial y}\right)^2\right] z_2 + \overline{\alpha}_2 + q_2 \tag{22}$$

consists of

$$\begin{aligned}\overline{\alpha}_2 = &-\hat{b}_m z_1 + \lambda v_1 + \frac{\partial \alpha_1}{\partial y}(\hat{\chi}_2 + \omega_0 + \omega^{\mathrm{T}}\hat{\theta}) \\ &+ \frac{\partial \alpha_1}{\partial \hat{\chi}}[A\hat{\chi} + K_o(y - \hat{\chi}_1) + l(\omega_0 + \omega^{\mathrm{T}}\hat{\theta})] \\ &+ \frac{\partial \alpha_1}{\partial \xi}(A_l\xi + B_l\phi) + \frac{\partial \alpha_1}{\partial \Xi}(A_l\Xi + B_l\Phi) \\ &+ \frac{\partial \alpha_1}{\partial \mu}[\Lambda\mu + e_1 v_1] + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r\end{aligned} \tag{23}$$

and the compensating function

$$q_2 = \frac{\partial \alpha_1}{\partial \hat{\theta}}\tau_2 + \frac{\partial \alpha_1}{\partial \hat{\varsigma}}\dot{\hat{\varsigma}} \tag{24}$$

where the second tuning function is designed as

$$\tau_2 = \tau_1 + \Gamma\left(-\frac{\partial \alpha_1}{\partial y}\omega + z_1 e_1\right) z_2. \tag{25}$$

Step $i(3 \le i \le \rho)$: Subsequent components of the state of the error system are defined as $z_i = v_{i-1} - \alpha_{i-1}$. The stabilizing functions

$$\alpha_i(X, v_1, \cdots, v_{i-1}, t) = -\left[c + d\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2\right] z_i + \overline{\alpha}_i + q_i \tag{26}$$

consist of

$$\begin{aligned}\overline{\alpha}_i = &-z_{i-1} + \lambda v_{i-1} + \frac{\partial \alpha_{i-1}}{\partial y}(\hat{\chi}_2 + \omega_0 + \omega^{\mathrm{T}}\hat{\theta}) \\ &+ \frac{\partial \alpha_{i-1}}{\partial \hat{\chi}}[A\hat{\chi} + K_o(y - \hat{\chi}_1) + l(\omega_0 + \omega^{\mathrm{T}}\hat{\theta})] \\ &+ \frac{\partial \alpha_{i-1}}{\partial \xi}(A_l\xi + B_l\phi) + \frac{\partial \alpha_{i-1}}{\partial \Xi}(A_l\Xi + B_l\Phi) \\ &+ \frac{\partial \alpha_{i-1}}{\partial \mu}[\Lambda\mu + e_1 v_1] \\ &+ \sum_{j=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial v_j}(-\lambda v_j + v_{j+1}) + \sum_{j=1}^{i}\frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)}\end{aligned} \tag{27}$$

and the compensating functions

$$\begin{aligned}q_i &= \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\tau_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\varsigma}}\dot{\hat{\varsigma}} - \sum_{k=2}^{i-1} z_k \sigma_{ki} \\ \sigma_{ki} &= \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\Gamma\omega\frac{\partial \alpha_{i-1}}{\partial y}\end{aligned} \tag{28}$$

and the ith tuning function is designed as

$$\tau_i = \tau_{i-1} - \Gamma\frac{\partial \alpha_{i-1}}{\partial y}\omega z_i. \tag{29}$$

At the end of the recursion, the last stabilizing function α_ρ is used for the actual control law

$$u = \frac{1}{\sigma(y)}\alpha_\rho. \tag{30}$$

The update laws are designed as

$$\dot{\hat{\varsigma}} = -\gamma_\varsigma \operatorname{sgn}(b_m)\alpha_\varsigma z_1 \tag{31}$$

$$\dot{\hat{\theta}} = \tau_\rho = \Gamma\left\{\omega\left[1, -\frac{\partial \alpha_1}{\partial y}, \cdots, -\frac{\partial \alpha_{\rho-1}}{\partial y}\right] \cdot z - \hat{\varsigma}\alpha_\varsigma e_1 z_1 + \gamma_\varepsilon \omega \varepsilon_1\right\} \tag{32}$$

with $\gamma_\varepsilon > \rho/2dq_o$ and q_o specified in the proof of Theorem 3.1.

By noting that $\omega^{\mathrm{T}}\theta = b_m v_1 + \overline{\omega}^{\mathrm{T}}\theta$ and

$$\dot{y} = \hat{\chi}_2 + \omega_0 + \omega^{\mathrm{T}}\theta + \varepsilon_2 \tag{33}$$

it is straightforward to verify that the resulting system, called the error system, is

$$\begin{aligned}\dot{z} &= A_z(z,t)z + W_\varepsilon(z,t)\varepsilon_2 + W_\theta(z,t)^{\mathrm{T}}\tilde{\theta} - b_m\alpha_\varsigma e_1\tilde{\varsigma}, \\ & z \in \mathbf{R}^\rho\end{aligned} \tag{34}$$

where (35) is shown at the bottom of the page, and

$$\begin{aligned}W_\varsigma &= \left[1, -\frac{\partial \alpha_1}{\partial y}, \cdots, -\frac{\partial \alpha_{\rho-1}}{\partial y}\right]^T \\ W_\theta^T &= W_\varsigma(z,t)\omega^T - \hat{\zeta}\alpha_\varsigma e_1 e_1^T\end{aligned} \tag{36}$$

and $p \triangleq q + m + 1$. We see that $\dot{\hat{\theta}}$ is absent from (34).

Theorem 3.1 (Tuning Functions Scheme): All the signals in the closed-loop adaptive system consisting of the plant (1), the filters (3)–(6), the observer (15), the update laws (31) and (32), and the control law (30) are globally uniformly bounded for all $t \ge 0$, and global asymptotic tracking is achieved: $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$.

Proof: See [12].

IV. Strengthened Observer and Controller

Observer: We strengthen the Marino–Tomei observer by adding the stability enhancing, nonlinear term $\kappa_o|\omega|^2 l(y - \hat{\chi}_1)$ in (15)

$$\dot{\hat{\chi}} = A\hat{\chi} + K_o(y - \hat{\chi}_1) + \kappa_o|\omega|^2 l(y - \hat{\chi}_1) + l(\omega_0 + \omega^{\mathrm{T}}\hat{\theta}) \tag{37}$$

so that the observer error system becomes

$$\dot{\varepsilon} = (A_o - \kappa_o|\omega|^2 l e_1^{\mathrm{T}})\varepsilon + l\omega^{\mathrm{T}}\tilde{\theta}. \tag{38}$$

Lemma 4.1: Suppose in (38) that ω and $\tilde{\theta}$ are piecewise continuous on $[0, t_f)$. If $\tilde{\theta} \in \mathcal{L}_\infty[0, t_f)$, then $\varepsilon \in \mathcal{L}_\infty[0, t_f)$.

Proof: Since $e_1^{\mathrm{T}}(sI - A_o)^{-1}l = 1/(s + c_o)$ is SPR, then there exist $P_o = P_o^{\mathrm{T}} > 0$ and $q_o > 0$ such that

$$A_o^{\mathrm{T}}P_o + P_oA_o \le -q_oI, \qquad P_ol = e_1. \tag{39}$$

Therefore, along the solutions of (38) we have

$$\begin{aligned}\frac{d}{dt}(|\varepsilon|_{P_o}^2) &\le -q_o|\varepsilon|^2 - 2\kappa_o|\omega|^2\varepsilon^{\mathrm{T}}P_o l e_1^{\mathrm{T}}\varepsilon \\ &\quad + 2\varepsilon^{\mathrm{T}}P_o l\omega^{\mathrm{T}}\tilde{\theta} \\ &= -q_o|\varepsilon|^2 - 2\kappa_o|\omega|^2\varepsilon_1^2 + 2\varepsilon_1\omega^{\mathrm{T}}\tilde{\theta} \\ &\le -q_o|\varepsilon|^2 + \frac{1}{2\kappa_o}|\tilde{\theta}|^2\end{aligned} \tag{40}$$

$$A_z = \begin{bmatrix} -c-d & \hat{b}_m & 0 & \cdots & 0 \\ -\hat{b}_m & -c-d\left(\frac{\partial \alpha_1}{\partial y}\right)^2 & 1+\sigma_{23} & \cdots & \sigma_{2,\rho} \\ 0 & -1-\sigma_{23} & \ddots & \vdots & \\ \vdots & \vdots & \ddots & \ddots & 1+\sigma_{\rho-1,\rho} \\ 0 & -\sigma_{2,\rho} & \cdots & -1-\sigma_{\rho-1,\rho} & -c-d\left(\frac{\partial \alpha_{\rho-1}}{\partial y}\right)^2 \end{bmatrix} \tag{35}$$

which implies that $\varepsilon \in \mathcal{L}_\infty[0, t_f)$, whenever $\tilde{\theta} \in \mathcal{L}_\infty[0, t_f)$. □

Assumption 4.1: In addition to $\operatorname{sgn} b_m$, a positive constant ϱ_m is known such that $|b_m| \geq \varrho_m$.

Controller: We only spell out the differences from the tuning functions design. The stabilizing functions α_i are designed to render the state $[z_1, \cdots, z_i]^{\mathrm{T}}$ bounded whenever $\tilde{\theta}$ is bounded and $\dot{\hat{\theta}}$ is either bounded or square-integrable. This is achieved with the nonnegative nonlinear damping functions s_i

$$s_1 = d + \kappa\left|\overline{\omega} + \frac{\overline{\alpha}_1}{\hat{b}_m} e_1\right|^2 \tag{41}$$

$$s_2 = d\left(\frac{\partial \alpha_1}{\partial y}\right)^2 + \kappa\left|\frac{\partial \alpha_1}{\partial y}\omega - z_1 e_1\right|^2 + g\left|\frac{\partial \alpha_1}{\partial \hat{\theta}}^{\mathrm{T}}\right|^2 \tag{42}$$

$$s_i = d\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 + \kappa\left|\frac{\partial \alpha_{i-1}}{\partial y}\omega\right|^2 + g\left|\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}^{\mathrm{T}}\right|^2 \tag{43}$$

which consist of three terms, each counteracting the effects of disturbances $\varepsilon_2, \tilde{\theta}, \dot{\hat{\theta}}$. The nonlinear damping functions appear in the modified stabilizing functions[1]

$$\alpha_1 = -\frac{\operatorname{sgn} b_m}{\varrho_m}(c + s_1) z_1 + \frac{1}{\hat{b}_m}\overline{\alpha}_1 \tag{44}$$

$$\alpha_i = -(c + s_i) z_i + \overline{\alpha}_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\chi}} \kappa_o |\omega|^2 l (y - \hat{\chi}_1), \quad i = 2, \cdots, \rho. \tag{45}$$

The term $(\partial\alpha_{i-1}/\partial\hat{\chi})\kappa_o|\omega|^2 l(y - \hat{\chi}_1)$ in (45) accommodates for the strengthening in the observer.

It is straightforward to verify that the resulting error system is

$$\dot{z} = A_z^*(z,t) z + W_\varepsilon(z,t)\varepsilon_2 + W_\theta^*(z,t)^{\mathrm{T}}\tilde{\theta} + D(z,t)\dot{\hat{\theta}}, \quad z \in \mathbb{R}^\rho \tag{46}$$

where

$$A_z^*(z,t) = \begin{bmatrix} -\frac{|b_m|}{\varrho_m}(c + s_1) & b_m & 0 & \cdots & 0 \\ -b_m & -(c + s_2) & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 & -(c + s_\rho) \end{bmatrix} \tag{47}$$

$$W_\varepsilon(z,t) = \begin{bmatrix} 1 \\ -\frac{\partial \alpha_1}{\partial y} \\ \vdots \\ -\frac{\partial \alpha_{\rho-1}}{\partial y} \end{bmatrix}$$

$$W_\theta^*(z,t)^{\mathrm{T}} = \begin{bmatrix} \overline{\omega}^{\mathrm{T}} + \frac{\overline{\alpha}_1}{\hat{b}_m} e_1^{\mathrm{T}} \\ -\frac{\partial \alpha_1}{\partial y}\omega^{\mathrm{T}} + z_1 e_1^{\mathrm{T}} \\ -\frac{\partial \alpha_2}{\partial y}\omega^{\mathrm{T}} \\ \vdots \\ -\frac{\partial \alpha_{\rho-1}}{\partial y}\omega^{\mathrm{T}} \end{bmatrix} \tag{48}$$

$$D(z,t) = \left[0, -\frac{\partial \alpha_1}{\partial \hat{\theta}}, \cdots, -\frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}}\right]^{\mathrm{T}}. \tag{49}$$

[1] Our identifiers will guarantee $|\hat{b}_m(t)| \geq \varrho_m, \forall t \geq 0$.

Lemma 4.2 (Input-to-State Properties): Consider the system (1), (3)–(6), (37), (30). If $\tilde{\theta} \in \mathcal{L}_\infty[0, t_f)$ and either $\dot{\hat{\theta}} \in \mathcal{L}_\infty[0, t_f)$ or $\dot{\hat{\theta}} \in \mathcal{L}_2[0, t_f)$, then $\hat{\chi}, \xi, \Xi, \mu, v, x, u \in \mathcal{L}_\infty[0, t_f)$.

Proof (Outline): Differentiating $\frac{1}{2}|z|^2$ along the solutions of (46) and using the definitions of the nonlinear damping functions (41)–(43), by completing squares one can show that

$$\frac{d}{dt}\left(\frac{1}{2}|z|^2\right) \leq -c|z|^2 + \frac{\rho}{4}\left(\frac{1}{d}\varepsilon_2^2 + \frac{1}{\kappa}|\tilde{\theta}|^2 + \frac{1}{g}|\dot{\hat{\theta}}|^2\right). \tag{50}$$

Since $\tilde{\theta} \in \mathcal{L}_\infty[0, t_f)$, by Lemma 4.1, $\varepsilon \in \mathcal{L}_\infty[0, t_f)$. If either $\dot{\hat{\theta}} \in \mathcal{L}_\infty[0, t_f)$ or $\dot{\hat{\theta}} \in \mathcal{L}_2[0, t_f)$, it is easy to see that $z \in \mathcal{L}_\infty[0, t_f)$. The boundedness of y_r and z_1 implies that y is bounded. Therefore ξ and Ξ are bounded. To prove boundedness of $\hat{\chi}$, let us first rewrite (37) as

$$\dot{\hat{\chi}} = A_o\hat{\chi} + K_o y + l[\kappa_o|\omega|^2(y - \hat{\chi}_1) + \omega_0 + \omega^{\mathrm{T}}\hat{\theta}] \tag{51}$$

and note that the boundedness of ε_1 and y implies that $\hat{\chi}_1$ is bounded. To prove that the remaining components of $\hat{\chi}$ are bounded, we employ the similarity transformation

$$\begin{bmatrix} \hat{\chi}_1 \\ \eta \end{bmatrix} \triangleq \begin{bmatrix} \hat{\chi}_1 \\ T\hat{\chi} \end{bmatrix} \triangleq \begin{bmatrix} e_1^{\mathrm{T}} \\ A_l e_1, I_{n-1} \end{bmatrix}\hat{\chi} \tag{52}$$

and, from (51), we obtain the system

$$\dot{\eta} = A_l\eta - A_l^2 e_1 y \tag{53}$$

which shows that η is independent of the input $\kappa_o|\omega|^2(y - \hat{\chi}_1) + \omega_0 + \omega^{\mathrm{T}}\hat{\theta}$. To arrive at the last equation, we have used the identities

$$Tl = 0, \quad K_o = c_o l - \begin{bmatrix} A_l e_1 \\ 0 \end{bmatrix}, \quad TA_o = A_l T \tag{54}$$

which hold for K_o such that $A_o = A - K_o e_1^{\mathrm{T}}$ satisfies (16) and are straightforward to verify. Because of the boundedness of y and the Hurwitzness of A_l, (53) proves that η is bounded. In view of the similarity transformation (52), it follows that $\hat{\chi}$ is bounded. The boundedness of μ, v, x, and u is established as in [15]. □

V. Observer-Based Scheme

We choose the parameter update law as

$$\dot{\hat{\theta}} = \Gamma\omega\varepsilon_1. \tag{55}$$

Although not indicated in (55), the standard projection is employed with $\hat{b}_m(0)\operatorname{sgn} b_m \geq \varrho_m$ only to guarantee that $|\hat{b}_m(t)| \geq \varrho_m, \forall t \geq 0$ (no other *a priori* parameter knowledge is needed).

Lemma 5.1: If ω is piecewise continuous on $[0, t_f)$, then (55) guarantees that $\tilde{\theta}, \varepsilon \in \mathcal{L}_\infty[0, t_f)$ *and* $\varepsilon, \dot{\hat{\theta}} \in \mathcal{L}_2[0, t_f)$.

Proof (Outline): With (39), and using the standard properties of projection [18], [19], [2], along the solutions of (38) and (55) we have

$$\frac{d}{dt}(|\varepsilon|_{P_o}^2 + |\tilde{\theta}|_{\Gamma^{-1}}^2) \leq -q_o|\varepsilon|^2 - 2\kappa_o|\omega|^2\varepsilon_1^2 \leq -q_o|\varepsilon|^2 - 2\frac{\kappa_o}{\overline{\lambda}(\Gamma)^2}|\dot{\hat{\theta}}|^2 \tag{56}$$

which proves that $\varepsilon, \tilde{\theta}$ are bounded and $\varepsilon, \dot{\hat{\theta}}$ are square-integrable. □

Theorem 5.1 (Observer-Based Scheme): All the signals in the closed-loop adaptive system consisting of the plant (1), the filters (3)–(6), the observer (37), the update law (55), and the control law (30) are globally uniformly bounded for all $t \geq 0$, and global asymptotic tracking is achieved $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$.

Proof: Combining Lemmas 4.2 and 5.1, it is straightforward to show that all the signals are globally uniformly bounded. To prove the tracking, we first rewrite (46) as

$$\dot{z} = A_z(z,t)z + W_\varepsilon(z,t)\varepsilon_2 + W_\theta(z,t)^{\mathrm{T}}\tilde{\theta} + D(z,t)\dot{\hat{\theta}} \tag{57}$$

where A_z^* is obtained from A_z by replacing b_m by $\hat{b}_m$, and $W_\theta(z,t)^{\mathrm{T}} = W_\varepsilon(z,t)\omega^{\mathrm{T}}$. Let us define $M(z,t) = W_\varepsilon(z,t)(l^{\mathrm{T}}/|l|^2)$ and consider $\zeta \triangleq z - M\varepsilon$ along the solutions of (57) and (38)

$$\dot{\zeta} = A_z\zeta + [W_\varepsilon e_2^{\mathrm{T}} - \dot{M} - M(A_o - \kappa_o|\omega|^2 l e_1^{\mathrm{T}}) + A_z M]\varepsilon + D\dot{\hat{\theta}} \tag{58}$$

where the bracketed expression and D are bounded. It is now straightforward to derive

$$\frac{d}{dt}(|\zeta|^2) \le -c|\zeta|^2 + \frac{1}{c}|W_\varepsilon e_2^{\mathrm{T}} - \dot{M} - M(A_o - \kappa_o|\omega|^2 l e_1^{\mathrm{T}}) + A_z M|_2^2|\varepsilon|^2 + \frac{\rho-1}{2g}|\dot{\hat{\theta}}|^2. \tag{59}$$

Since $\varepsilon, \dot{\hat{\theta}} \in \mathcal{L}_2$, it follows by [1, Theorem IV.1.9] that $\zeta \in \mathcal{L}_2$. Thus $z \in \mathcal{L}_2$. From (57) we can see that $\dot{z} \in \mathcal{L}_\infty$ which, along with $z \in \mathcal{L}_\infty \cap \mathcal{L}_2$ by Barbalat's lemma, proves that $z(t) \to 0$ as $t \to \infty$. Since $z_1 = y - y_r$, this proves the tracking. □

VI. Swapping-Based Scheme

A swapping-based identifier for (38) would seem to require filters of dynamic order $n(p+1)$. However, with a special choice of K_o in $A_o = A - K_o e_1^{\mathrm{T}}$ such that $\det(sI - A_o) = (s + c_o)(s+\lambda)^{n-1}$, we are able to design an identifier of the minimal dynamic order $p+1$. With this choice of K_o, (54), and the similarity transformation

$$\begin{bmatrix}\varepsilon_1\\ \zeta\end{bmatrix} \triangleq \begin{bmatrix}\varepsilon_1\\ T\varepsilon\end{bmatrix} \triangleq \begin{bmatrix} e_1^{\mathrm{T}} \\ A_l e_1, I_{n-1}\end{bmatrix}\varepsilon \tag{60}$$

(38) is decomposed into the scalar equation for ε_1

$$\dot{\varepsilon}_1 = -(c_o + \kappa_o|\omega|^2)\varepsilon_1 + \omega^{\mathrm{T}}\tilde{\theta} + \zeta_1 \tag{61}$$

and the $(n-1)$-dimensional uncontrollable exponentially stable part

$$\dot{\zeta} = A_l\zeta. \tag{62}$$

Now we design a parameter identifier only for (61). We introduce the filters

$$\dot{\Omega} = -(c_o + \kappa_o|\omega|^2)\Omega + \omega, \qquad \Omega \in \mathbb{R}^p \tag{63}$$

$$\dot{\overline{\Omega}} = -(c_o + \kappa_o|\omega|^2)\overline{\Omega} + \omega^{\mathrm{T}}\hat{\theta}, \quad \overline{\Omega} \in \mathbb{R} \tag{64}$$

and the estimation error

$$\epsilon = \varepsilon_1 + \overline{\Omega} - \Omega^{\mathrm{T}}\hat{\theta}. \tag{65}$$

Substituting (61), (63), and (64) into (65), we get

$$\epsilon = \Omega^{\mathrm{T}}\tilde{\theta} + \tilde{\epsilon} \tag{66}$$

where $\tilde{\epsilon}$ is governed by

$$\dot{\tilde{\epsilon}} = -(c_o + \kappa_o|\omega|^2)\tilde{\epsilon} + \zeta_1. \tag{67}$$

The parameter update law is either the gradient

$$\dot{\hat{\theta}} = \Gamma\frac{\Omega\epsilon}{1 + \nu|\Omega|_\Gamma^2}, \qquad \nu \ge 0 \tag{68}$$

or the least squares

$$\dot{\hat{\theta}} = \Gamma\frac{\Omega\epsilon}{1 + \nu|\Omega|_\Gamma^2}, \quad \dot{\Gamma} = -\Gamma\frac{\Omega\Omega^{\mathrm{T}}}{1 + \nu|\Omega|_\Gamma^2}\Gamma, \qquad \nu \ge 0 \tag{69}$$

where by allowing $\nu = 0$, we encompass unnormalized update laws.

Lemma 6.1: If Ω is piecewise continuous on $[0, t_f)$, then (68) and (69) guarantee that $\tilde{\theta}, \epsilon, \dot{\hat{\theta}} \in \mathcal{L}_\infty[0, t_f)$ and $\epsilon, \dot{\hat{\theta}} \in \mathcal{L}_2[0, t_f)$.

Proof (Outline): It is readily shown that

$$\frac{d}{dt}\left(\frac{1}{2}|\Omega|^2\right) \le -c_o|\Omega|^2 + \frac{1}{4\kappa_o} \tag{70}$$

which proves that Ω is bounded. For both the gradient and the least-squares update laws it can be shown that

$$\frac{d}{dt}\left(\frac{1}{c_o^2}|\zeta|_{P_l}^2 + \frac{1}{c_o}\tilde{\epsilon}^2 + \frac{1}{2}|\tilde{\theta}|_{\Gamma^{-1}}^2\right) \le -\frac{1}{2}\frac{\epsilon^2}{1 + \nu|\Omega|_\Gamma^2} \tag{71}$$

where $P_l = P_l^{\mathrm{T}} > 0$ satisfies $P_l A_l + A_l^{\mathrm{T}} P_l = -I$. The conclusions of the lemma are immediate from (71) and (70). □

Theorem 6.1 (Swapping-Based Scheme): All the signals in the closed-loop adaptive system consisting of the plant (1), the filters (3)–(6), the observer (37), the identifier filters (63)–(64), either the gradient (68) or the least-squares (69) update law, and the control law (30) are globally uniformly bounded for all $t \ge 0$, and global asymptotic tracking is achieved $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$.

Proof: With Lemmas 4.2 and 6.1, we establish the global uniform boundedness of all the signals. To prove the tracking, we first show that ε is square-integrable. Let us consider $\psi \triangleq \overline{\omega} - \Omega^{\mathrm{T}}\hat{\theta}$ which satisfies

$$\dot{\psi} = -(c_o + \kappa_o|\omega|^2)\psi - \Omega^{\mathrm{T}}\dot{\hat{\theta}}. \tag{72}$$

It can easily be seen that

$$\frac{d}{dt}\left(\frac{1}{2}\psi^2\right) \le -\frac{c_o}{2}\psi^2 + \frac{1}{2c_o}|\Omega^{\mathrm{T}}\dot{\hat{\theta}}|^2. \tag{73}$$

Since $\Omega^{\mathrm{T}}\dot{\hat{\theta}} \in \mathcal{L}_2$, then by [1, Theorem IV.1.9], $\psi \in \mathcal{L}_2$. In view of the fact that $\epsilon = \varepsilon_1 + \psi$, this proves that $\varepsilon_1 \in \mathcal{L}_2$. From (60) it follows that $\varepsilon = l\varepsilon_1 + \left[\begin{smallmatrix}0\\ I_{n-1}\end{smallmatrix}\right]\zeta$ which proves that $\varepsilon \in \mathcal{L}_2$. Following the same arguments as in the proof of Theorem 5.1, [cf. (57)–(59)], we show that $z \in \mathcal{L}_2$. The tracking is deduced via Barbalat's lemma. □

The flexibility to incorporate any of the standard parameter update laws in the swapping-based scheme is achieved at the expense of additional filters for the identifier.

VII. Transient Performance Analysis

We first derive $\mathcal{L}_\infty$-bounds for estimation-based schemes. To simplify the analysis, we let $\Gamma = \gamma I$, as well as $c_o = c$ and $\kappa_o = \kappa$. For the same reason, we implement $\hat{\chi}_1(0) = y(0)$ to get $\varepsilon_1(0) = 0$.

Theorem 7.1 (Observer-Based Scheme): In the adaptive system (1), (3)–(6), (37), (55), (30), the following inequality holds:

$$|z(t)| \le \frac{1}{\sqrt{c}}(M|\tilde{\theta}(0)| + N|\varepsilon(0)|) + |z(0)|\mathrm{e}^{-ct/2} \tag{74}$$

where M and N are nonincreasing functions of c, d, κ, g.

Proof: First, we note from (61) that

$$\frac{d}{dt}\left(\frac{1}{2}\varepsilon_1^2\right) \le -\frac{c}{2}\varepsilon_1^2\frac{\kappa}{2}|\omega|^2\varepsilon_1^2 + \frac{1}{2c}\zeta_1^2 + \frac{1}{2\kappa}|\tilde{\theta}|^2. \tag{75}$$

Combining (50) an (75) we compute

$$\begin{aligned}\frac{d}{dt}&\left(\frac{1}{2}|z|^2 + \frac{\rho\gamma^2}{4\kappa g}\varepsilon_1^2\right)\\ &\le -c|z|^2 + \frac{\rho}{4}\left(\frac{1}{d}\varepsilon_2^2 + \frac{1}{\kappa}|\tilde{\theta}|^2 + \frac{1}{g}|\dot{\hat{\theta}}|^2\right)\\ &\quad - c\frac{\rho\gamma^2}{4\kappa g}\varepsilon_1^2 - \frac{\rho\gamma^2}{4g}|\omega\varepsilon_1|^2 + \frac{\rho\gamma^2}{4c\kappa g}\zeta_1^2 + \frac{\rho\gamma^2}{4\kappa^2 g}|\tilde{\theta}|^2\\ &\le -\frac{c}{2}\left(|z|^2 + \frac{\rho\gamma^2}{2\kappa g}\varepsilon_1^2\right) + \frac{\rho}{4g}(|\dot{\hat{\theta}}|^2 - |\gamma\omega\varepsilon_1|^2)\\ &\le +\frac{\rho}{4}\left[\frac{1}{d}\varepsilon_2^2 + \frac{1}{\kappa}\left(1 + \frac{\gamma^2}{\kappa g}\right)|\tilde{\theta}|^2 + \frac{\gamma^2}{c\kappa g}\zeta_1^2\right]\end{aligned} \tag{76}$$

and, since the second term in the parentheses is zero, it is straightforward to obtain

$$|z(t)| \le \sqrt{\frac{\rho}{2c}}\left[\frac{1}{d}||\varepsilon_2||_\infty^2 + \frac{1}{\kappa}\left(1+\frac{\gamma^2}{\kappa g}\right)||\tilde{\theta}||_\infty^2 + \frac{\gamma^2}{c\kappa g}||\zeta_1||_\infty^2\right]^{1/2} + |z(0)|e^{-ct/2} \tag{77}$$

where we have used $\varepsilon_1(0) = 0$. Now we determine bounds on $||\varepsilon_2||_\infty, ||\tilde{\theta}||_\infty$ and $||\zeta_1||_\infty$. First, from (62) we have

$$\frac{d}{dt}(|\zeta|_{P_l}^2) = -|\zeta|^2 \tag{78}$$

which implies that $||\zeta_1||_\infty^2 \le [1/\underline{\lambda}(P_l)]|\zeta(0)|_{P_l}^2$. Since $\varepsilon_1(0) = 0$, then $\zeta(0) = T\varepsilon(0) = [0, I_{n-1}]\varepsilon(0)$ and it follows that:

$$||\zeta_1||_\infty^2 \le \frac{\overline{\lambda}(P_l)}{\underline{\lambda}(P_l)}|\varepsilon(0)|^2. \tag{79}$$

Along the solutions of (61), (62), and (55) we have

$$\frac{1}{2}\frac{d}{dt}\left(\varepsilon_1^2 + \frac{1}{c}|\zeta|_{P_l}^2 + \frac{1}{\gamma}|\tilde{\theta}|^2\right) \le -\frac{c}{2}\varepsilon_1^2 - \frac{\kappa}{\gamma^2}|\dot{\hat{\theta}}|^2 \tag{80}$$

which yields

$$||\tilde{\theta}||_\infty^2 \le |\tilde{\theta}(0)|^2 + \frac{\gamma\overline{\lambda}(P_l)}{c}|\varepsilon(0)|^2 \tag{81}$$

$$||\varepsilon_1||_\infty^2 \le \frac{1}{\gamma}\left(|\tilde{\theta}(0)|^2 + \frac{\gamma\overline{\lambda}(P_l)}{c}|\varepsilon(0)|^2\right). \tag{82}$$

To obtain a bound on $||\varepsilon_2||_\infty$, from (60) we recall that $\varepsilon_2 = (n-1)\lambda\varepsilon_1 + \zeta_1$ which, by virtue of (79) and (82), shows that

$$||\varepsilon_2||_\infty^2 \le \frac{2(n-1)^2\lambda^2}{\gamma}|\tilde{\theta}(0)|^2 + \left(\frac{2(n-1)^2\lambda^2\overline{\lambda}(P_l)}{c} + 2\frac{\overline{\lambda}(P_l)}{\underline{\lambda}(P_l)}\right)|\varepsilon(0)|^2. \tag{83}$$

Substituting (79), (81), and (83) into (77), we arrive at (74) with

$$M = \sqrt{\rho}\left[\frac{1}{2\kappa}\left(1+\frac{\gamma^2}{\kappa g}\right) + \frac{(n-1)^2\lambda^2}{d\gamma}\right]^{1/2} \tag{84}$$

$$N = \sqrt{\rho\overline{\lambda}(P_l)}\left[\frac{\gamma M^2}{c\rho} + \frac{1}{\underline{\lambda}(P_l)}\left(\frac{1}{d} + \frac{\gamma^2}{2c\kappa g}\right)\right]^{1/2}. \tag{85}$$

Now we consider the swapping-based scheme. For simplicity, we initialize $\overline{\Omega}(0) = -\varepsilon_1(0) = 0$ and $\Omega(0) = 0$ to set $\tilde{\epsilon}(0), \Omega(0)$ to zero.

Theorem 7.2 (Swapping-Based Scheme): In the adaptive system (1), (3)–(6), (37), (63)–(64), (68), (30), the following inequality holds:

$$|z(t)| \le \frac{1}{\sqrt{c}}(M|\tilde{\theta}(0)| + N|\varepsilon(0)|) + |z(0)|e^{-ct} \tag{86}$$

where M and N are nonincreasing functions of c, d, κ, g.

Proof: We derive an $\mathcal{L}_\infty$-bound on z using (50) rewritten as

$$|z(t)| \le \sqrt{\frac{\rho}{4c}}\left(\frac{1}{d}||\varepsilon_2||_\infty^2 + \frac{1}{\kappa}||\tilde{\theta}||_\infty^2 + \frac{1}{g}||\dot{\hat{\theta}}||_\infty^2\right)^{1/2} + |z(0)|e^{-ct}. \tag{87}$$

It remains to determine bounds on $||\varepsilon_2||_\infty, ||\tilde{\theta}||_\infty$ and $||\dot{\hat{\theta}}||_\infty$. First, from (71), using $\tilde{\epsilon}(0) = 0$ and $|\zeta(0)|_{P_l}^2 \le \overline{\lambda}(P_l)|\varepsilon(0|^2$, we get

$$||\tilde{\theta}||_\infty^2 \le |\tilde{\theta}(0)|^2 + \frac{2\gamma\overline{\lambda}(P_l)}{c^2}|\varepsilon(0)|^2. \tag{88}$$

Noting that (61) yields

$$\frac{d}{dt}\left(\frac{1}{2}\varepsilon_1^2\right) \le -\frac{c}{2}\varepsilon_1^2 + \frac{1}{4\kappa}|\tilde{\theta}|^2 + \frac{1}{2c}\zeta_1^2 \tag{89}$$

we obtain

$$||\varepsilon_1||_\infty^2 \le \frac{1}{2c\kappa}||\tilde{\theta}||_\infty^2 + \frac{1}{c^2}||\zeta_1||_\infty^2. \tag{90}$$

Recalling that $\varepsilon_2 = (n-1)\lambda\varepsilon_1 + \zeta_1$, using (79) and (90), we get

$$||\varepsilon_2||_\infty^2 \le \frac{(n-1)^2\lambda^2}{c\kappa}||\tilde{\theta}||_\infty^2 + 2\left(1+\frac{(n-1)^2\lambda^2}{c^2}\right)\cdot\frac{\overline{\lambda}(P_l)}{\underline{\lambda}(P_l)}|\varepsilon(0)|^2. \tag{91}$$

From (68) we write

$$|\dot{\hat{\theta}}|^2 \le \gamma^2\frac{|\Omega|^2\epsilon^2}{(1+\nu|\Omega|^2)^2} \le \gamma^2||\Omega||_\infty^2\frac{\epsilon^2}{(1+\nu|\Omega|^2)^2} \le \gamma^2||\Omega||_\infty^2||\epsilon||_\infty^2 \tag{92}$$

and by substituting (66) obtain

$$||\dot{\hat{\theta}}||_\infty^2 \le 2\gamma^2||\Omega||_\infty^2(||\Omega||_\infty^2||\tilde{\theta}||_\infty^2 + ||\tilde{\epsilon}||_\infty^2). \tag{93}$$

From (70), using $\Omega(0) = 0$, it follows:

$$||\Omega||_\infty^2 \le \frac{1}{4c\kappa}. \tag{94}$$

To obtain a bound on $||\tilde{\epsilon}||_\infty$, along (67) and (62) we consider

$$\frac{d}{dt}\left(\frac{1}{2}|\tilde{\epsilon}|^2 + \frac{1}{4c}|\zeta|_{P_l}^2\right) \le -c\tilde{\epsilon}^2 + \tilde{\epsilon}\zeta_1 - \frac{1}{4c}|\zeta|^2 \le 0 \tag{95}$$

which yields

$$||\tilde{\epsilon}||_\infty^2 \le \frac{\overline{\lambda}(P_l)}{2c}|\varepsilon(0)|^2. \tag{96}$$

By substituting (88) into (90) and also (88), (94), and (96) into (93), and then the two results, along with (88) into (87), we arrive at (86) with

$$M = \sqrt{\frac{\rho}{4\kappa}}\left(1 + \frac{\gamma^2}{8c^2\kappa g} + \frac{(n-1)^2\lambda^2}{cd}\right)^{1/2} \tag{97}$$

$$N = \sqrt{\frac{\rho\overline{\lambda}(P_l)}{2}\left[\frac{4\gamma M^2}{c^2\rho} + \frac{\gamma^2}{8c^2\kappa g} + \frac{1}{\underline{\lambda}(P_l)}\cdot\left(1+\frac{(n-1)^2\lambda^2}{c^2}\right)\right]^{1/2}}. \tag{98}$$

□

Although the initial states $z_2(0), \cdots, z_\rho(0)$ may depend on c and d, this dependence can be removed by setting $z(0) = 0$ with the standard trajectory or reference model initialization explained in [10]. From (74) and (86) it is evident that the $\mathcal{L}_\infty$-performance bounds in both the observer-based scheme and the swapping-based scheme can be made as small as desired by initializing $z(0) = 0$ and increasing c.

To obtain a similar $\mathcal{L}_\infty$-bound for the tuning functions scheme, the design has to be augmented with nonlinear damping terms. Unlike for the estimation-base schemes, for the tuning functions scheme one can also derive an $\mathcal{L}_2$-bound

$$||z||_2 \le \frac{1}{\sqrt{2c}}\left(|z(0)|^2 + \gamma_\varepsilon|\varepsilon(0)|_{P_o}^2 + |\tilde{\theta}(0)|_{\Gamma^{-1}}^2 + \frac{|b_m|}{\gamma_\varsigma}\tilde{\varsigma}(0)^2\right)^{1/2}. \tag{99}$$

VIII. Conclusions

The adaptive schemes proposed in this paper advance the state-of-the-art of adaptive nonlinear output-feedback control in several directions. They remove the main drawbacks of the original Marino–Tomei design. Only the minimal number of parameters is updated, and any standard update law can be incorporated in the swapping-based scheme. The estimation-based approach can now be used for adaptive nonlinear output-feedback control without any growth restrictions. The modifications made in the Marino–Tomei controller make it possible to systematically improve the transient performance by increasing certain design parameters.

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Adaptive Control of a Class of Decentralized Nonlinear Systems

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***Abstract*—Within this brief paper, a stable indirect adaptive controller is presented for a class of interconnected nonlinear systems. The feedback and adaptation mechanisms for each subsystem depend only upon local measurements to provide asymptotic tracking of a reference trajectory. In addition, each subsystem is able to adaptively compensate for disturbances and interconnections with unknown bounds. The adaptive scheme is illustrated through the longitudinal control of a string of vehicles within an automated highway system (AHS).**

I. Introduction

Decentralized control systems often arise from either the physical inability for subsystem information exchange or the lack of computing capabilities required for a single central controller. Furthermore, difficulty and uncertainty in measuring parameter values within a large-scale system may call for adaptive techniques. Since these restrictions encompass a large group of applications, a variety of decentralized adaptive techniques have been developed. Model reference adaptive control (MRAC)-based designs for decentralized systems have been studied in [1]–[4] for the continuous time case and in [5] and [6] for the discrete time case. These approaches, however, are limited to decentralized systems with linear subsystems and possibly nonlinear interconnections. Decentralized adaptive controllers for robotic manipulators were presented in [7]–[9], while a scheme for nonlinear subsystems with a special class of interconnections was presented in [10].

Our objective is to present adaptive controllers for a class of decentralized systems with nonlinear subsystems, unknown nonlinear interconnections, and disturbances with unknown bounds. This paper is organized as follows: In Section II, the details of the problem statement for the decentralized system are presented. The adaptive algorithms for each subsystem using only local information are presented, and composite system stability is established in Section III. An illustrative example is then used in Section IV to demonstrate the effectiveness of the decentralized adaptive technique.

II. Problem Statement

Our objective is to design an adaptive control system for each subsystem which will cause the output, y_{p_i}, of a relative degree r_i subsystem, S_i, to track a desired output trajectory, y_{m_i}, in the presence of interconnections, I_{ij}, and unknown disturbances using only local measurements (see Fig. 1). The desired output trajectory, y_{m_i}, may be defined by a signal external to the control system so that the first r_i derivatives of the ith subsystem's reference signal y_{m_i} may be measured or by a reference model with relative degree greater than or equal to r_i which characterizes the desired performance. It is thus assumed that the desired output trajectory and its derivatives $y_{m_i}, \cdots, y_{m_i}^{(r_i)}$ for the ith subsystem, S_i, are measurable and bounded (let $y_{m_i}^{(r_i)}$ denote the r_ith derivative of y_{m_i} with respect to time). Within this paper an "output error indirect adaptive controller" is

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