Adaptive Regularization Algorithm Paired with Image Segmentation

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Abstract. This paper presents the subsequent improvements of our adaptive regularization algorithm. The current version of the algorithm contains an additional stabilizer. This stabilizer includes the coefficient, which utilizes prior information about segmentation of an image to be processed. We demonstrate that our adaptive regularization algorithm can exploit only the observed image segmentation for both the adaptive coefficient calculation and successful restoration of the original image segmentation.

Keywords: image restoration, Tikhonov regularization, adaptation, image segmentation

1 Introduction

In the model experiments, see [1-4], we demonstrated, that our adaptive regularization algorithm can successfully restore and provide more deblurred and denoisy intensity of the original model image. In this paper, we improve our algorithm, which is based on the results from [5-7], by an additional stabilizer. This stabilizer includes the coefficient, which utilizes some information about segmentation of an image to be restored.

Most artificial objects are made of surfaces, resulting in images with several subimages (or a single subimage) together with their boundary lines.

These facts motivate us to use ideas from image segmentation in order to improve our algorithm. In the sequel, we describe the stabilizer coefficient construction, using ideas from image segmentation.

Image segmentation algorithms are based on properties of intensity values: discontinuity and similarity, see [8-11]. Starting with the observed object intensity, setting $\beta(x, y) \equiv 0$ in our adaptive algorithm, we successfully apply the original approach for both sub-images and their boundaries. In the next iterations, the adaptive algorithm utilizes this initial result for calculation of the coefficient β.

Thus, we consider image segmentation both as result and tool in every image restoration.

We prove numerically that the proposed algorithm can reveal the latent image segmentation of observed images.

The paper is organized as follows. In Section 2, we describe regularizing functionals and present our adaptive technique. In Section 3, we describe the calculation process of our stabilizer coefficient. The last section concludes the paper.

2 Mathematical Model for Image Deblurring

We consider the following two-dimensional Fredholm integral equation of the first kind:

$$Au \equiv \int_0^1 \int_0^1 K(x-\xi, y-\eta) u(x,y) dx, dy = f(\xi,\eta). \tag{1}$$

In image restoration, the estimation of u from the observation of f is referred to as the two-dimensional image deblurring problem.

We construct and develop the novel technique for numeric solution of equation (1). Abstract methods with convergence analysis of regularization for this problem are presented in [3-7].

The foundation of the regularization method is given by

$$\min \left\{ ||A_h u - f_\delta||_{L_2}^2 + \alpha \left(||u||_{L_2}^2 + J(u) \right) : \ u \in U \right\}, J(u) = \int_D |\nabla u| \, dx, \tag{2}$$

where ∇u denotes the gradient of smooth function u, $(u \in W_1^1(D))$, $J(u)$ is the total variation of the function u on D. The practical implementation of this method requires minimization of the functional in (2).

The novelty was that we proposed to add the version of the iterative technique, containing additional parameters $\beta_{i,j}$, $\beta_{i,j} \geq 0$, see [1-4]:

$$\mathbf{u}^k = \arg\min \{ \varPhi_N^\alpha(\mathbf{u}) + \sum_{i,j} \beta_{i,j} \, (u_{i,j} - u_{i,j}^{k-1})^2 \ : \mathbf{u} \in R^N \} \,. \tag{3}$$

Here, $N = n^2$, $\varPhi_N^\alpha(\mathbf{u})$ is the discrete form of the functional in (2).

$\sum\limits_{i,j} \beta_{i,j} \, (u_{i,j} - u_{i,j}^{k-1})^2$ is a discrete form for the additional stabilizer in (3).

We use the iterative subgradient method in order to compute $\mathbf{u}^k$ defined in (3), see details in [1-4].

3 Stabilizer coefficient selection

3.1 Image Segmentation: Motivation

The previous version of the algorithm (3) used the constant β in every mesh point of discrete models. The low resulting accuracy of this two-dimensional model led us to change the way of $\{\beta_{i,j}\}$ selection.

As we said above, most man-made objects are made of surfaces, resulting in images with a single image or some sub-images. The quality of the images restoration features the quality of sub-images restoration together with sub-image boundary lines restoration.

In [1], we proposed the more general form of the stabilizer I^β:

$$I^\beta = \int\limits_Q \beta(x,y)[u(x,y) - u^k(x,y)]^2 \, dxdy, \tag{4}$$

we used

$$\begin{cases} \beta(x,y) = u_{\max}, & \text{for} \quad (x,y) \in Q, \\ \beta(x,y) = 0, & \text{for} \quad \mathrm{(x,y) \in \Pi/Q}, \quad \text{where} \end{cases} \tag{5}$$

$$u_{\max} = \max\{u_{\text{true}}(x,y)\,, (x,y) \in \mathit{\Pi} = [0,1] \times [0,1]\}, \tag{6}$$

$$(x,y) \in Q \Leftrightarrow u_{\text{true}}(x,y) = u_{\max}. \tag{7}$$

So, in [1] our regularization algorithm stabilizer includes the coefficient β, which uses some information about true image segmentation, see (6) and (7). This facts motivates us to exclude true image parameters. We approach this purpose using image segmentation ideas.

3.2 Image Segmentation as Tool

In this paper we propose to begin calculations with the parameter $\beta \equiv 0$.

So, in this case, we use standard regularization method:

$$\min\left\{||A_h u - f_\delta||^2_{L_2} + \alpha\,(||u||^2_{L_2} + J(u)): \; u \in U\right\}\,, J(u) = \int\limits_D |\nabla u| \, dx, \tag{8}$$

where ∇u denotes the gradient of smooth function u, $J(u)$ is the total variation of the function u on D.

After obtaining the solution of problem (8), we get approximate $\tilde{u}^0(x,y)\,, Q^0\,, \mathit{\Gamma}_{Q^0}$, and we use them for the $\beta_{i,j}$ construction:

$$\beta_{i,j} \in \{o;c\},\; c = max\{\tilde{u}^0(x,y), \quad (x,y) \in Q^0 \cup \mathit{\Gamma}_{Q^0}\}. \tag{9}$$

3.3 Image Segmentation as Result

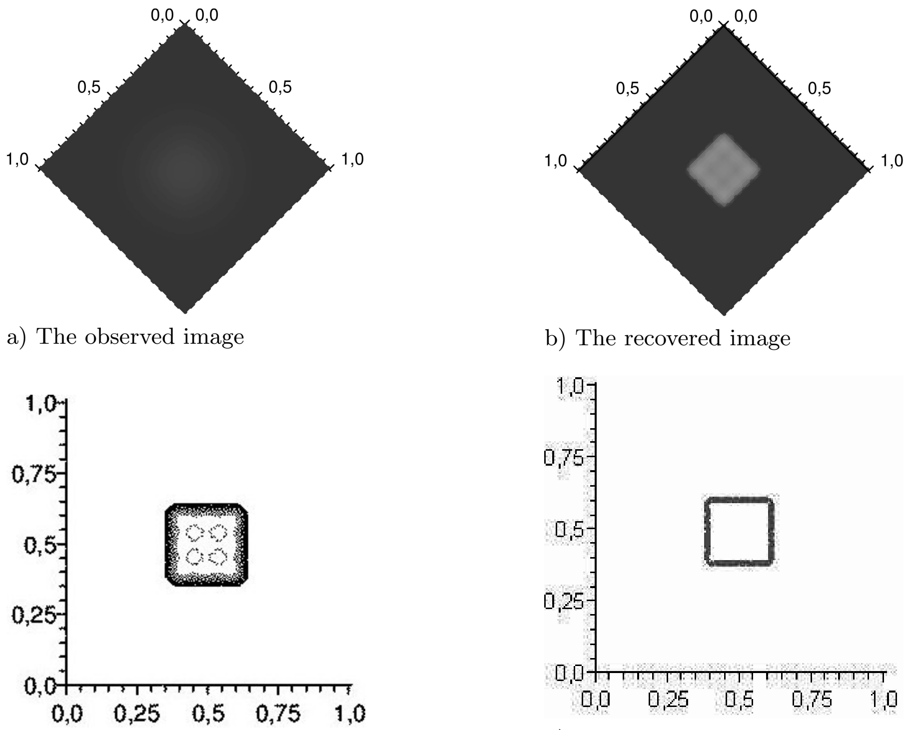
Now, using β calculated in (9), the algorithm computes the final image restoration including its final image segmentation.

Numerical experiments have confirmed our ideas. Thus, we see in Figure 1:

(a) the observed image; (b) the restoration image;

(c) the central image segment restoration, $Q^0 \cup \Gamma_{Q^0}$, calculated with $\beta \equiv 0$;

(b) the central image segment restoration, $\tilde{Q} \cup \tilde{\Gamma}_{\tilde{Q}}$, calculated with adapted β.



a) The observed image

b) The recovered image

c) The model segment restoration for $\beta(x, y) \equiv 0$

d) The model segment restoration for the adapted stabilizer coefficient $\beta(x, y)$

Fig. 1. Model experiments

It is clear, that we can write the final image segmentation of the recovered image in the form:

$$\Pi = \{\Pi / \tilde{Q}\} \cup \tilde{Q}, \tag{10}$$

were $\tilde{Q} \cup \tilde{\Gamma}_{\tilde{Q}}$ we see in Figure 1.(d), $\Pi = [0, 1] \times [0, 1]$.

3.4 Conclusion

In this paper, we have described and verified the image segmentation approach for the construction of stabilizer coefficient β.

Starting with the observed object intensity and setting the stabilizer coefficient $\beta \equiv 0$ in the regularization algorithm, we successfully apply the initial approach for both the sub-image and the sub-image boundary, Q^0, Γ_{Q^0}.

After that, we use calculated results for construction of the adaptive stabilizer coefficient β. At this step the image segmentation is a tool.

Using the calculated coefficient β, the algorithm computes the final recovered image. We obtain a good localization of segments, $\tilde{Q}$, $\Gamma_{\tilde{Q}}$, and we see a more deblurred final image segmentation.

As a result, in this paper, we have described and verified that our adaptive algorithm does not need a priori information. For successful image restoration and restoration of image segmentation, our adaptive algorithm can utilize the image of an observed object only.

We plan the following modifications and experiments:

- inclusion of the solution estimate for equation $u(x, y, \Gamma_Q(x, y)) = c$ to the estimate list;
- validation of β^k inclusion in (4), where k is the iteration number;
- processing of thin lines and narrow segments.

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