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Improvement Least-Distance Measure Model with Coplanar DMU on Strong Hyperplanes

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Abstract Technique of Data Envelopment Analysis (DEA) involves methods conducted for desirable objective management of Decision Making Unit (DMU) that is same increasing of efficiency level. Data envelopment analysis furthermore determines the efficiency level, provides situation, removes inefficiency with evaluated benchmarking information. In this paper the use of the improvement Least-Distance measure with relation previous model by coplanar DMU, is proposed for computational dissipation at assess distance on these interior combinations, for determination the shortest projection from a considered unit to the strongly efficient production frontier. Therefore locate nearest path to improvement efficiency the evaluated DMU.

Keywords Data Envelopment Analysis, Least Distance, Coplanar DMU, Benchmarking.

1 Introduction

Data Envelopment Analysis (DEA), first proposed by Charnes, is a non-parametric approach to evaluate the performance or efficiency of various organizations in public and private sectors with multiple inputs and multiple outputs. DEA is a mathematical programming approach that uses the production frontiers to evaluated relative efficiency. If Decision making unit (DMU) lies on production frontiers, DMU is efficient, otherwise DMU is inefficient. Each inefficient DMU afforded to be efficient, efforts a manager with find at efficient frontier improvement inefficiency of unit. Some models in DEA practice with delineate inefficient DMU upon efficient frontier. The efficiency is evaluated by distance between the observed DMU and the reference DMU, which serves as a benchmarking target.

The DEA models may be generally classified into radial and non-radial models. The radial models include the CCR ratio form (Charnes-Cooper-Rhodes) and BCC model (Banker-Charnes-Cooper). Meanwhile, the non-radial models include an additive model, multiplication model, range-adjusted measure and slack-based measure [1]. Look like possible scheme range of delineate, base of distance, regarding choices more like DMU on frontier for reference, for this reason proposed by C. Beak and J. Lee Least-Distance measure [2]. Least-Distance Measure practices computations by definition strong efficient set, against other models which work with supporting hyperplanes or pareto efficient faces.

Really each inefficient DMU seeks for improvement situation for comparison and achievement to position after an efficient DMU, which efficient DMU on efficient surfaces

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and preferably in this process effort inefficient DMU relative adjusted with pareto efficient frontier. In regarded to the Least-Distance model (part 3 of paper) evaluated the all combinations of m+s components from pareto efficient unit and calculated the distance of non-efficient of all combination. In this paper, try to elimination the excess combinations of pareto efficient units, with introduction coplanar DMU on the PPS defining hyperplanes [3]. Gouyeia point out that the efficiency measures of previous DEA models are very dankly in that their explanation are not intuitive, concerning the l_1-norm properties [4]. Sapienza present the efficiency of learning will be formulated by the degree to which there is an overlap between the knowledge bases of the two firms involved in the learning relationship [5]. Yi require the similarity of benchmarks [6], and calculate the Euclidian distance for determine the degree of similarity of benchmarks. Gonzalez and Alvarez introduce the shortest path to the efficient subset [7]. Bogetoft and Houggard suggest that the closest DMU is the reference point [8]. Post and Spronk suggest the use of interactive DEA [9] and Coelli introduce the multi-stage DEA for finding the nearest efficient point [10]. Also, Frei and Harker extend the Least-norm projection for the inefficient DMU of the efficient frontier [11].

The reminder of this paper is organized as follows. Section 2 introduces the DEA models, section 3 present the Least-Distance Measure for evaluated the benchmarking. In section 5 an illustrative application is presented and discussed in section 6.

2 DEA

Suppose we have a set of units, $DMU_j\,(j=1,...,n)$. Each DMU uses m inputs $x_{ij}\,(i=1,...,m)$ to produces s outputs $y_{rj}\,(r=1,...,s)$.Then the efficiency of DMU_j can be expressed as

$$E_j=\frac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}}$$

Where u_r and v_i are output and input multipliers, respectively. In DEA, E_j is obtained by solving the following additive model [1].

$$\begin{aligned}
&Max\quad es^- + es^+\\
&s.t\quad X\lambda + s^- = x_o,\\
&\qquad Y\lambda + s^+ = y_o,\\
&\qquad e\lambda = 1,\\
&\qquad \lambda \ge 0,\ s^- \ge 0,\ s^+ \ge 0.
\end{aligned}\tag{1}$$

Theorem 1. DMU_o is ADD-efficient, if and only if is the BCC-efficient [1].

3 Least-Distance measure

Least-Distance measure practice computations by definition strong efficient set, against another models which works with supporting hyperplanes or pareto efficient faces.

Definition 1. The Production Possibility Set (pps) will be represented as

$$T = \{(x, y) | \text{y can produced by x}\}$$

Definition 2. The set of observations satisfying the pareto efficiency conditions is defined as a strongly efficient set, E, such that;

$$E = \{(x, y) | \max(e^t s^- + e^t s^+) = 0$$
$$s.t \quad (s^+, s^-) = (x - X\lambda, Y\lambda + y), e^t\lambda = 1, \lambda \geq 0\}$$

Where

$$e^t = (1,1,...,1),$$
$$e^t s^+ = \sum_{r=1}^{s} s_r^+, e^t s^- = \sum_{i=1}^{m} s_r^-$$

The objective function of Least-Distance Measure introduce the distance between the evaluated DMU (x^o, y^o), and the strongly efficient set (E), into an efficiency measure, and can described as follows,[13]

$$\theta = \max\left[1 - \frac{1}{m+s}\left\{\sum_{i=1}^{m}\left(\frac{x_i - x_i^o}{R_i^-}\right)^2 + \sum_{r=1}^{s}\left(\frac{y_r - y_r^o}{R_r^+}\right)^2\right\}^{1/2}\right]$$
$$s.t \quad (x, y) \in E$$

where $R_i^- = \max_j \{x_{ij}\} - \min_j \{x_{ij}\}$

$$R_r^+ = \max_j \{y_{rj}\} - \min_j \{y_{rj}\} \quad (2)$$

where m is the number of input variables, s is the number of output variables, x_i^o is i-th input of input vector x^o and y_r^o is rth output of output vector y^o.

In Least-Distance Measure describe the strongly efficient set primarily and then obtain distances of evaluated DMU at all combinations of m+s components of set E (this is based on the reality that a point on a facet of the production frontier of the m+s dimension can be showed by a linear combination of m+s members of set E [12]), and put into Least-Distance Measure, the first efficient benchmark with nearest distance and calculated measure of θ.

4 Least-Distance measure by coplanar DMU

Like as mentioned, in Least-Distance Measure find distances of evaluated DMU at all combinations of m+s components the strongly efficient set which also involved combinations inside of Production Possibility Set. Same as presented in Fig. 1 will be some of this combinations into of PPS, whereof in Least-Distance Measure selected if efficient point for benchmark, therefore with add the result projection upon this combinations to Production Possibility Set, will be again inefficient DMUs and they can't considered for benchmarking. Hence this model will be involve computational dissipation at assess distance on this interior combinations. At first, the proposed method found coplanar DMUs on strong defining hyperplanes [1], then just calculated distance of the evaluated DMU to defining hyperplanes in Least-Distance Measure model. Certainly after sorting the array in increasing order based on objective value (distances of evaluated DMU at defining hyperplanes), is selected initial point for chosen benchmark.

Theorem 2. Let (x_p, y_p) and (x_q, y_q) be observed DMUs that lie on a strong supporting hyperplane, then each convex combination of them is on the same hyperplane [1].

Theorem 3. Consider (x_p, y_p) and (x_q, y_q) are two observed DMUs lie on different hyperplanes (excluding their intersection, if it is not empty). Then every point (virtual DMU) which obtained by strict convex combination of them is an interior point of PPS. In other words this virtual DMU is radial inefficient [1].

4.1 The algorithm of Least-Distance measure by coplanar DMU

Step 1. Considered n DMUs, solve the additive DEA model for each DMU and divided each DMU as either pareto efficient or inefficient. Locate indexes of pareto efficient DMUs in F and suppose $|F| = L$.

Step 2. Evaluate for each index $p, q \in F$ that $p \neq q$.

$$DMU_k = \frac{1}{2} DMU_p + \frac{1}{2} DMU_q$$

If DMU_k is efficient, $p \in F_p, q \in F_q$.

F_p , set indexes of coplanar DMUs with DMU_p . F_q , set indexes of coplanar DMUs with DMU_q .

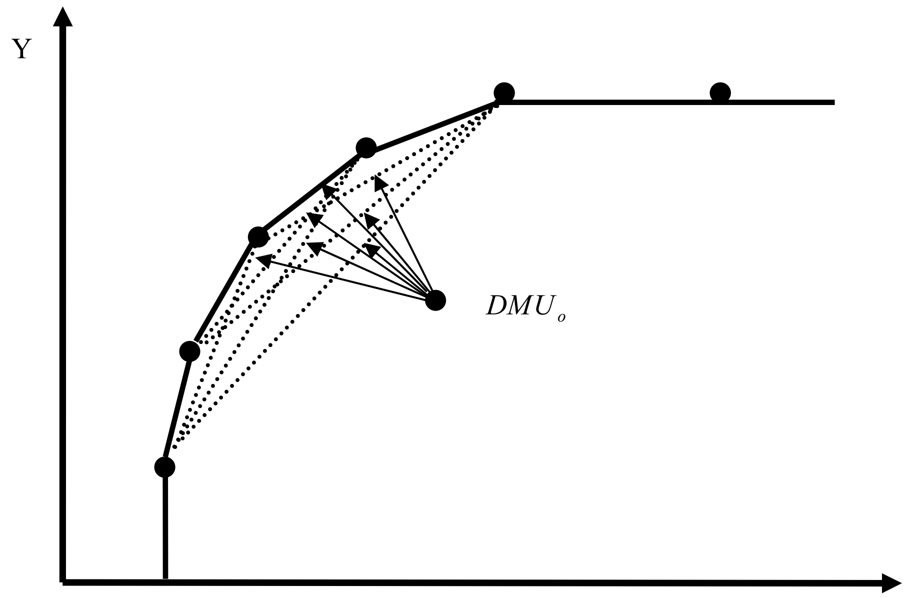

Fig. 1 Projection evaluated DMU upon pareto points interior combinations

Step 3. Evaluate the, $\bar{F}_j = F - F_j \quad j = 1,...,L$.

Step 4. Select the arbitrary m+s members of F such that neither of them belongs to some others $\bar{F}$. Introduce set $D = \{j_1, j_2, ..., j_{m+s}\}$ that input and output matrixes can be described as follows.

$$\begin{pmatrix} x_{1j_1} & \cdots & x_{1j_{m+s}} \\ \vdots & \ddots & \vdots \\ x_{mj_1} & \cdots & x_{mj_{m+s}} \end{pmatrix} , \begin{pmatrix} y_{1j_1} & \cdots & y_{1j_{m+s}} \\ \vdots & \ddots & \vdots \\ y_{mj_1} & \cdots & y_{mj_{m+s}} \end{pmatrix}$$

$x_{ij_t} (i = 1,...,m; t = 1,...,m+s)$ i-th input of DMU_{j_t}, $y_{rj_t} (r = 1,...,s; t = 1,...,m+s)$ r-th output of DMU_{j_t}. Presently have the coplanar DMUs on strong supporting hyperplanes.

Step 5. Compute the following model for each combination.

$$\begin{aligned} \text{Min} = & \sum_{i=1}^{m} \left(\frac{x_i - x_i^o}{R_i^-} \right)^2 + \sum_{r=1}^{s} \left(\frac{y_r - y_r^o}{R_r^+} \right)^2 \\ s.t \quad & x = x_k^F \lambda \\ & y = y_k^F \lambda \\ & e^t \lambda = 1 \\ & \lambda \ge 0 \end{aligned} \qquad (3)$$

where x_k^F input matrix of k-th combination of set F and y_k^F output matrix of k-th combination of set F.

Step 6. Sort the optimal value of step 5 in increasing, then selected initial point (x, y) correspondent of chosen objective value, for benchmarking.

Step 7. (x, y) is the nearest projection point from evaluated DMU to the strongly efficient set F, so put in model (3) and obtain the efficiency measure of the Least-Distance Measure.

5 Example

We apply our approach to the data set of 23 public libraries in Tokio: L1 to L23 [5], in total, 4 inputs and 2 outputs were employed. The inputs were floor area (unit=1000 m^2) [Area], the number of books (unit=1000) [Book], staffs (unit=1000) [staff], and the population of the area (unit=1000) [Population]. The outputs were the number of registered residents (unit=1000) [register] and the number of borrowed books (unit=1000) [borrow]. Table 1 provides the data for example.

By following the algorithm of the Least-Distance Measure, libraries 1,2,5,6,9,15,17,19,23 constituted the set F in initial. 49 combinations for each inefficient DMU are obtained. Present calculated the distance of evaluated DMU (inefficient DMU) at all combinations and selected for benchmarking wherein existence least measure with objective value. In Table (2) presented the result of Least-Distance Measure.

Table 1 Public library data

Library	Part	Area	Book	Staff	Population	Register	Borrow
L1	Chiyoda	2249	163523	26	49196	5561	105321
L2	Chuo	4617	338671	30	78599	18106	314682
L3	Taito	3873	281655	51	176381	16498	542349
L4	Arakawa	5541	400993	78	189397	30810	847872
L5	Minato	11381	363116	69	192235	57279	758704
L6	Bunkyo	10086	541658	114	194091	66137	1438746
L7	Sumida	5435	508141	61	228535	35295	839597
L8	Shibuya	7524	338804	74	238691	33188	540821
L9	Megura	5077	511467	84	267385	65391	1562274
L10	Toshima	7029	393815	68	277402	41197	978117
L11	Shinjuku	11121	509682	96	330609	47032	930437
L12	Nakano	7072	527457	92	332609	56064	1345185
L13	Shinagawa	9348	601594	127	356504	69536	1164801
L14	Kita	7781	528799	96	365844	37467	1348588
L15	Koto	6235	394158	77	389894	57727	1100779
L16	Katushika	10593	515624	101	417513	46160	1070488
L17	Itabashi	10866	566708	118	503914	102967	1707645
L18	Edogawa	6500	467617	74	517318	47236	1223026
L19	Suginama	11469	768484	103	537746	84510	2299694
L20	Nerima	10868	669996	107	590601	69576	1901465
L21	Adachi	10717	844949	120	622550	89401	1909698
L22	Ota	19716	1258981	242	660164	97941	3055193
L23	Setagaya	10888	1148863	202	808369	191166	4096300

For example , in the case of library 4 (arakawa) come namely the benchmarking, 5913 floor area, 430925 the number of books, 63 number of staffs, 166771 the population of the area, 42795 number of registered residents and 892155 the number of borrowed books.

Table 2 The result of the Least-Distance Measure

Library	θ	Benchmarking value						Combination used
		Area	Book	Staff	Population	Register	Borrow	
L1	1	2249	163523	26	49196	5561	105321	
L2	1	4617	338671	30	78599	18106	314682	
L3	0.9693	4617	338671	30	78599	18106	314682	{1,2,9,15,17,19}
L4	0.9823	5913	430925	63	166771	42795	892155	{2,5,6,9,17,23}
L5	1	11381	363116	69	192235	57279	758704	
L6	1	10086	541658	114	194091	66137	1438746	
L7	0.9831	5511	421525	60	208323	45109	922672	{2,5,6,9,15,17}
L8	0.9833	7239	329436	62	229797	45001	738535	{1,5,6,9,15,17}
L9	1	5077	511467	84	267385	65391	1562274	
L10	0.9833	6866	437896	68	237647	53699	847954	{1,2,6,9,15,19}
L11	0.9653	11148	477886	94	345512	81412	1281234	{1,2,5,17,19,23}
L12	0.9910	7328	499820	86	325933	61661	1411905	{1,2,6,9,15,19}
L13	0.9803	9896	550526	113	338326	82375	1309262	{2,5,6,9,17,23}
L14	0.9775	7899	509151	80	343362	57302	1391158	{1,2,6,9,15,19}
L15	1	6235	394158	77	389894	57727	1100779	
L16	0.9705	9809	506136	83	368875	66779	1373958	{2,5,9,15,17,19}
L17	1	10866	566708	118	503914	102967	1707645	
L18	0.9717	6793	434047	80	405649	60581	1228537	{2,9,15,17,19,23}
L19	1	11469	768484	103	537746	84510	2299694	
L20	0.9773	10601	692259	100	511725	81987	2060387	{1,2,9,15,17,19}
L21	0.9719	11026	769781	113	554609	95593	2369728	{2,5,15,17,19,23}
L22	0.7424	10675	987464	179	645090	157932	3389904	{1,5,6,9,17,23}
L23	1	10888	1148863	202	808369	191166	4096300	

With Least-Distance Measure previous, must have estimated 84 combinations, while the present method has better efficiency in high dimensions. Hence obtaining for example to finding the nearest distance 686 models, whereas evaluate for previous model the 1176 model.

6 Conclusion

Efficiency value and benchmarking information can be obtained by DEA models which provid the situation for eliminating inefficiency and improvement of the state of inefficient unit for manager decision that is a process for finding unit upon the efficiency frontier the Least-Distance Measure model.

Used at concept the combinations of m+s to find the Projection evaluated DMU (inefficiency DMU) upon the strongly efficient frontier in the Least-Distance Measure model, located into the PPS space the number of this combination and is inefficient the Projection of evaluated DMU upon their. Therefore by introduction the coplanar DMU upon the strongly efficient hyperplanes obviated this failure .The provided discussions can be a development in fuzzy and interval data.

For example in data on 14 general hospital from Tones article [7], must be evaluated the distance of 70 combination with the Least-Distance Measure of model (3), whereas need to the evaluated the distance of 57 combination, that solve less than of 78 model, with the rendered model in this article.

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