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Derivation of Stress Intensification Factors for a Special, Contoured, Integrally Reinforced Branch Connection

The results of an extensive experimental investigation of a contoured, integrally reinforced branch connection in a cylindrical pressure vessel (or run pipe) have been reported [1].[1] One size model, specifically a 12 in. (0.375) × 6 in. (0.280) standard weight header was studied by three-dimensional photoelasticity using the stress-freezing and slicing technique. Loads applied were internal pressure, plus in-plane and out-of-plane bending moments on the branch; one model was used for each mode of loading. In addition, carbon steel headers were fatigue tested by longitudinal and transverse moments cyclically applied to the branch pipes. A model was required for each mode of loading for each level of amplitude of applied nominal stress. Stress concentration factors (stress indices) were derived from the photoelastic tests, whereas, the fatigue tests produced stress intensification factors. The stress indices and stress intensification factors derived from the tests apply only to 12 × 6 standard weight headers, or geometrically identical headers, with the particular type of branch connection. This paper describes how generalized stress intensification factor equations were derived to cover a broad range of sizes and thicknesses of headers incorporating the same type of branch fitting. In this paper the term "header" applies to a single branch connection in a pipe remote from all other discontinuities.

Introduction

In 1952 Markl published a paper on the bending fatigue behavior of piping components such as (*a*) elbows, (*b*) curved and miter bends, (*c*) reinforced and unreinforced branch connections and (*d*) straight sections of pipe containing a girth buttweld [2].[1] It was found that the component consisting of two sections of pipe joined by an as-welded buttweld could be described by the formula:

$$S_a = 245{,}000\, N^{-0.20} \qquad (1)$$

In addition, it was possible to correlate the bending fatigue life of all components tested by the application of a so-called stress intensification factor i to the same equation so that:

$$iS_a = 245{,}000\, N^{-0.20} \qquad (2)$$

When i is set equal to unity, the equation predicts the fatigue life of an as-welded girth buttweld between two pieces of straight pipe (since Markl considered such a component as his reference standard and compared other components with it). Accordingly, i-factors for other piping components give the fatigue strength of those components in relation to that of a typical, as-welded girth weld.

The bending fatigue tests by Markl were conducted on 4-in. pipe size components which included full size B16.9 tees [3], plus full size pad, saddle and unreinforced fabricated connections. The results were extrapolated by means of empirical relationships developed by Markl to cover a wide range of sizes and conditions.

Stress intensification factors for certain types of piping components and full-size branch connections were first introduced into the ANSI Code for Pressure Piping in 1955 when the Code was identified as ASA B31.1-1955 [4]. The stress intensification factors were based almost entirely on the results of bending fatigue tests by Markl [2, 5, 6] and Markl and George [7]. In July, 1963, Piping Code Case No. 53 was published to provide stress intensification factors for reducing outlet branch connections and the provisions of the Case were subsequently incorporated in the 1967 Edition of USAS B31.1.0.

[1] Numbers in brackets designate References at end of Paper.

Contributed by the Pressure Vessels and Piping Division and presented at the Petroleum Mechanical Engineering Conference with Pressure Vessel and Piping Conference, New Orleans, La., September 17–21, 1972, of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, February 2, 1972. Paper No. 72-PVP-1.

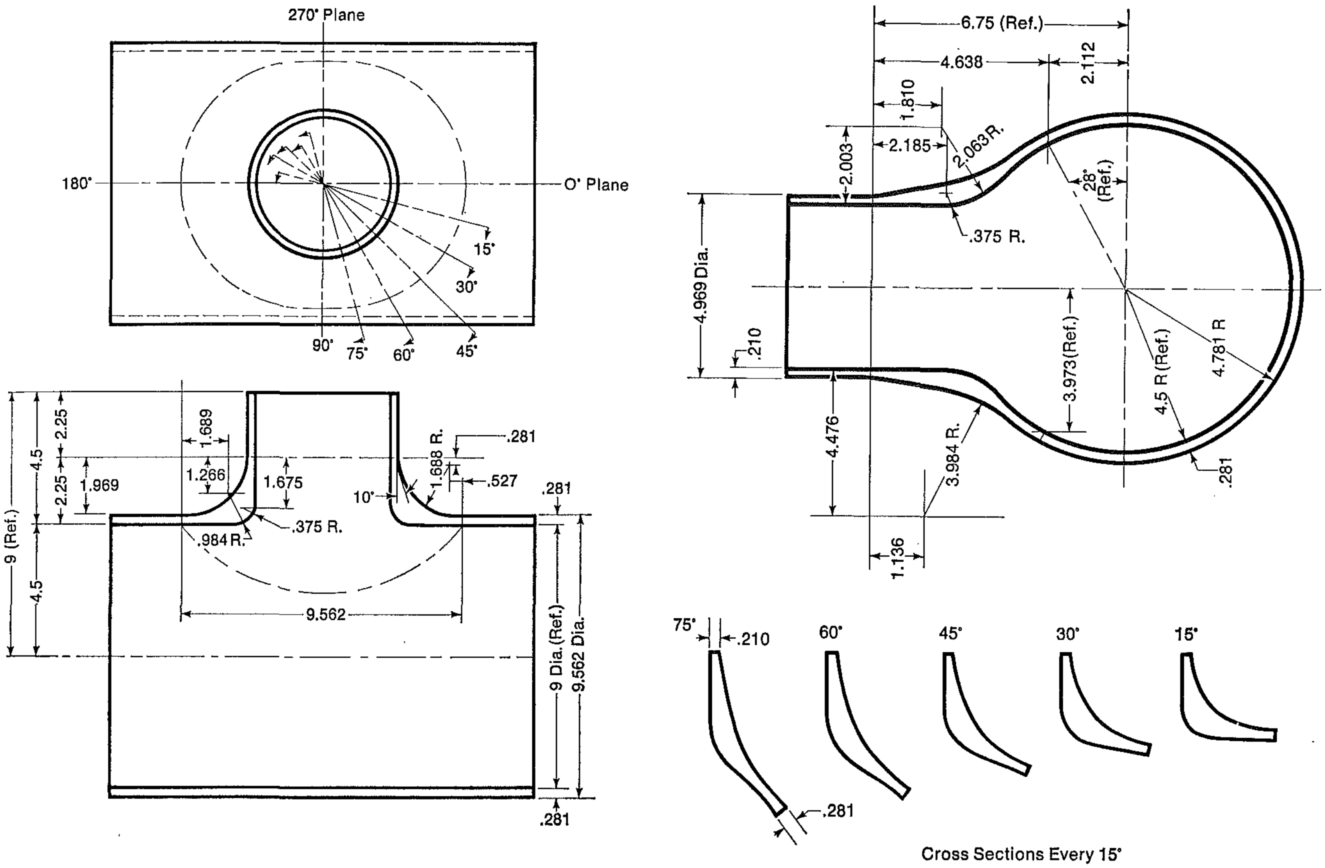

Fig. 1 Nominal dimensions of 3/4 scale, 12(0.375) × 6(0.280) photoelastic models

Stress intensification factors are used in the various ANSI Codes for Pressure Piping for estimating if the desired fatigue life can be expected to be realized as a result of thermal expansion. Forces and moments acting at anchors, connected equipment and on piping components are found by means of a so-called "flexibility analysis;" nominal stresses throughout the piping system can then be calculated. The maximum stress in a particular component is then found by applying a suitable stress intensification factor to the calculated nominal stress; for complex loading the stresses are combined in a special way.

Data from an extensive investigation of a single size contoured, integrally reinforced branch connection in a cylindrical shell are reported in the open technical literature [1]. Dimensionless parameters which describe the size of the test headers are $r/R = 0.513$, $R/T = 16.5$, $t/T = 0.747$.

The purpose of this paper is to show how the available test data from the subject headers plus analytical and extrapolation techniques can be combined to yield generalized equations which give the i-factors of such fittings over a broad range of sizes and thicknesses.

Summary of Pertinent Test Data. Fig. 1 shows the 3/4-scale epoxy models used for the photoelastic studies and Fig. 2 illustrates the 12 in. (0.375) × 6 in. (0.280) carbon steel headers used in the Markl-type bending fatigue tests. No attempt was made to replicate any welds in the photoelastic models. Although the welds between branch fitting, run pipe, and branch pipe in the

Nomenclature

F_1 = stress concentration factor for the condition of the insert or Zone A weld

i = stress intensification factor

i_{X3} = stress intensification factor for an out-of-plane bending moment on the branch

i_{Z3} = stress intensification factor for an in-plane bending moment on the branch

M_L = in-plane (longitudinal) bending moment applied to branch, lb-in.

M_T = out-of-plane (transverse) bending moment applied to branch, lb-in.

N = fatigue life, cycles

r = mean radius of branch pipe, in.

R = mean radius of run pipe, in.

S_a = nominal stress amplitude, psi

t = thickness of branch pipe, in.

T = thickness of run pipe, in.

$(\sigma_m)_B$ = maximum calculated stress using Bijlaard's theory, psi

$(C_{X3})_B$ = stress index calculated using Bijlaard's theory for an out-of-plane moment on the branch

$(C_{Z3})_B$ = stress index calculated using Bijlaard's theory for an in-plane moment on the branch

Insert weld = the butt weld which joins the branch connection to the run pipe

Branch weld = the butt weld which joins the branch pipe to the branch connection

Zone A = general area in the vicinity of the insert weld

Zone B = general area in the vicinity of the branch weld

Table 1 Summary of available test data

Model	Load	Loading	Max. SCF	Avg. i	Point of Maximum Stress or Fatigue Failure
Photoelastic	P	Steady	2.06[1]	—	0° plane at inside corner radius
Photoelastic	M_L	Steady	1.33[2]	—	0° plane at junction of branch pipe and fitting (Zone B)
Photoelastic	M_T	Steady	2.83[2]	—	90° plane in general area of run pipe to branch fitting intersection (Zone A)
Carbon Steel	M_L	Cyclic	—	0.85	0° plane at junction of branch pipe and fitting (Zone B)
Carbon Steel	M_T	Cyclic	—	1.22	90° plane in general area of run pipe to branch fitting weld (Zone A)

[1] Ratio of maximum stress to the nominal stress in the run pipe due to pressure
[2] Ratio of maximum stress to the nominal stress in the branch pipe due to the applied moment

① 6-inch × 150-pound ASA W.N. Flange, ASTM A181-Grade 1
② 6-inch ASA Standard Weight Carbon Steel Pipe, ASTM A106-Grade B
③ 12 (.375) × 6 (.280)-inch contoured, integrally Reinforced Branch Connection, ASTM A350, Grade LF1
④ 12-inch ASA Standard Weight Carbon Steel Pipe, ASTM A106-Grade B
⑤ 12-inch × 300-pound ASA W.N. Flange, ASTM A181-Grade 1

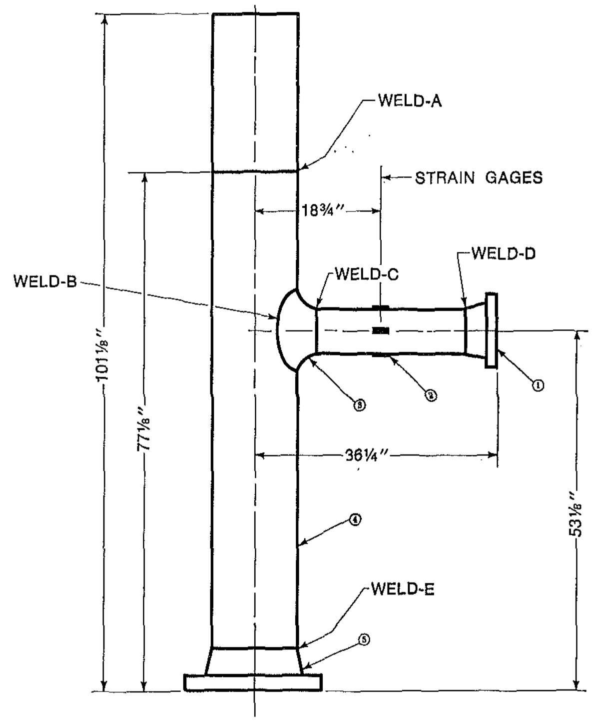

Fig. 2 12(0.375) × 6(0.280) bending fatigue test header

carbon steel headers were dressed inside and outside to remove weld ripples; a slight amount of reinforcement or crown remained on the exterior surface and intersection of weld and base metal was less than ideal.

Pertinent test data are summarized in Table 1, however, for complete details the reader is referred to Ref. 1.

Stress Intensification Factors for Out-of-Plane Bending

Derivation of Generalized Equation for Zone *A*. To be of value, generalized equations for calculating the stress intensification factor of any type of branch connection for any specified mode of loading must include the "size" and "shape" of the header in terms of dimensionless parameters. Guidance for extrapolating the test data with respect to R/T comes from equations given in the various non-nuclear ANSI Piping Codes such as ASA B31.1.0-1967[2]. These codes specify that the stress intensification factor i for full-size welding tees per B16.9 [3] can be calculated by the equation:

$$i = i_{X3} = 0.335 \left(\frac{R}{T}\right)^{2/3} \qquad (3)$$

The use of $(R/T)^{2/3}$ as an extrapolation parameter for B16.9 tees and other branch connections was originally introduced by Markl [2] who noted a similarity in fatigue failure location between those in elbows and those in full-size B16.9 tees.

For a contoured, integrally reinforced, insert branch connection of the type shown in Figs. 1 and 2, hereafter referred to as a "contoured fitting," equation (3) takes the form:

$$i_{X3} = A \left(\frac{R}{T}\right)^{2/3} \qquad (4)$$

where A is a constant to be derived later using both the available experimental data and Bijlaard's theory [8]. Bijlaard's theory gives stresses in a cylinder subjected to surface loads distributed in a particular manner and it is perhaps obvious that the theory is only indirectly applicable to branch connections in general. However, Bijlaard's theory is frequently used for estimating stresses due to moments imposed on nozzles in pressure vessels or branch connections in piping and therefore it would seem to have merit for extrapolating the test data available on contoured fittings.

Often the stress index for moment loading on the branch is taken as the ratio of the maximum calculated or measured stress to the nominal bending stress in the branch thus indicating incorrectly that the maximum stress is a function of the branch wall thickness. Actually when an out-of-plane moment M_T is applied to the branch, the maximum stress occurs either near or on the transverse plane at the branch pipe to fitting junction or down on the skirt near the run pipe where it is obvious that the maximum stress is insensitive to the branch thickness. This is taken into account by writing an equation for the stress index as:

$$(C_{X3})_B = \frac{(\sigma_m)_B}{\dfrac{M}{\pi r^2 t} \cdot \dfrac{t}{T}} \qquad (5)$$

and subsequently specifying a minimum value which is the stress index of the welded joint between the branch and branch fitting. Now an indication of the suitability of $(R/T)^{2/3}$ as an extrapolation parameter is seen by comparing the slope of the curves in Fig. 3. The set of curves is a plot of R/T versus $(C_{X3})_B$ for nozzles over a range of r/R ratios calculated according to Bijlaard's theory and the remaining curve is R/T versus $2i_{X3}$ for full size B16.9 tees calculated according to equation (3). Since a stress index $(C_{X3})_B$ is approximately double a stress intensification factor (i_{X3}) [9], it is necessary to make the comparison on a consistent basis; therefore the axis of the abscissa of Fig. 3 is $(C_{X3})_B$ and $2i_{X3}$.

There is also a significant r/R effect which should be taken into account to avoid excessive conservatism. Similar to the above concerning $(R/T)^{2/3}$ as an extrapolation parameter, Bijlaard's theory can be used to show that the variation of stress, over the range of R/T, is reasonably well represented by $(r/R)^{1/2}$, therefore equation (4) now becomes:

$$i_{X3} = A \left(\frac{R}{T}\right)^{2/3} \left(\frac{r}{R}\right)^{1/2} \qquad (6)$$

As discussed previously, the maximum stress in Zone A due to an out-of-plane moment on the branch is independent of the branch thickness; further, in using Bijlaard's theory, nowhere does the thickness of the imaginary nozzle appear. Therefore,

[2] The next edition will carry the ANSI prefix, i.e., ANSI B31.1.0-.

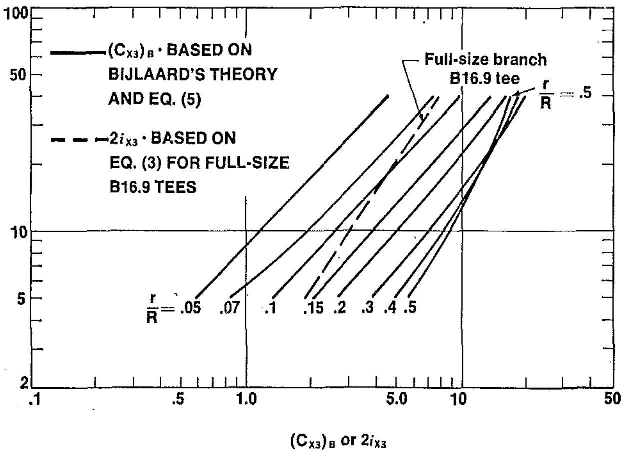

Fig. 3 Stress indices for branch fittings and stress intensification factors for full-size branch B16.9 tees for out-of-plane moment on branch

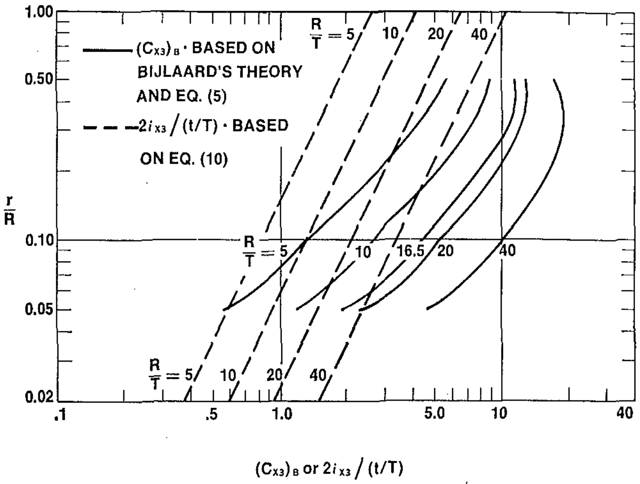

Fig. 4 Stress indices for contoured branch fittings by equation (10) compared with Bijlaard's theory

the equation for the i-factor of a contoured fitting should be modified as follows to reflect this by using t/T as a multiplier:

$$i_{X3} = A\left(\frac{R}{T}\right)^{2/3}\left(\frac{r}{R}\right)^{1/2}\left(\frac{t}{T}\right) \tag{7}$$

The value of the constant A can now be determined using the test data from Table 1 for an out-of-plane moment M_T on the branch. The fatigue tests gave $i_{X3} = 1.22$, whereas, using the rule of thumb and dividing the SCF from the photoelastic analysis by two gives $i_{X3} \simeq C_{X3}/2 = 1.42$. Substituting the larger value of i_{X3} into equation (7) along with the dimensional size parameters of the test headers yields:

$$1.42 = A(16.5)^{2/3}(0.513)^{1/2}(0.747) \tag{8}$$

which, when solved for A, gives $A = 0.409$. A second criterion will be used for calculating the constant A and conservatively the larger of the values will be taken as the constant. If Fig. 3 is entered with the dimensional parameters of the test headers, namely, $R/T = 16.5$, $r/R = 0.513$ and $t/T = 0.747$ one finds that $(C_{X3})_B$ according to Bijlaard's theory is 11.65. Multiplying $(C_{X3})_B$ by t/T yields $(C_{X3})_B(t/T) = 11.65\ (0.747) = 8.70$ and this value can now be compared with either $2i_{X3} = 2(1.22) = 2.44$ from the fatigue tests or 2.83 from the photoelastic tests for M_T loading. The second criterion for determining the constant A is that the stress (or i-factor) from equation (7) should never (for any value of R/T or r/R) be less than one-half of the stress[3] from Bijlaard's theory multiplied by the ratio 2.44/8.70. For the range of parameters covered, the governing combination is $R/T = 40$ and $r/R = 0.2$ where $(C_{X3})_B = 16.55$. Therefore, $i_{X3} \simeq \frac{(C_{X3})_B}{2} \cdot \frac{2.44}{8.70}$ and equation (7) becomes:

$$\frac{16.55}{2} \cdot \frac{2.44}{8.70} = A(40)^{2/3}(0.2)^{1/2} \tag{9}$$

and, thus according to our second criterion expressed as equation (9) the constant A becomes 0.45 which is larger than A from equation (8).

A generalized equation for the stress intensification factor of headers with contoured fittings loaded by an out-of-plane moment on the branch can now be written using $A = 0.45$ and equation (7). It is:

$$i_{X3} = 0.45\left(\frac{R}{T}\right)^{2/3}\left(\frac{r}{R}\right)^{1/2}\left(\frac{t}{T}\right) \tag{10}$$

[3] Bijlaard's theory gives the equivalent of a stress concentration factor and dividing by two applies the rule of thumb for converting a SCF to i.

Conservatism. Fig. 4 compares $(C_{X3})_B$ with $2i_{X3}/(t/T)$ over a wide range of the dimensional parameters r/R and R/T. $(C_{X3})_B$ is a stress concentration factor from Bijlaard's theory and i_{X3} is a stress intensification factor by equation (10). Multiplying i_{X3} by 2 is the rule of thumb for converting i to a SCF and dividing by t/T is required to make the term consistent with $(C_{X3})_B$ of equation (5). Ideally the ratio of $(C_{X3})_B(t/T)/2i_{X3}$ should be about $8.70/2.44 = 3.56$ over the range of parameters shown in Fig. 4. For $R/T = 40$ and $r/R = 0.2$ this ratio is about 3.56; elsewhere the ratio is less than 3.56 and at $R/T = 5$ and $r/R = 0.05$ it becomes unity indicating that the degree of conservatism is very high in some instances.

For the test headers covered by Table 1, equation (10) gives:

$$i_{X3} = 0.45(16.5)^{2/3}(0.513)^{1/2}(0.747) = 1.56 \tag{11}$$

This value is moderately conservative with respect to the actual average fatigue test i_{X3} of 1.22 and to the photoelastic test result (converted to an i-factor using the rule of thumb) of $C_{X3}/2 = 2.83/2 = 1.415$.

Effect of Insert Weld. Out-of-plane fatigue tests reported in Table 1 resulted in failures in Zone A. While the insert welds were not ideal, they had been dressed to remove weld ripples and were blended reasonably well into the adjacent base metal. Accordingly, equation (10) is applicable when insert welds are dressed or ground flush.

The "stress intensifying" effect of an "as-welded" insert weld can be accounted for by adding a multiplying factor F_1 to equation (10) so that the equation for i_{X3} becomes:

$$i_{X3} = 0.45\left(\frac{R}{T}\right)^{2/3}\left(\frac{r}{R}\right)^{1/2}\left(\frac{t}{T}\right) F_1 \tag{12}$$

Some work [10] has been reported on the relative fatigue strength of flush welds versus as-welded welds with a tensile stress normal to the weld. This work indicates that the fatigue strength of a weld with "good overfill shape" is about 5/8 of a flush weld. This suggests that the basic i-factor for Zone A should be multiplied by 8/5 or 1.6 for an as-welded insert weld, i.e., $F_1 = 1.6$. Markl's data [2] on a typical butt weld for which $i = 1.0$, and on plain straight pipe for which $i = 0.64$, also suggests a multiplier F_1 equal to 1.6 for an as-welded insert weld ($1/0.64 \simeq 1.6$).

Effect of Branch Weld. The fatigue tests with in-plane bending on the branch resulted in failures in Zone B, i.e., at the weld between the branch and branch connection, giving an average value of i_{Z3} of 0.85 for a dressed weld. This value of i indicates that the fatigue strength would not have improved significantly had the Zone B weld been flush as compared to the dressed weld actually used. For "as-welded" welds in Zone B, it is appro-

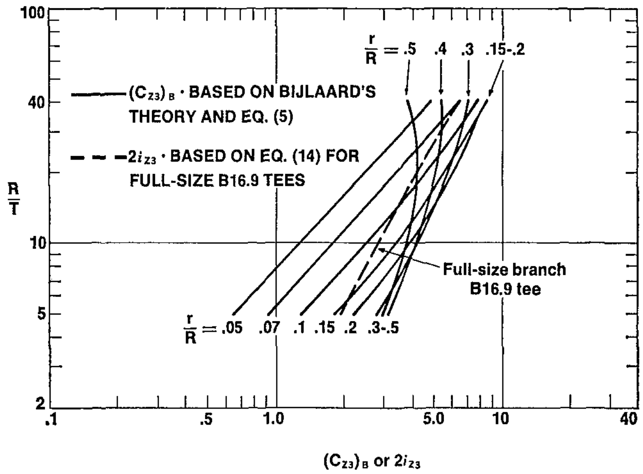

Fig. 5 Stress indices for branch fittings and stress intensification factors for full-size branch B16.9 tees for in-plane moment on branch

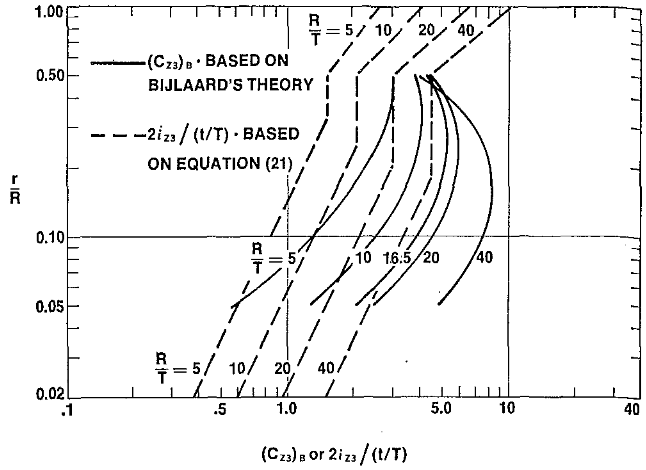

Fig. 6 Stress indices for contoured branch fittings by equation (21) compared with Bijlaard's theory

priate to use the same i-factor assigned to a girth butt weld between the branch pipe and a B16.9 tee. This factor is unity This is consistent with Markl's work where he took two pieces of straight pipe joined by an "as-welded" butt weld as his reference standard and then assigned it an i-factor of unity. Accordingly, when i_{X3} calculated according to equation (12) is less than the i-factor of the Zone B weld (0.85 if dressed or 1.0 if as-welded) it means that the Zone B weld controls and $i_{X3} = 0.85$ or 1.0. In other words, $(i_{X3})_{\min.} = 0.85$ or 1.0 depending upon the condition of the branch to branch connection weld.

Stress Intensification Factors for In-Plane Bending

Derivation of Generalized Equation for Zone A. Par. 119.6.4 of USAS B31.1.0-1967 [11] gives the equivalent of the following equation for calculating the maximum bending stress due to an in plane moment on the branch of a full size or reducing B16.9 tee:-

$$S_b = i_{Z3} \frac{M_{Z3}}{Z_b} \quad (13)$$

where:

$i_{Z3} = 0.75 i_{X3} + 0.25$

$Z_b = \pi r^2 t_s$

t_s = lesser of T and $i_{X3} t$

however; using equation (3) the expression for i_{Z3} becomes:

$$i_{Z3} = 0.75 \left[0.335 \left(\frac{R}{T}\right)^{2/3} \right] + 0.25 = 0.25 \left(\frac{R}{T}\right)^{2/3} + 0.25 \quad (14)$$

and when T is less than $i_{X3} t$ equation (13) reduces to:

$$S_b = \left[0.25 \left(\frac{R}{T}\right)^{2/3} + 0.25 \right] \frac{M}{\pi r^2 t} \cdot \frac{t}{T} \quad (15)$$

Equation (15) gives the Zone A stresses for the case of an in-plane bending moment on the branch of a full-size or reducing B16.9 tee.

Fig. 5 is a plot of $(C_{Z3})_B$ calculated according to Bijlaard's theory for in-plane moment loading (on the branch) over a range of the dimensional parameters R/T and r/R. Superposed is the curve $2i_{Z3}$ for full-size B16.9 tees calculated according to equation (14).

Table 1 shows that in-plane bending fatigue tests of the 12(0.375) × 6(0.280) headers resulted in failures at Zone B and not in Zone A which we wish to investigate. However, it is not necessarily true that the critical location cannot be Zone A for certain of the parameters R/T and r/R. According to Fig. 5 the calculated value of $(C_{Z3})_B$ for R/T and r/R corresponding to the test model is 4.10. For $R/T = 40$ and $r/R = 0.2$, the value of $(C_{Z3})_B$ is 8.30. Accordingly, if stresses in contoured fittings vary with R/T and r/R in the same way as indicated by Bijlaard's theory, then for some values of R/T and r/R the critical location will be Zone A. This follows from the observation that the maximum stress index in Zone A from the photoelastic test [1] was 0.96; therefore, 0.96 (8.30/4.10) gives an estimated value of $C_{Z3} = 1.96$ for the header with $R/T = 40$ and $r/R = 0.2$. An i_{Z3} factor of 0.85 is assigned to a flush or dressed Zone B weld, therefore,[4] since 2×0.85 is less than $C_{Z3} = 1.96$, the possibility of Zone A failures for a branch carrying an in-plane moment exists. Accordingly, it is necessary to develop an equation for calculating i_{Z3} in Zone A.

Fig. 5, a comparison of $2i_{Z3}$ of full-size B16.9 tees with $(C_{Z3})_B$ from Bijlaard's theory, indicates that:

1 There is a similarity in the "trend" of the $2i_{Z3}$ and $(C_{Z3})_B$ curves.

2 While there is a significant r/R effect, it is difficult to formulate in any simple way because the trend is in one direction for small values of the ratio R/T and in the opposite direction for large values of R/T

3 The code formula for in-plane bending applied to the branch of a contoured fitting can be shown to be more conservative than the Code formula for out-of-plane bending. Equations (14) and (15) yield the following for i_{Z3} of full-size and reducing B16.9 tees:

$$i_{Z3} = \left[0.25 \left(\frac{R}{T}\right)^{2/3} + 0.25 \right] \frac{t}{T} \quad (16)$$

and when applied to the test models gives $i_{Z3} = 1.395$ or $C_{Z3} \simeq 2i_{Z3} = 2.79$ as compared to the photoelastic results for Zone A of $C_{Z3} = 0.96$. The code formula for i_{X3} for full-size and reducing B16.9 tees is simply:

$$i_{X3} = 0.335 \left(\frac{R}{T}\right)^{2/3} \left(\frac{t}{T}\right) \quad (17)$$

and when this code formula is applied to the test model $i_{X3} = 1.62$ or $C_{X3} \simeq 2i_{X3} = 3.24$ as compared to the photoelastic result of $C_{X3} = 2.83$. Comparing $2i_{Z3}$ with C_{Z3} determined photoelastically and $2i_{X3}$ with C_{X3} determined photoelastically provides an indication of the relative conservatism of the Code formula for B16.9 tees with respect to i_{Z3} and i_{X3}.

Equation (16) will be used as an extrapolation equation for

[4] Multiplying i by two is the rule of thumb for converting an i-factor to a SCF.

Table 2 Stress intensification factors for contoured branch fittings[1]

Loading	Equation for Stress Intensification Factor[2] (i)	i_{min}[3] F or D	A-W
Out-of-plane moment on branch (M_T or M_{X3})	$i_{X3} = 0.45\left(\frac{R}{T}\right)^{2/3}\left(\frac{r}{R}\right)^{1/2}\left(\frac{t}{T}\right)(F_1)$	0.85	1.0
In-plane moment on branch (M_L or M_{Z3})	(a) For $\frac{r}{R} \leq 0.5$ Lesser of: $i_{Z3} = 0.45\left(\frac{R}{T}\right)^{2/3}\left(\frac{r}{R}\right)^{1/2}\left(\frac{t}{T}\right)(F_1)$ and $i_{Z3} = [0.17\left(\frac{R}{T}\right)^{2/3} + 0.25]\left(\frac{t}{T}\right)(F_1)$	0.85	1.0
	(b) For $\frac{r}{R} > 0.5$ Interpolate between: $\frac{r}{R} = 0.5, i_{Z3} = [0.17\left(\frac{R}{T}\right)^{2/3} + 0.25]\left(\frac{t}{T}\right)(F_1)$ $\frac{r}{R} = 1.0, i_{Z3} = 0.45\left(\frac{R}{T}\right)^{2/3}\left(\frac{t}{T}\right)(F_1)$	0.85	1.0

[1] These factors are intended for use with USAS B31.1.0-1967, Par. 119.6.4 except that for the branch (leg 3)

$$S_b = \frac{\sqrt{(i_{X3}M_T)^2 + (i_{Z3}M_L)^2}}{\pi r^2 t}$$

and similarly for other codes based on the stress intensification factor concept.

[2] $F_1 = 1.0$ for flush or dressed insert welds.

$F_1 = 1.6$ for as-welded insert welds.

[3] The minimum values of i depend upon the type of girth butt weld between the branch fitting and branch pipe. F or D stands for flush or dressed; A-W stands for as-welded.

contoured fittings for r/R to 0.5 except that some excess conservatism will be removed by replacing the coefficient of $(R/T)^{2/3}$ with a suitably smaller coefficient "A." From the photoelastic tests under in-plane bending, Zone A $C_{Z3} = 0.96$ whereas, Fig. 5 based on Bijlaard's theory yields a value of $(C_{Z3})_B(t/T) = 3.06$. To find "A," the stress intensification factor from equation (16) shall never be less than one-half of the stress from Bijlaard's theory multiplied by the ratio 0.96/3.06; therefore, since the "worst case" is for $R/T = 5$ and $r/R = 0.4$, for which $(C_{Z3})_B = 3.0$ the equation for calculating "A" is:

$$A(5)^{2/3} + 0.25 = \frac{3.0}{2} \cdot \frac{0.96}{3.06} \qquad (18)$$

however, 0.96/3.06 will be replaced by 1/2 for additional conservatism because the fatigue tests of Table 1 did not produce any Zone A failures. Therefore:

$$A(5)^{2/3} + 0.25 = \frac{3.0}{2} \cdot \frac{1}{2} \qquad (19)$$

Solving equation (19) yields $A = 0.171$ so that a generalized equation for i_{Z3} for contoured fittings becomes:

$$i_{Z3} = \left[0.17\left(\frac{R}{T}\right)^{2/3} + 0.25\right]\frac{t}{T} \qquad (20)$$

however, equation (20) is not proposed as the complete expression for i_{Z3}. The equation can be improved by considering the relationship between i_{X3} and i_{Z3}; Bijlaard's theory suggests that the ratio i_{X3}/i_{Z3} depends upon R/T and r/R and tends to become a maximum in the general range of r/R between 0.2 and 0.7. At very small values of r/R, Bijlaard's theory indicates that $i_{Z3} \simeq i_{X3}$ which is reasonable because when r/R is small the curvature effect of the run pipe would become negligible. On the other hand, the fatigue tests by Markl [2] on branch connections with $r/R = 1.0$ indicate that i_{X3}/i_{Z3} may be fairly close to unity. Equation (20) does not reflect these trends in two respects:

1. For small values of r/R, equation (20) gives values of i_{Z3} that are significantly higher than values of i_{X3} from equation (10). According to Bijlaard's theory i_{Z3} should be essentially equal to or somewhat less than i_{X3}.

2. For $r/R = 1.0$, equation (20) gives values of i_{Z3} that are significantly lower than values of i_{X3} from equation (10). For example, at $R/T = 10$ and $r/R = 1.0$, equation (20) gives $i_{Z3} = 1.04(t/T)$, whereas, equation (10) yields $i_{X3} = 2.09(t/T)$. Test data and Markl's work [2] suggest that at $r/R = 1.0$, $i_{Z3} < i_{X3}$ but not by a factor of two.

Table 3 Comparison of stress intensification factors from derived equations with experimental values

Loading	Maximum Stress Intensification Factor	
	Experimental (Table 1)	Calculated (Table 2)
M_T or M_{X3}	$i_{X3} = 1.22$ (fatigue tests) $i_{X3} = C_{X3}/2 = 1.42$ (photoelastic analysis)	$i_{X3} = 1.56$
M_L or M_{Z3}	$i_{Z3} = 0.85$ (fatigue tests) $i_{Z3} = C_{Z3}/2 = 0.67$ (photoelastic analysis)	$i_{Z3} = 1.04$

In view of the preceding, the suggested expressions for i_{Z3} become:

For $r/R \leqq 0.5$
i_{Z3} = lesser of:

$$i_{Z3} = 0.45\left(\frac{R}{T}\right)^{2/3}\left(\frac{r}{R}\right)^{1/2}\left(\frac{t}{T}\right)F_1 \qquad (21a)$$

$$i_{Z3} = \left[0.17\left(\frac{R}{T}\right)^{2/3} + 0.25\right]\left(\frac{t}{T}\right)F_1 \qquad (21b)$$

For $r/R > 0.5$

i_{Z3} is to be determined by linear interpolation (with respect to r/R) between the value of i_{Z3} by equation (21(b)) and the value obtained by equation (21(a)) using $r/R = 1.0$ (21c)

A multiplier F_1 appears in equations (21(a)) and (21(b)) to correct for the stress intensifying effect of the insert weld for the same reasons as discussed above in the section: "Effect of Insert Weld." Regardless of the value of i_{Z3} calculated using equation (21), there will be a minimum value (0.85 or 1.0) depending upon the condition of the branch weld as discussed above in the section titled: "Effect of Branch Weld."

Conservatism. Fig. 6 compares $2i_{Z3}/(t/T)$ based on equation (21) with $(C_{Z3})_B$ calculated using Bijlaard's theory. For the test headers covered by Table 1, equation (21(c)) gives $i_{Z3} = 1.04$. This value is conservative by a factor of about two with respect to the Zone A photoelastic test results of $C_{Z3}/2 = 0.96/2 = 0.48$.

Discussion and Conclusions

Generalized equations for calculating the stress intensification factors for in-plane and out-of-plane bending moments applied to the branch of a contoured branch fitting have been derived and are presented in Table 2. The equations are based on test data using Bijlaard's theory for extrapolation along with extrapolation equations derived by Markl [2] and first introduced into the code ASA B31.1-1955 [4].

Table 3 compares stress intensification factors determined experimentally for the test headers with the corresponding value calculated according to Table 2.

The stress intensification factors calculated according to Table 2 for the test headers are somewhat conservative with respect to the actual test data. However, one may find considerably more conservatism at other values of the dimensional parameters R/T and r/R. This conservatism is introduced, in part, by the use of relatively simple equations of no more than simple power functions of dimensional parameters to cover a wide range of dimensional parameters. Also, the equations have been adjusted so that they cover the most adverse combination of dimensions.

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