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Research Article

Robust H_∞ Control of Uncertain T-S Fuzzy Time-Delay System: A Delay Decomposition Approach

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This paper is concerned with the problem of robust H_∞ control for a class of uncertain time-delay fuzzy systems with norm-bounded parameter uncertainties. By utilizing the instrumental idea of delay decomposition, the decomposed Lyapunov-Krasovskii functional is introduced to uncertain T-S fuzzy system, and some delay-dependent conditions for the existence of robust controller are formulated in the form of linear matrix inequalities (LMIs). When these LMIs are feasible, a controller is presented. A numerical example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

It is well known that time delay is built-in features in various nonlinear systems such as tandem mills, remote control systems, long transmission lines in pneumatic systems, and chemical system. The time delay is recognized to be a source of instability and performance deterioration of control systems. Therefore, stability analysis and controller synthesis for time-delay system have been one of the most hot research area in the control community over the past years [1–14].

Fuzzy systems in the form of the Takagi-Sugeno (T-S) model have attracted rapidly growing interest in recent years. It has been shown that the T-S model method is a simple and effective way to represent complex nonlinear systems by a set of simple local linear dynamic systems with their linguistic description [12, 15–19]. Over the past few years, most work has been devoted to analysis and synthesis of T-S fuzzy control systems. See the survey papers [16, 17] and the reference citied therein for the most recent advances on this topic. The appeal and superiority of T-S fuzzy models is that the analysis and synthesis of the overall fuzzy systems can be carried out in the Lyapunov-function-based framework. To mention a few, by using LMI, Cao and Frank presented controller design for a class of fuzzy dynamic systems with time delay in both continuous and discrete cases in [20, 21]. Wu et al. studied the model approximation problem and L_2-L_∞ control problem for nonlinear time-delay systems in [22, 23]. Moreover, great attention from researchers has been drawn to the study of stability analysis and controller design for T-S fuzzy systems with time delays [24–28]. On the other hand, type-2 fuzzy mode are considered in [29, 30].

Recently, many scholars studied the stability problem based on the piecewise Lyapunov-Krasovskii functional [31–33]. Reference [31] investigated the linear continuous/discrete systems with time-varying delay and divided the variation interval of the time delay into several subintervals. based on this method, [32]addressed the problem of the robust H_∞ filtering for singular linear parameter varying (LPV). Reference [33] researched the stability of linear time-invariant systems and divided the delay interval into N subintervals. The simulations show these methods can lead to much less conservative results than those in the existing references.

Motivated by the above observations, in this paper, we will investigate the problem of robust H_∞ control of uncertain T-S fuzzy systems with constant delay. Attention is focused on the design of robust H_∞ controllers via the parallel distributed compensation scheme such that the closed-loop fuzzy time-delay system is asymptotically stable and the H_∞ disturbance attenuation is below a prescribed level.

Based on delay decomposition approach [33], the decomposed Lyapunov-Krasovskii functional is introduced, and some delay-dependent conditions have been obtained. These conditions are formulated in the form of LMIs, and the controller design is cast into a convex optimization problem subject to LMI constraints, which can be readily solved via standard numerical software. Finally, a numerical example is provided to show the effectiveness and less conservatism of the proposed results.

The rest of this paper is organized as follows. In Section 2, the model description and problem are first formulated. The main results for delay-dependent robust H_∞ controller are presented in Section 3. Illustrative examples are given in Section 4, and the paper is concluded in Section 5.

Notations. The notations used throughout this paper are fairly standard. The superscript "T" stands for matrix transpose, and the notation $P > 0$ ($P \geq 0$) means that matrix P is real symmetric and positive (or being positive semidefinite). I and 0 are used to denote appropriate dimensions identity matrix and zero matrix, respectively. The notation * in a symmetric always denotes the symmetric block in the matrix. The parameter diag$\{\cdots\}$ denotes a block-diagonal matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. System Descriptions and Preliminaries

Consider the uncertain nonlinear system with state delay that is described by the following T-S model with uncertain parameter matrices.

Plant Rule i. IF $s_1(t)$ is F_{i1} and $s_2(t)$ is F_{i2} and ... and $s_n(t)$ is F_{in} THEN

$$\begin{aligned}\dot{x}(t) &= (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\tau) \\ &\quad + (B_{1i} + \Delta B_{1i})u(t) + B_{2i}\omega(t), \\ z(t) &= C_i x(t), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \quad i = 1, 2, \ldots, r,\end{aligned} \tag{1}$$

where $s_1(t), s_2(t), \ldots, s_n(t)$ are the premise variables that are measurable and each F_{ij} $(j = 1, 2, \ldots, n)$ is fuzzy set. $x(t) \in R^n$ is the state vector and $u(t) \in R^m$ is the control input vector. $z(t)$ is the output vector. $\omega(t) \in R^q$ is the disturbance input vector belongs to $L_2[0, \infty)$. r is the number of IF-THEN rules, τ is the constant delay in the state. $\phi(t)$ is a vector-valued initial continuous function.

The matrices ΔA_i, ΔA_{di}, and ΔB_{1i} denote the parameters uncertainties, which are assumed of the form

$$[\Delta A_i, \Delta A_{di}, \Delta B_{1i}] = \mathrm{MF}(t)[E_i, E_{di}, E_{1i}], \tag{2}$$

where M, E_i, E_{di}, and E_{1i} are known constant matrices and $F(t)$ is an unknown time-varying matrix function satisfying $F^T(t)F(t) \leq I$.

For simplicity, introduce the following notations:

$$\overline{A}_i = A_i + \Delta A_i \qquad \overline{A}_{di} = A_{di} + \Delta A_{di} \qquad \overline{B}_{1i} = B_{1i} + \Delta B_{1i}. \tag{3}$$

By using a center-average defuzzier, product fuzzy inference, and a singleton fuzzifier, the following global T-S fuzzy model can be obtained:

$$\begin{aligned}\dot{x}(t) &= \left(\sum_{i=1}^{r}\alpha_i(s(t))\left[(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\tau) + (B_{1i} + \Delta B_{1i})u(t) + B_{2i}\omega(t)\right]\right) \times \left(\sum_{i=1}^{r}\alpha_i(s(t))\right)^{-1} \\ &= \sum_{i=1}^{r}\mu_i(s(t))\left[(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\tau) + (B_{1i} + \Delta B_{1i})u(t) + B_{2i}\omega(t)\right], \\ z(t) &= \sum_{i=1}^{r}\mu_i(s(t))C_i x(t) \\ &= C(t)x(t), x(t) \\ &= \phi(t) \quad t \in [-\tau, 0],\end{aligned} \tag{4}$$

where $\alpha_i(s(t)) = \prod_{j=1}^{n}F_{ij}(s_j(t))$, $\mu_i(s(t)) = \alpha_i(s(t))/\sum_{i=1}^{r}\alpha_i(s(t))$, and $F_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in F_{ij}, and it is assumed that $\alpha_i(s(t)) \geq 0$, $i = 1, 2, \ldots, r$ $\sum_{i=1}^{r}\alpha_i(s(t)) > 0$ for all t. Therefore, $\mu_i(s(t)) \geq 0$ and $\sum_{i=1}^{r}\mu_i(s(t)) = 1$ for all t.

In this paper, employing the idea of parallel distributed compensation (PDC), the T-S fuzzy-model-based controller via the PDC can be constructed as follows.

Controller Rule i. IF $s_1(t)$ is F_{i1} and $s_2(t)$ is F_{i2} and ... and $s_n(t)$ is F_{in} THEN

$$u(t) = K_i x(t), \tag{5}$$

where K_i $(i = 1, 2, \ldots, r)$ are the controller gains of (5) to be determined.

Then, the overall output of the controller rules is given by

$$u(t) = \sum_{i=1}^{r}\mu_i(s(t))K_i x(t). \tag{6}$$

Substituting (6) into (4), the closed-loop system can be given as

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(s(t))\mu_j(s(t)) \times \left[(\overline{A}_i + \overline{B}_{1i}K_j)x(t) + \overline{A}_{di}x(t-\tau) + B_{2i}\omega(t)\right], \\ z(t) &= \sum_{i=1}^{r}\mu_i(s(t))C_i x(t),\end{aligned} \tag{7}$$

with its compact form

$$\dot{x}(t) = \left(\overline{A}(t) + \overline{B}_1(t) K(t)\right) x(t) + \overline{A}_d(t) x(t-\tau) + B_2(t)\omega(t), \quad (8)$$

$$z(t) = C(t) x(t), \quad (9)$$

where

$$\overline{A}(t) = A(t) + \Delta A(t) = \sum_{i=1}^{r}\mu_i(s(t))\left[A_i + \Delta A_i(t)\right],$$
$$\overline{A}_d(t) = A_d(t) + \Delta A_d(t) = \sum_{i=1}^{r}\mu_i(s(t))\left[A_{di} + \Delta A_{di}(t)\right],$$
$$\overline{B}_1(t) = B_1(t) + \Delta B_1(t) = \sum_{i=1}^{r}\mu_i(s(t))\left[B_{1i} + \Delta B_{1i}(t)\right],$$
$$B_2(t) = \sum_{i=1}^{r}\mu_i(s(t)) B_{2i}, \qquad C(t) = \sum_{i=1}^{r}\mu_i(s(t)) C_i,$$
$$K(t) = \sum_{i=1}^{r}\mu_i(s(t)) K_i. \quad (10)$$

Before ending this section, we introduce the following definitions and lemmas, which will be used in the derivation of our main results.

Definition 1 (*H_∞ performance*). Given a scalar $\gamma > 0$ and under zero initial condition, the system (1) is said to be asymptotically stable with γ-disturbance attenuation if the system (4) is asymptotically stable and the output $z(t)$ satisfies

$$\|z(t)\|_2 \le \gamma\|\omega(t)\|_2, \quad (11)$$

that is,

$$\int_0^\infty \left[z^T(t) z(t) - \gamma^2 \omega^T(t)\omega(t)\right] dt \le 0, \quad (12)$$

for all nonzero $\omega(t) \in L_2[0,\infty)$.

Lemma 2 (see [34]). *For any constant matrix $W \in R^{n\times n}$, $W = W^T > 0$, scalar $r > 0$, and vector-valued function $\dot{x} : [-r, 0] \to R^n$ such that the following integration is well defined; then*

$$-\int_{t-r}^{t} \dot{x}^T(s)(rW)\dot{x}(s)\,ds \le \left(x^T(t) \quad x^T(t-r)\right)\begin{pmatrix} -W & W \\ W & -W\end{pmatrix}\begin{pmatrix} x(t) \\ x(t-r)\end{pmatrix}. \quad (13)$$

Lemma 3 (see [35]). *Given a symmetric matrix M and matrices D, $F(t)$, and E of compatible dimensions, then, for $F^T(t)F(t) \le I$, the inequality*

$$M + DF(t)E + (DF(t)E)^T < 0 \quad (14)$$

holds if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + \varepsilon DD^T + \varepsilon^{-1}E^T E < 0. \quad (15)$$

3. Main Results

In this section, some delay-dependent sufficient conditions on the existence of robust H_∞ controller for T-S fuzzy system (7) will be presented. A Lyapunov-Krasovskii functional, based on the idea of delay decomposition approach, will be introduced, which can potentially reduce the conservatism of the results.

To this end, we first consider the following nominal closed-loop system:

$$\dot{x}(t) = \left(A(t) + B_1(t) K(t)\right) x(t) + A_d(t) x(t-\tau) + B_2(t)\omega(t), \quad (16)$$

$$z(t) = C(t) x(t). \quad (17)$$

Firstly, the sufficient condition of H_∞ performance analysis for the unforced case of system (16) is established in Proposition 4.

Proposition 4. *For some prescribed $\gamma > 0$ and $\tau > 0$, the unforced case of system* (16) *is asymptotically stable with a guaranteed H_∞ performance γ, if there exist matrices $P > 0$, $R_l > 0$, and $Q_l > 0$ $(l = 1, 2, \ldots, N)$ such that the following LMIs hold for $i = 1, 2, \ldots, r$:*

$$\Sigma = \begin{bmatrix} \Sigma^{(1)} & \Sigma^{(2)} & \Sigma^{(3)} \\ * & -\gamma^2 I & \Sigma^{(4)} \\ * & * & \Sigma^{(5)} \end{bmatrix} < 0, \quad (18)$$

where

$$\Sigma^{(1)} = \begin{bmatrix} \Sigma_{11}^{(1)} & R_1 & 0 & \cdots & 0 & PA_{di} \\ * & \Sigma_{22}^{(1)} & R_2 & \cdots & 0 & 0 \\ * & * & \Sigma_{33}^{(1)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & \Sigma_{NN}^{(1)} & R_N \\ * & * & * & \cdots & * & \Sigma_{N+1\,N+1}^{(1)} \end{bmatrix},$$

$$\Sigma_{11}^{(1)} = Q_1 - R_1 + A_i^T P + PA_i + C_i^T C_i,$$
$$\Sigma_{22}^{(1)} = Q_2 - Q_1 - R_1 - R_2,$$
$$\Sigma_{33}^{(1)} = Q_3 - Q_2 - R_2 - R_3,$$
$$\vdots$$
$$\Sigma_{NN}^{(1)} = Q_N - Q_{N-1} - R_{N-1} - R_N,$$
$$\Sigma_{N+1\,N+1}^{(1)} = -Q_N - R_N,$$

$$\Sigma^{(2)} = \begin{bmatrix} PB_{2i} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Sigma^{(3)} = \begin{bmatrix} hA_i^T R_1 & hA_i^T R_2 & \cdots & hA_i^T R_N \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ hA_{di}^T R_1 & hA_{di}^T R_2 & \cdots & hA_{di}^T R_N \end{bmatrix}$$

$$\Sigma^{(4)} = \begin{bmatrix} hB_{2i}^T R_1 & hB_{2i}^T R_2 & \cdots & hB_{2i}^T R_N \end{bmatrix},$$

$$\Sigma^{(5)} = \operatorname{diag}\{-R_1, -R_2, \ldots, -R_N\}. \tag{19}$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$
$$V_1(t) = x^T(t) P x(t),$$
$$V_2(t) = \sum_{l=1}^{N} \int_{t-lh}^{t-(l-1)h} x^T(s) Q_l x(s)\, ds, \tag{20}$$
$$V_3(t) = \sum_{l=1}^{N} \int_{-lh}^{-(l-1)h} \int_{t+\theta}^{t} \dot{x}^T(s)\left(hR_l\right)\dot{x}(s)\, ds\, d\theta,$$

where $h = \tau/N$ and N is the partitioning number of time delay τ.

Taking the derivative of $V_i(t)$, for $i = 1, 2, 3$, with respect to t along the trajectory of unforced case of system (16) yields

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) \\
&= \left(A(t)x(t) + A_d(t)x(t-\tau) + B_2(t)\omega(t)\right)^T \\
&\quad \times P x(t) + x^T(t) P\left(A(t)x(t)\right. \\
&\quad \left. + A_d(t)x(t-\tau) + B_2(t)\omega(t)\right), \\
\dot{V}_2(t) &= \sum_{l=1}^{N}\left(x^T(t-(l-1)h)\right. \\
&\quad \left. \times Q_l x(t-(l-1)h) - x^T(t-lh) Q_l x(t-lh)\right) \\
&= \sum_{l=1}^{N} x^T(t-(l-1)h) Q_l x(t-(l-1)h) \\
&\quad - \sum_{l=1}^{N} x^T(t-lh) Q_l x(t-lh), \\
\dot{V}_3(t) &= \sum_{l=1}^{N} h\dot{x}^T(t)\left(hR_l\right)\dot{x}(t) \\
&\quad - \sum_{l=1}^{N}\int_{t-lh}^{t-(l-1)h} \dot{x}^T(s)\left(hR_l\right)\dot{x}(s)\, ds.
\end{aligned} \tag{21}$$

By using Lemma 2, we obtain that

$$\begin{aligned}
&\dot{V}_3(t) \\
&\le \sum_{l=1}^{N} h\dot{x}^T(t)\left(hR_l\right)\dot{x}(t) \\
&\quad + \sum_{l=1}^{N}\left(x^T(t-(l-1)h) \quad x^T(t-lh)\right) \\
&\qquad \times \begin{pmatrix} -R_l & R_l \\ R_l & -R_l \end{pmatrix} \begin{pmatrix} x(t-(l-1)h) \\ x(t-lh) \end{pmatrix} \\
&= \sum_{l=1}^{N}\left(A(t)x(t) + A_d(t)x(t-\tau) + B_2(t)\omega(t)\right)^T \\
&\qquad \times \left(h^2 R_l\right)\left(A(t)x(t) + A_d(t)x(t-\tau) + B_2(t)\omega(t)\right) \\
&\quad + \sum_{l=1}^{N}\left(x^T(t-(l-1)h) \quad x^T(t-lh)\right) \\
&\qquad \times \begin{pmatrix} -R_l & R_l \\ R_l & -R_l \end{pmatrix} \begin{pmatrix} x(t-(l-1)h) \\ x(t-lh) \end{pmatrix} \\
&= \left(A(t)x(t) + A_d(t)x(t-\tau) + B_2(t)\omega(t)\right)^T \\
&\quad \times \left(\sum_{l=1}^{N} h^2 R_l\right)\left(A(t)x(t) + A_d(t)x(t-\tau) + B_2(t)\omega(t)\right) \\
&\quad + \sum_{l=1}^{N}\left(x^T(t-(l-1)h) \quad x^T(t-lh)\right)\begin{pmatrix} -R_l & R_l \\ R_l & -R_l \end{pmatrix}\begin{pmatrix} x(t-(l-1)h) \\ x(t-lh) \end{pmatrix} \\
&= \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} A^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)A(t) & A^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)A_d(t) & A^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)B_2(t) \\ A_d^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)A(t) & A_d^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)A_d(t) & A_d^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)B_2(t) \\ B_2^T(t)\left(\sum_{l=1}^{N} h^2 R_j\right)A(t) & B_2^T(t)\left(\sum_{j=1}^{N} h^2 R_l\right)A_d(t) & B_2^T(t)\left(\sum_{l=1}^{N} h^2 R_l\right)B_2(t) \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix} \\
&\quad + \sum_{l=1}^{N}\left(x^T(t-(l-1)h) \quad x^T(t-lh)\right) \\
&\quad \times \begin{pmatrix} -R_l & R_l \\ R_l & -R_l \end{pmatrix}\begin{pmatrix} x(t-(l-1)h) \\ x(t-lh) \end{pmatrix}.
\end{aligned} \tag{22}$$

Define the variable $\eta^T(t) = [x^T(t), x^T(t-h), \ldots, x^T(t-(N-1)h), x^T(t-\tau)]$ and by simple manipulation, we have

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \\ &\leq \xi^T(t)(\Pi_1 + \Pi_2 + \Pi_3)\xi(t),\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\Pi_1 &= \begin{bmatrix} A^T(t)P + PA(t) & \cdots & PA_d(t) & PB_2(t) \\ \vdots & \ddots & \vdots & \vdots \\ A_d^T(t)P & \cdots & 0 & 0 \\ B_2^T(t)P & \cdots & 0 & 0 \end{bmatrix}, \\
\Pi_2 &= \begin{bmatrix} Q_1 & 0 & \cdots & 0 & 0 \\ * & Q_2 - Q_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & Q_N - Q_{N-1} & 0 \\ * & * & \cdots & * & -Q_N \end{bmatrix}, \\
\Pi_3 &= \begin{bmatrix} -R_1 & R_1 & 0 & \ldots & 0 & 0 & 0 \\ * & -R_1 - R_2 & R_2 & \ldots & 0 & 0 & 0 \\ * & * & -R_2 - R_3 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & * & \ldots & -R_{N-1} - R_N & R_N & 0 \\ * & * & * & \ldots & * & -R_N & 0 \\ * & * & * & \ldots & * & * & 0 \end{bmatrix} \\
&\quad - \begin{bmatrix} \Sigma^{(3)} \\ \Sigma^{(4)} \end{bmatrix} \left(\Sigma^{(5)}\right)^{-1} \begin{bmatrix} \Sigma^{(3)} \\ \Sigma^{(4)} \end{bmatrix}^T, \\
\xi(t) &= \left[\eta^T(t) \quad \omega^T(t)\right]^T.
\end{aligned} \tag{24}$$

First, we prove the asymptotic stability of the system in (16). To this end, assume $\omega(t) \equiv 0$, and thus $\Pi_1 + \Pi_2 + \Pi_3$ in (23) reads

$$\widetilde{\Pi}_1 + \widetilde{\Pi}_2 + \widetilde{\Pi}_3, \tag{25}$$

where

$$\begin{aligned}
\widetilde{\Pi}_1 &= \begin{bmatrix} A^T(t)P + PA(t) & \cdots & PA_d(t) \\ \vdots & \ddots & \vdots \\ * & \cdots & 0 \end{bmatrix}, \\
\widetilde{\Pi}_2 &= \begin{bmatrix} Q_1 & 0 & \cdots & 0 \\ * & Q_2 - Q_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ * & * & \cdots & Q_N - Q_{N-1} \end{bmatrix}, \\
\widetilde{\Pi}_3 &= \begin{bmatrix} -R_1 & R_1 & 0 & \cdots & 0 & 0 \\ * & -R_1 - R_2 & R_2 & \cdots & 0 & 0 \\ * & * & -R_2 - R_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -R_{N-1} - R_N & 0 \\ * & * & * & \cdots & * & -R_N \end{bmatrix} \\
&\quad - \left(\Sigma^{(3)}\right)\left(\Sigma^{(5)}\right)^{-1}\left(\Sigma^{(3)}\right)^T.
\end{aligned} \tag{26}$$

From (18), we know that

$$\widetilde{\Pi}_1 + \widetilde{\Pi}_2 + \widetilde{\Pi}_3 < 0, \tag{27}$$

which guarantees $\dot{V}(t) < 0$ for all non-zero $\eta(t)$. Thus one can always find a sufficiently small $\upsilon > 0$ such that $\dot{V}(t) < -\upsilon\|x(t)\|^2$. The asymptotic stability of the considered system is proved.

Next, assuming that $\omega(t) \neq 0$ and $\phi(t) = 0$, $t \in [-\tau, 0]$, we consider H_∞ performance of the system in Definition 1.

Considering $\mu_i(s(t)) \geq 0$, $\sum_{i=1}^{r} \mu_i(s(t)) = 1$, we obtain that

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t) \leq \sum_{i=1}^{r} \mu_i(s(t))\xi^T(t)\Psi\xi(t), \tag{28}$$

where

$$\Psi = \begin{bmatrix} \Sigma^{(1)} & \Sigma^{(2)} \\ * & -\gamma^2 I \end{bmatrix} - \begin{bmatrix} \Sigma^{(3)} \\ \Sigma^{(4)} \end{bmatrix} \left[\Sigma^{(5)}\right]^{-1} \begin{bmatrix} \Sigma^{(3)} \\ \Sigma^{(4)} \end{bmatrix}^T. \tag{29}$$

Applying Schur complement, guarantees $\Psi < 0$. From (28), we can get that

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t) \leq 0. \tag{30}$$

Integrating the preceding inequality from 0 to ∞, it is easy to get that

$$\int_0^\infty z^T(t)z(t)\,dt \leq \int_0^\infty \gamma^2\omega^T(t)\omega(t)\,dt + V(x_0) - V(x_\infty). \tag{31}$$

Since $V(x_0) = 0$ and $V(x_\infty) \geq 0$, we have

$$\int_0^\infty z^T(t)z(t)\,dt \leq \int_0^\infty \gamma^2\omega^T(t)\omega(t)\,dt. \tag{32}$$

Then, according to Definition 1, the H_∞ performance of the system in (16) is established. This completes the proof. □

In the following, based on Proposition 4, we design robust state feedback H_∞ controller for the system (7).

Theorem 5. *For some prescribed $\gamma > 0$, $\tau > 0$, $\delta > 0$, and N is a positive integer, if there exist scalar $\varepsilon > 0$, matrices $X > 0$, $V_l > 0$ $(l = 2, 3, \ldots, N)$, and $Q_l > 0$ $(l = 1, 2 \ldots N)$, and appropriate dimension matrices W_j $(j = 1, 2, \ldots, r)$ such that the following LMIs simultaneously hold for i, $j = 1, 2, \ldots, r$:*

$$\begin{gathered}\Omega_{ii} < 0, \quad (i = 1, 2, \ldots, r), \\ \frac{\left(\Omega_{ij} + \Omega_{ji}\right)}{2} < 0, \quad \left(1 \le i < j \le r\right), \\ V_i R_i = I, \quad (i = 1, 2, \ldots, N),\end{gathered} \tag{33}$$

where

$$\Omega_{ij} = \begin{bmatrix} \Upsilon_{ij} & \varepsilon U & T_{ij}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix}, \qquad \Upsilon_{ij} = \begin{bmatrix} \Pi_{ij}^{(1)} & \Pi_i^{(2)} & \Pi_{ij}^{(3)} & \Pi_i^{(6)} \\ * & -\gamma^2 I & \Pi_i^{(4)} & 0 \\ * & * & \Pi^{(5)} & 0 \\ * & * & * & -I \end{bmatrix},$$

$$U = \begin{bmatrix} H & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & N & 0 \\ * & * & * & 0 \end{bmatrix}, \qquad T_{ij} = \begin{bmatrix} S_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ L_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} M & 0 & 0 & \cdots & 0 & 0 \\ * & 0 & 0 & \cdots & 0 & 0 \\ * & * & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & 0 & 0 \\ * & * & * & \cdots & * & 0 \end{bmatrix},$$

$$S_{ij} = \begin{bmatrix} E_i X + E_{1i} W_j & 0 & 0 & \cdots & 0 & E_{di} \\ * & 0 & 0 & \cdots & 0 & 0 \\ * & * & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & 0 & 0 \\ * & * & * & \cdots & * & 0 \end{bmatrix},$$

$$L_{ij}^T = \begin{bmatrix} hXE_i^T + hW_j^T E_{1i}^T & hXE_i^T + hW_j^T E_{1i}^T & \cdots & hXE_i^T + hW_j^T E_{1i}^T \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ hE_{di}^T & hE_{di}^T & \cdots & hE_{di}^T \end{bmatrix},$$

$$\Pi_{ij}^{(1)} = \begin{bmatrix} \Pi_{11ij}^{(1)} & \delta I & 0 & \ldots & 0 & A_{di} \\ * & \Pi_{22}^{(1)} & R_2 & \cdots & 0 & 0 \\ * & * & \Pi_{33}^{(1)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & \Pi_{NN}^{(1)} & R_N \\ * & * & * & \cdots & * & \Pi_{N+1\ N+1}^{(1)} \end{bmatrix},$$

$$N^T = \begin{bmatrix} M^T & 0 & 0 & \cdots & 0 & 0 \\ * & M^T & 0 & \cdots & 0 & 0 \\ * & * & M^T & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & M^T & 0 \\ * & * & * & \cdots & * & M^T \end{bmatrix},$$

$$\Pi_{11ij}^{(1)} = XQ_1X - \delta X + XA_i^T + W_j^T B_{1i}^T + A_i X + B_{1i} W_j,$$

$$\Pi_{22}^{(1)} = Q_2 - Q_1 - \delta P - R_2,$$

$$\Pi_{33}^{(1)} = Q_3 - Q_2 - R_2 - R_3,$$

$$\Pi_{NN}^{(1)} = Q_N - Q_{N-1} - R_{N-1} - R_N,$$

$$\Pi_{N+1\ N+1}^{(1)} = -Q_N - R_N,$$

$$\Pi_i^{(4)} = \left[hB_{2i}^T \quad hB_{2i}^T \quad \cdots \quad hB_{2i}^T\right],$$

$$\Pi^{(5)} = \operatorname{diag}\left\{-\delta^{-1} X, -V_2, \ldots, -V_N\right\},$$

$$\Pi_i^{(2)} = \begin{bmatrix} B_{2i} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Pi_{ij}^{(3)} = \begin{bmatrix} hXA_i^T + hW_j^T B_{1i}^T & hXA_i^T + hW_j^T B_{1i}^T & \cdots & hXA_i^T + hW_j^T B_{1i}^T \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ hA_{di}^T & hA_{di}^T & \cdots & hA_{di}^T \end{bmatrix}$$

$$\Pi_i^{(6)} = \begin{bmatrix} XC_i^T \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{34}$$

Then the closed-loop system (16) *is asymptotically stable with the H_∞ performance index γ. Moreover, if the above condition is feasible, the gain matrices of a desired controller in the form of* (6) *can be designed by*

$$K_j = W_j X^{-1}. \tag{35}$$

Proof. The proof of this theorem is divided into two parts. First, we design the state-feedback H_∞ controller of the nominal case of closed-loop system (7).

According to Proposition 4, it is easy to know that the H_∞ performance requirement of the nominal case of closed-loop system (7) implies

$$\widetilde{\Sigma} < 0, \quad (36)$$

where $\widetilde{\Sigma}$ is a matrix derived from (18) by changing the term A_i to $A_i + B_{1i}K_j$.

Introduce the following matrix variables:

$$\begin{gathered} X = P^{-1}, \qquad R_1 = \delta P, \\ V_2 = R_2^{-1}, \dots, V_N = R_N^{-1}, \qquad K_j X = W_j. \end{gathered} \quad (37)$$

Combined with Schur complement, and pre- and post-multiply (36) by $\operatorname{diag}\{X, I, \delta^{-1}X, V_2, \dots, V_N, I\}$ and its transpose, respectively, we get

$$\Upsilon_{ij} < 0. \quad (38)$$

Next, we investigate robust state feedback H_∞ controller of the closed-loop system (7).

Replace A_i, A_{di}, and B_i with $A_i + \Delta A_i$, $A_{di} + \Delta A_{di}$, and $B_{1i} + \Delta B_{1i}$, respectively; then we obtain from (2) and (38) that

$$\Upsilon_{ij} + UF(t)\,T_{ij} + T_{ij}^T F(t)\,U^T < 0. \quad (39)$$

By Lemma 3, we can know (39) holds, if and only if the following inequality holds:

$$\Upsilon_{ij} + \varepsilon U U^T + \varepsilon^{-1} T_{ij}^T T_{ij} < 0, \quad (40)$$

where ε is a positive scalar.

Applying Schur complement to (40), we have that

$$E_{ij} = \begin{bmatrix} \Upsilon_{ij} & \varepsilon U & T_{ij}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \quad (41)$$

Noting $\mu_i(s(t)) \geq 0$, $\sum_{i=1}^{r} \mu_i(s(t)) = 1$,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(s(t))\,\mu_j(s(t))\,E_{ij} = \sum_{i=1}^{r} \mu_i^2(s(t))\,E_{ii} + \sum_{i<j}^{r} \mu_i(s(t))\,\mu_j(s(t))\left(E_{ij} + E_{ji}\right). \quad (42)$$

Therefore, condition (33) can guarantee that condition (41) holds. This completes the proof. □

It should be noted that the obtained conditions in Theorem 5 are not strict LMI conditions due to the existence of nonlinear term XQ_1X in (33). It cannot be directly solved by standard LMI Toolbox. In the following, we present an approach to solving the condition in Theorem 5.

Introduce additional matrix variable $G > 0$ such that

$$XQ_1X \leq G. \quad (43)$$

By Schur complement, it follows from (43) that

$$\begin{bmatrix} -G & X \\ X & -Q_1^{-1} \end{bmatrix} \leq 0. \quad (44)$$

Then, we readily obtain the following theorem.

Theorem 6. *For some prescribed $\gamma > 0$, $\tau > 0$, $\delta > 0$, and N is a positive integer, if there exist scalar $\varepsilon > 0$, matrices $G > 0$, $P > 0$, $X > 0$, $V_l > 0$ $(l = 2, 3, \dots, N)$, $Q_l > 0$, $(l = 1, 2 \dots N)$, and $\overline{Q}_1 > 0$, and appropriate dimension matrices W_j $(j = 1, 2 \dots, r)$ such that the following LMIs simultaneously hold for $i, j = 1, 2, \dots, r$:*

$$\begin{gathered} \widehat{\Omega}_{ii} < 0, \quad (i = 1, 2, \dots, r), \\ \frac{\left(\widehat{\Omega}_{ij} + \widehat{\Omega}_{ji}\right)}{2} < 0, \quad (1 \leq i < j \leq r), \end{gathered} \quad (45)$$

$$\begin{bmatrix} -G & X \\ X & -\overline{Q}_1 \end{bmatrix} \leq 0, \quad (46)$$

$$Q_1\overline{Q}_1 = I, \qquad PX = I, \qquad V_i R_i = I, \quad (i = 1, 2, \dots, N), \quad (47)$$

where $\widehat{\Omega}_{ij}$ is a matrix derived from Ω_{ij} by replacing the term

$$\Pi_{11ij}^{(1)} = XQ_1X - X + XA_i^T + W_j^T B_{1i}^T + A_i X + B_{1i} W_j \quad (48)$$

with

$$\widehat{\Pi}_{11ij}^{(1)} = G - \delta X + XA_i^T + W_j^T B_{1i}^T + A_i X + B_{1i} W_j, \quad (49)$$

then the closed-loop system (25) is asymptotically stable with the H_∞ performance index γ. Moreover, if the above condition is feasible, the gain matrices of a desired controller in the form of (6) can be designed by

$$K_j = W_j X^{-1}. \quad (50)$$

Remark 7. Note that the obtained conditions in Theorem 6 are not all in LMI form due to equality constraints, which cannot be solved directly using standard LMI procedures. However, via the result in [35] which has been widely used by many scholars [36, 37], we can solve these nonconvex feasibility problems by formulating them into some sequential optimization problems subject to LMIs constraints.

Now using the approach [35], we suggest the following minimization problem involving LMI conditions instead of the original nonconvex feasibility problem formulated in Theorem 6.

Problem HSFCD (*H_∞ state feedback controller design*). Consider the following:

$$\text{Minimize trace}\quad \left(PX + G\overline{G} + \sum_{j=2}^{N} R_j V_j\right)$$
$$\text{subject to}\quad (45), (46)\ \text{and}\ \begin{bmatrix} P & I \\ I & X \end{bmatrix} \geq 0, \tag{51}$$
$$\begin{bmatrix} Q_1 & I \\ I & \overline{Q}_1 \end{bmatrix} \geq 0, \qquad \begin{bmatrix} R_j & I \\ I & V_j \end{bmatrix}_{j=2}^{N} \geq 0.$$

When minimize Trace $(PX + Q_1\overline{Q}_1 + \sum_{j=2}^{N} R_j V_j) = (N+1)n$, then the conditions in Theorem 6 are solvable. Algorithm 1 in [35] can be easily adapted to solve Problem HSFCD.

4. Numerical Example

In this section, we use an example to show the applicability of the results proposed in this paper.

Example 8. Consider the truck trailer system borrowed from [38], which can be represented by the following uncertain time-delay T-S fuzzy model.

Rule 1. If $\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t-\tau)$ is about 0, then

$$\begin{aligned} \dot{x}(t) &= (A_1 + \Delta A_1)x(t) + (A_{d1} + \Delta A_{d1})x(t-\tau) \\ &\quad + (B_{u1} + \Delta B_{u1})u(t) + B_{w1}\omega(t), \\ z(t) &= C_1 x(t). \end{aligned} \tag{52}$$

Rule 2. If $\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t-\tau)$ is about π or $-\pi$, then

$$\begin{aligned} \dot{x}(t) &= (A_2 + \Delta A_2)x(t) + (A_{d2} + \Delta A_{d2})x(t-\tau) \\ &\quad + (B_{u2} + \Delta B_{u2})u(t) + B_{w2}\omega(t) \\ z(t) &= C_2 x(t), \end{aligned} \tag{53}$$

where

$$a = 0.7, \quad v = -1, \quad \bar{t} = 2.0, \quad L = 5.5, \quad \tau = 0.5,$$
$$A_1 = \begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ -0.5091 & -4 & 0 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.2182 & 0 & 0 \end{bmatrix},$$
$$B_{u1} = \begin{bmatrix} -1.4286 \\ 0 \\ 0 \end{bmatrix}, \qquad B_{w1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ -0.8102 & -6.3662 & 0 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.3472 & 0 & 0 \end{bmatrix}, \qquad B_{u2} = \begin{bmatrix} -1.4286 \\ 0 \\ 0 \end{bmatrix},$$
$$B_{w1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$
$$[\Delta A_i, \Delta A_{di}, \Delta B_{ui}] = MF(t)[E_i, E_{di}, E_{ui}],$$
$$C_1 = C_2 = [0 \;\; 1 \;\; 0],$$
$$M = \text{diag}\{0.05, 0.05, 0.05\},$$
$$E_1 = \begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.5091 & 0 & 0 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.8107 & 0 & 0 \end{bmatrix},$$
$$E_{d1} = \begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.2182 & 0 & 0 \end{bmatrix}, \qquad E_{d2} = \begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.3472 & 0 & 0 \end{bmatrix},$$
$$E_{u1} = \begin{bmatrix} -0.3571 \\ 0 \\ 0 \end{bmatrix}, \qquad E_{u2} = \begin{bmatrix} -0.3571 \\ 0 \\ 0 \end{bmatrix}. \tag{54}$$

For this example, the prescribed H_∞ performance level is chosen as $\gamma = 0.5$. In order to design a robust H_∞ state feedback controller for the given T-S fuzzy model, choose $\delta = 1$, $N = 4$. According to Theorem 5, the gain matrix of controller is given as

$$\begin{aligned} K_1 &= [25.6229 \;\; -29.9005 \;\; 8.0667], \\ K_2 &= [23.8882 \;\; -31.9243 \;\; 8.1041]. \end{aligned} \tag{55}$$

According to [38], take the membership function as

$$\begin{aligned} h_1 &= \left(1 - \frac{1}{1 + \exp(-3(\theta(t) - 0.5\pi))}\right) \\ &\quad \times \left(\frac{1}{1 + \exp(-3(\theta(t) + 0.5\pi))}\right), \\ h_2 &= 1 - h_1. \end{aligned} \tag{56}$$

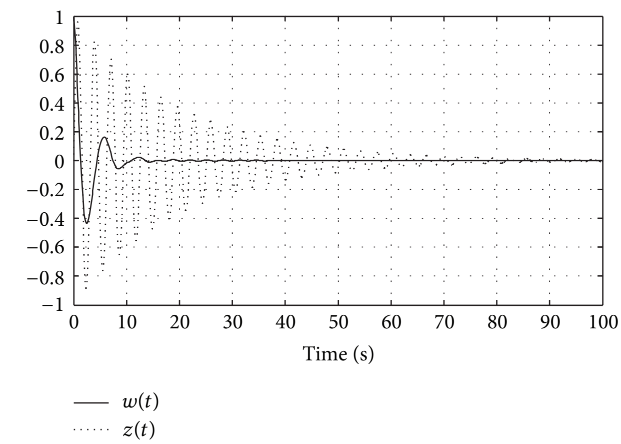

Figure 1: Controlled output $z(t)$ and the disturbance input $w(t)$.

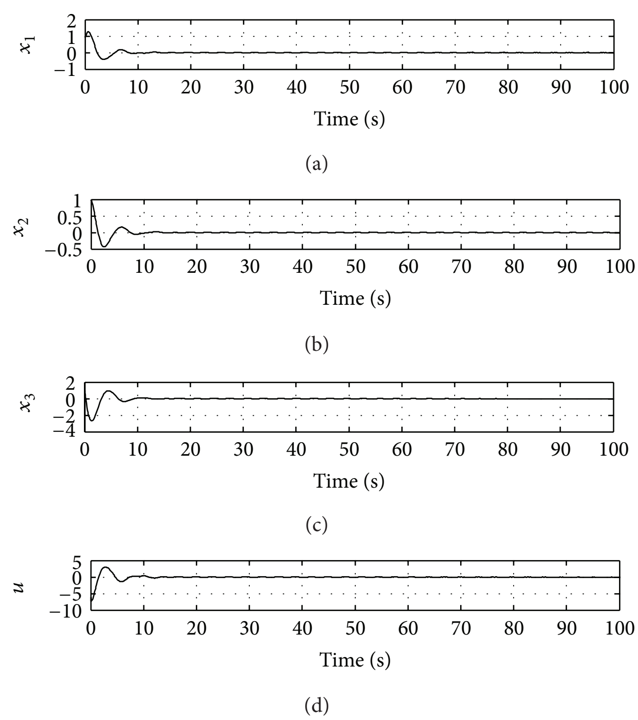

Figure 2: Response of the closed-loop system and controller input.

Let disturbance input $\omega(t) = \sin(2t)e^{-0.05t}$ and initial condition $\phi(t) = [1 \quad 1 \quad 1]^T$, $t \in [-0.5, 0]$, and simulation time is 100 s.

Figure 1 shows the controlled output $z(t)$ and the disturbance input $\omega(t)$. According to Figure 1, the resulting output energy of the robust H_∞ controller is $\int_0^{100} z^2(t)dt = 0.832$, while the input energy is $\int_0^{100} \omega^2(t)dt = 5$. Simulation result for the ratio of the output energy to the disturbance energy is 0.1664, and the l_2-norm is $\sqrt{0.1664} = 0.41 < \gamma = 0.5$ (due to the fact that the state has been stable for a long time, we can regard the value 0.41 as the l_2-norm).

State response of the closed-loop system and controller input are shown in Figure 2.

The simulation results show that the algorithm proposed in this paper is effective for robust H_∞ control problem of the truck trailer system with time delay.

5. Conclusion

The problem of robust H_∞ controller design has been addressed for a class of T-S fuzzy-model-based systems with constant delay and norm-bounded parameter uncertainty. Based on the Lyapunov-Krasovskii functional approach, a sufficient condition for the existence of the robust H_∞ controller, which robustly stabilizes the T-S fuzzy-model-based uncertain systems and guarantees a prescribed level on disturbance attenuation, has been obtained in an LMI form. The given numerical example has shown the effectiveness of the proposed method. In addition, the filtering problems of T-S fuzzy delayed systems by using the delay decomposition approach are also challenging, which could be our further work.

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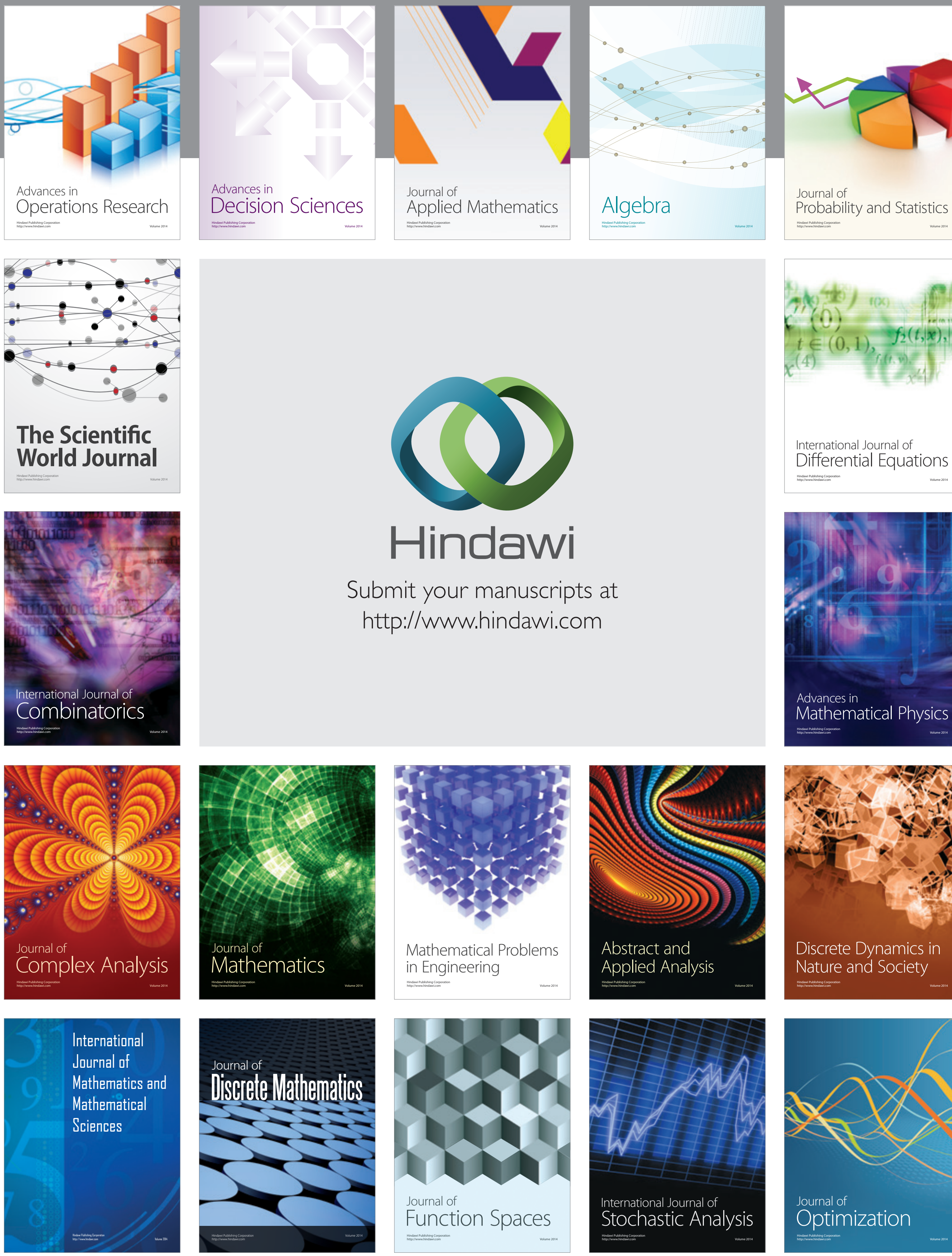

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