Artesian Landfill Liner System: Optimization and Numerical Analysis

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Abstract: Conventional landfill design attempts to control the downward seepage of leachate by using low permeability liners. The rate of leachate seepage into the underlying ground-water system can be controlled by decreasing the permeability of soil liners and/or by using synthetic membranes to form an additional barrier to leachate migration. However, loss of leachate from conventional landfills is likely to occur due to the inherent limitations of natural materials and the inevitable imperfections of installing synthetic liners. The artesian landfill liner system eliminates the downward seepage by reversing the direction of the hydraulic gradient so that seepage occurs into, and not out of, the landfill. A conceptual cost model incorporates the trade-offs between the capital cost of constructing robust liners and the operational costs of supplying recharge water and treating additional leachate produced by the artesian hydraulics. In addition, a two-dimensional, transient finite-element flow model demonstrates that the reverse hydraulic gradient limits the loss of leachate even if the integrity of the landfill liner is imperfect or deteriorates over time.

INTRODUCTION

A major environmental problem is the safe and secure disposal of solid and hazardous wastes. One common alternative is "sanitary landfilling"—the practice of burying waste materials in depressions or excavations in the ground. This practice includes the direct land disposal of unprocessed wastes and the ultimate land disposal of residues or residuals of processed wastes. Because of the prevalence of such wastes and the lack of economical, reliable, or socially acceptable disposal alternatives, the practice of waste landfilling is likely to continue.

Given that waste landfills exist and will continue to be built, the environmental problems associated with conventional landfill design will remain. The most severe of these problems is often regarded as being the seepage of leachate through the wetted perimeter of the landfill base. This contaminated leachate is known to be responsible for the pollution of ground water. Hence, a major issue in landfill design is the containment or control of leachate. The term "secure landfill" is used to describe a landfill in which leachate migration out of the landfill does not occur. In fact, conventional landfills are not secure in this sense. Landfill liners are never completely impervious. There is a lower limit to the permeability of soils as there is a practical upper limit to the thickness of liners. Moreover, natural soil liners, mechanically placed over large areas, are inevitably nonuniform. Similarly, thin synthetic liners, which are sometimes used to replace natural liners or in combination with them, may be subject to a loss of integrity through their installation over uneven soil surfaces, faulty seams between panels of the membrane, the movement of heavy equipment over the membrane, and environmental factors.

The end result of conventional landfill design practice is a significant probability of loss of leachate through the landfill liner system. Although the rate of leachate flow and its composition is uncertain, conventional sanitary landfills cannot be considered as a safe, secure, and acceptable waste disposal alternative. As an alternative, the artesian landfill concept is introduced, a cost optimization model is developed for its preliminary design, and a numerical model is presented to describe its behavior under normal operating conditions and possible failure modes.

ARTESIAN LANDFILL SYSTEM

Conventional landfill systems fail because they do not eliminate the basic problem—namely, the hydraulic potential of leachate acting on the liner. As long as this potential exists, leachate leakage will inevitably occur. Rather than working against nature, the artesian landfill system utilizes the same hydraulic forces to reverse the direction of leachate migration, thereby eliminating leakage from the landfill. Similar concepts for the control of leachate seepage have been advocated by Matich and Tao (1984), Sallfors and Peirce (1984), Amos (1985), and Adams and Karney (1988) and more recently in work on the "hydraulic trap" by Rowe et al. (1995).

In applying the upflow concept to the design of solid waste landfills, the hydraulic potential is not eliminated. Rather, the direction of the gradient, and hence the direction of leachate migration, is reversed. To accomplish this, a multilayered liner system (see Fig. 1) is constructed with the following four elements (Sallfors and Peirce 1984): (1) A leachate collection system; (2) a compacted clay liner with a low hydraulic conductivity (upper liner); (3) a porous "artesian" layer consisting of sand and gravel, in which drain pipes are installed; and (4) a lower liner consisting of either compacted clay or synthetic material, or both.

The artesian landfill system is essentially the same as the conventional landfill system above the upper impermeable liner. The key difference is the lower impermeable liner and the granular fill between the two liners that forms a confined aquifer. A piezometric surface can be maintained in this aquifer by the addition of water from recharge wells on the surface. When this piezometric surface lies above the upper liner, the aquifer becomes artesian, hence its name. This difference in piezometric surfaces, or pressure differential, prevents the downward flow of leachate through the upper liner. In fact, this pressure differential causes an upward flow from the artesian layer to the drainage layer through the upper liner. The upward flow rate is a function of the magnitude of the pressure differential. Operationally, the goal is to limit recharge to a

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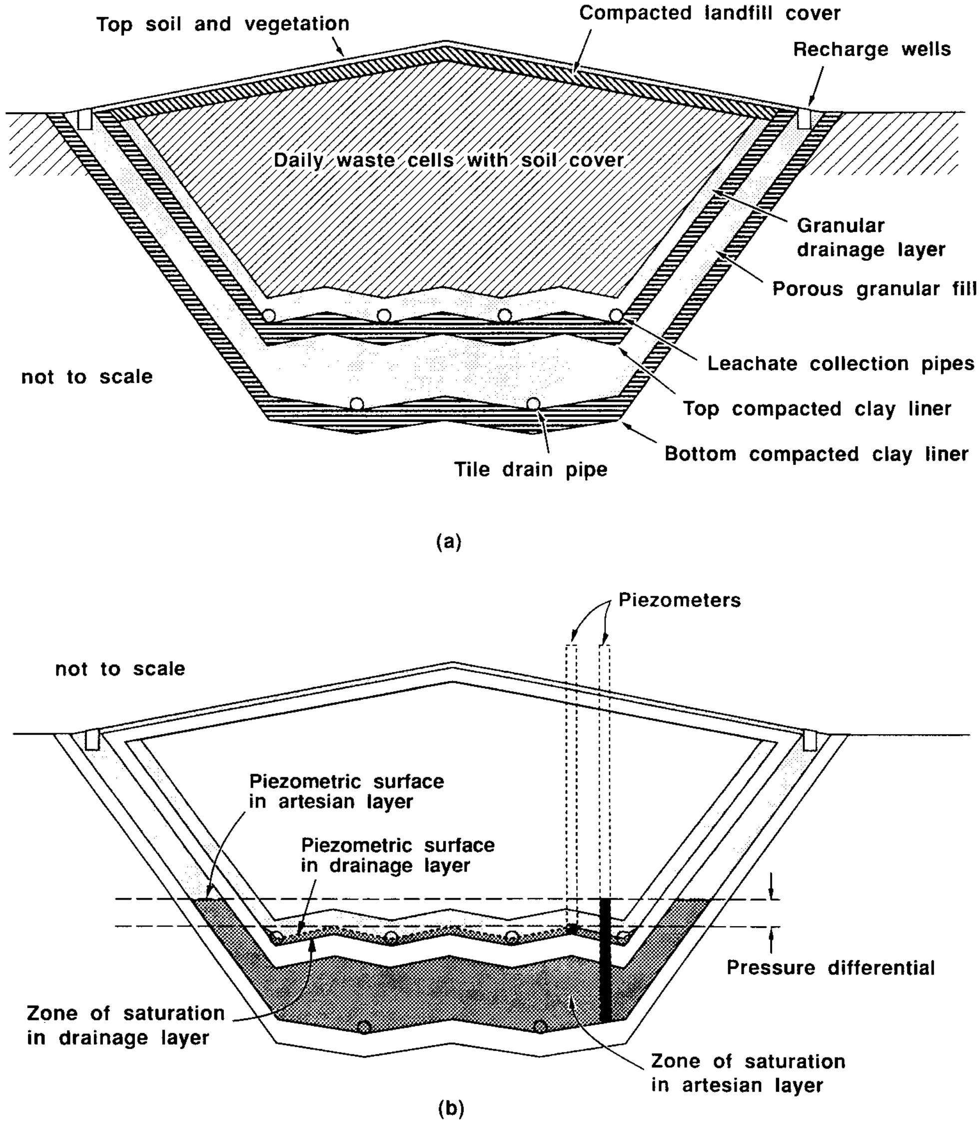

FIG. 1. Artesian Landfill System: (a) Functional Sketch; (b) Hydraulic Sketch

rate that causes a minimal pressure differential and, hence, a minimal upward flow rate.

The artesian landfill may be more expensive than the conventional sanitary landfill since it consists of two additional elements—a lower liner and a granular drainage layer. The granular drainage layer may be equipped with drain pipes, which would permit both the adjustment of pressure differentials and the sampling of the recharge water that creates the pressure differential. These features represent both larger excavation and material costs that are to some extent offset by less robust liners than would be required in conventional landfill design. However, these costs may be justified when considering the environmental benefits of improved leachate containment (Adams and Karney 1988) and the possible public acceptance of landfilling as a viable long-term waste disposal method (Sallfors and Peirce 1984). Moreover, multilayer leachate containment systems are often advocated in any case. In fact, it is possible to operate the artesian landfill system exactly as a conventional landfill system until the first breakthrough of contaminants occurs through the upper liner. After this breakthrough, operation in the artesian mode can begin. The remainder of this paper concentrates on the unique features of the artesian system.

In addition to the trade-offs between the conventional and the artesian landfill liner systems, there are trade-offs within the design of the artesian landfill liner system. For example, the permeability of the upper and lower liners are design variables. The greater the permeability of both liners, the greater the flux rate of recharge water both upward through the upper layer and downward through the lower layer; thus, the greater the volume of recharge water required. The flux rate of recharge water flowing upward through the upper liner is directly proportional to its permeability for a given pressure differential across the liner. When the marginal cost of decreasing the liner permeability exceeds the marginal cost of collecting and treating leachate, it is more economical to manage the greater leachate quantities than to decrease the permeability of the upper liner and vice versa. Thus, there is a trade-off between the capital cost of a more robust liner and the operational cost associated with leachate collection and treatment. These economic trade-offs are explored quantitatively in detail in the next section. Note, however, that the trade-offs within the artesian design are of a quite different nature than those associated with conventional landfill design. In a conventional design, reduced capital costs in the design must be traded-off against an increased environmental risk associated with liner failure and leachate loss; in the artesian system, the designer trades-off capital costs against operating costs, within the context of a secure containment system.

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Conceptual Cost Model

Although the actual conditions within the artesian landfill system are complex, a simple conceptual model allows the trade-offs between operating and capital costs to be investigated. The basic assumption is that the portion of the landfill above the upper liner, specifically the leachate collection system, is properly designed to maintain a reasonable head on the upper surface of the upper liner. Since such a leachate collection system is required for even the conventional landfill system, this assumption seems justified.

If leachate flow conditions are approximately steady, the seepage flux from the artesian layer into the landfill can then be estimated by Darcy's law as

$$Q_u = K_u \frac{H_u}{B_u} \tag{1}$$

where Q_u = specific discharge through the upper liner (m/s); K_u = average hydraulic conductivity of the upper liner (m/s); H_u = average difference in hydraulic potential acting across the upper liner (m); and B_u = upper liner thickness (m). The design variables are the selection of upper and lower liner thicknesses, B_u and B_l, respectively, the choice of liner materials and their associated hydraulic conductivities, K_u and K_l, and the operating hydraulic potential difference acting across the upper liner, H_u. The operating potential in the artesian layer is chosen to ensure that a minimum positive gradient exists at all locations in the upper liner. Thus, the gradient would be determined by geometrical factors to account for the slope of the drainage system and its piezometric surface. The operation and control of the artesian layer recharge system is largely determined by the minimum acceptable value of potential difference.

The selection of the thickness of the liners represents a compromise between operating and capital costs. As the thickness of the liners increases, the capital material cost P_l can be expected to increase in an approximately linear fashion. Thus

$$P_l = P_{cu} + P_{cl} = C_c B_u + C_c B_l \tag{2}$$

in which P_{cu} and P_{cl} = present value of the material cost of the upper and lower liners, respectively, per unit area; and C_c = cost of placing the liner material per unit volume. As the upper liner becomes thicker, the volume of additional leachate from the artesian zone that must be collected and treated decreases. Using (1), the annual cost of collecting and treating this additional leachate, C_l, is

$$C_l = C_1 K_u \frac{H_u}{B_u} \tag{3}$$

in which C_1 = constant converting flux rates into total annual collection and treatment costs.

Eqs. (2) and (3) can be combined to form the following:

$$y_1 = P_{cu} + P_{cl} + \sum_{t=0}^{N} \frac{C_l}{(1 + r)^t} = C_c B_u + C_c B_l + \sum_{t=0}^{N} \frac{C_1 K_u (H_u/B_u)}{(1 + r)^t} \tag{4}$$

in which y_1 = total present value of upper and lower liner placement costs plus additional leachate collection and treatment cost per unit area of landfill; r = annual discount rate; and N = period of analysis in years. The water flowing through the lower liner is not considered here in the collection and treatment costs since it need not be treated. If the cost is approximately constant and the period of analysis is sufficiently long, the geometric series can be summed to yield

$$\sum_{t=0}^{N} \frac{1}{(1 + r)^t} \cong \frac{1 + r}{r} \tag{5}$$

There is another potentially important trade-off involved in the implementation of the artesian landfill system. High permeabilities of both the upper and lower liners, K_u and K_l, respectively, translate to a high flux rate of recharge, Q_u and Q_l, through both liners; hence, high liner permeabilities are associated with a greater volume of recharge water required. Thus, a trade-off exists between the capital cost of providing robust upper and lower liners and the operational cost of supplying recharge water. Mathematically, the cost of providing recharge water can be formulated in a similar manner to that for the cost of treating leachate. Let C_2 represent the constant that converts flux rates to the total annual cost of recharge per unit area of landfill. If C_2 is relatively constant over time, the present worth y_2, of supplying recharge water to the landfill per unit area can be summed to

$$y_2 = C_2 \left[K_l \frac{H_l}{B_l} + K_u \frac{H_u}{B_u} \right] \frac{1 + r}{r} \tag{6}$$

Finally, the cost of placing a liner with high hydraulic conductivity is usually lower than that for placing a liner with low hydraulic conductivity. The increase in cost is assumed mainly due to the intrinsic properties of the material as well as the quality of work in placing the liners. The capital cost of placing a liner with a certain hydraulic conductivity per unit volume, C_k, is formulated as

$$C_k = C_3 K^{-b} \tag{7}$$

in which C_3 and b = constants relating the hydraulic conductivity to the capital cost of placing the liner per unit volume. Then, the present worth, y_3, of placing the upper and lower liners with specific hydraulic conductivities per unit area is

$$y_3 = C_k B_u + C_k B_l = C_3 K_u^{-b} B_u + C_3 K_l^{-b} B_l \tag{8}$$

In constructing the artesian landfill, one of the objectives remains to minimize the total cost of the facility. This minimization can thus be presented as the summation of the material cost of the upper and lower liners, P_{cu} and P_{cl}, respectively, the operational cost of collecting and treating the diluted leachate, the cost of supplying the recharge water to the artesian layer, and the cost of placing the upper and lower liners with specific hydraulic conductivities. Thus, the present value of all costs, y, per unit area is as follows:

$$y = C_c B_u + C_c B_l + C_3 K_u^{-b} B_u + C_3 K_l^{-b} B_l + (C_1 + C_2) K_u \frac{H_u}{B_u} \frac{1 + r}{r} + C_2 K_l \frac{H_l}{B_l} \frac{1 + r}{r} \tag{9}$$

It is assumed that other costs involved in the construction and operation of the landfill are constant and cannot be further minimized.

GEOMETRIC PROGRAMMING

A considerable amount of work has been done to optimize the performance of various ground-water systems ranging from simple linear programming applications [e.g., Ahlfeld and Heidari (1994)] to complex nonlinear optimization techniques [e.g., Jones et al. (1987) and Ahlfeld and Hill (1996)]. In the current context, linear programming is clearly inappropriate for (9) since it is nonlinear. However, the technique of geometric programming is a powerful nonlinear programming technique well suited to this kind of problem and can account for linear and nonlinear constraints. With this method, the optimal cost (indicated with a superscript *) of a design is first determined. If acceptable, a further calculation yields the optimal values of design variables and provides a sensitivity analysis (Woolsey and Swanson (1975). Since they are central to this paper, these techniques are briefly reviewed.

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The general mathematical statement of the posynomial geometric programming problem is as follows (Beightler and Philips 1976):

$$\text{minimize } y(\underline{\mathbf{x}}) = \sum_{t=1}^{T} M_t P_t(\underline{\mathbf{x}}) \tag{10}$$

where costs M_t = positive numbers; $\underline{\mathbf{x}} = (x_1, x_2, \ldots, x_N)$ = decision variables; and functions $P_t(\underline{\mathbf{x}})$ are defined as follows:

$$P_t(\underline{\mathbf{x}}) = \prod_{n=1}^{N} x_n^{a_{tn}} \tag{11}$$

where exponent a_{tn} = known real number. Should the polynomial have negative coefficients and/or have exponents that are positive integers only, a similar technique called signomial geometric programming could be used to optimize the function.

Posynomial geometric programming seeks the optimal way to distribute the total cost among the various terms of the objective function. To do so, the concept of optimal weights W_t^* is introduced using the minimum cost $y^*(\underline{\mathbf{x}}^*)$ and the optimal values of the independent variables $\underline{\mathbf{x}}^*$ as follows (Beightler and Philips 1976)

$$W_t^* = \frac{M_t P_t(\underline{\mathbf{x}}^*)}{y^*}, \quad t = 1, \ldots, T \tag{12}$$

By definition, these weights must satisfy the normality condition by adding up to unity

$$\sum_{t=1}^{T} W_t^* = 1 \tag{13}$$

The weights must also comply to an orthogonality condition obtained by setting the first derivative of the objective function with respect to the independent variables to zero

$$\frac{\partial y}{\partial x_k} = \sum_{t=1}^{T} M_t a_{tk} x_k^{a_{tn}-1} \prod_{n \neq k} x_n^{a_{tk}} = 0, \quad k = 1, \ldots, N \tag{14}$$

Since all $x_n > 0$, these equations are reduced to

$$\sum_{t=1}^{T} M_t a_{tk} \prod_{n=1}^{N} x_n^{a_{tn}} = 0 \tag{15}$$

By combining (12) and (15), the first derivative of the objective function is transformed to

$$\sum_{t=1}^{T} a_{tn} W_t^* y^* = 0, \quad n = 1, \ldots, N \tag{16}$$

and, since y^* must be positive

$$\sum_{t=1}^{T} a_{tn} W_t^* = 0, \quad n = 1, \ldots, N \tag{17}$$

which is the simplified form of the orthogonality condition.

From (13) and (15), it is apparent that the problem has T unknowns, one for each term in the objective function. Also, the optimization is now written as a system of $(N + 1)$ simultaneous linear equations, one equation arising from the normality condition, and N orthogonal equations arising from each variable x_n. The degree of freedom for solving the problem is $T - (N + 1)$. Clearly, the degree of difficulty in solving the problem is greatly magnified as its degree of freedom increases. Once the weights are determined, the optimal cost y^* can be obtained from the following (Beightler and Philips 1976):

$$y^* = \prod_{t=1}^{T} (y^*)^{W_t^*} = \prod_{t=1}^{T} \left(\frac{M_t P_t(\underline{\mathbf{x}}^*)}{W_t^*}\right)^{W_t^*} = \prod_{t=1}^{T} \left(\frac{M_t}{W_t^*}\right)^{W_t^*} \prod_{t=1}^{T} [P_t(\underline{\mathbf{x}}^*)]^{W_t^*} \tag{18}$$

But, recalling (17) and (18), y^* is simplified as follows:

$$\prod_{t=1}^{T} [P_t(\underline{\mathbf{x}}^*)]^{W_t^*} = \prod_{n=1}^{N} \prod_{t=1}^{T} (x_n^*)^{a_{tn} W_t^*} = \prod_{n=1}^{N} (x_n^*)^{\sum_{t=1}^{T} a_{tn} W_t^*} = 1 \tag{19}$$

and, thus

$$y^* = \prod_{t=1}^{T} \left(\frac{M_t}{W_t^*}\right)^{W_t^*} \tag{20}$$

Hence, the optimal cost y^* can be obtained if the values of the coefficients M_t and the values of the optimal weights W_t^* in the objective function are known. The optimization problem thus becomes one of finding the optimal values of the weights, which is accomplished by solving the dual function (20). The dual must be maximized to obtain the optimal weights; however, if the degree of freedom is greater than zero, the dual function is not readily solved.

Maximizing Dual Function

The change of focus from determining the values of the independent variables in the posynomial to determining the values of the weights in effect changes the problem to a linear form. Recall that the dual geometric programming problem is given as [from (20)]

$$\text{maximize } d(\underline{W}) = \prod_{t=1}^{T} \left(\frac{M_t}{W_t}\right)^{W_t} \tag{21}$$

subject to (13) and (17); that is, the weights must sum to one and the weight functions are orthogonal and must satisfy the nonnegativity condition $W_t \geq 0$.

The dual constraint set forms a convex region since all constraints are linear. Rather than working with $d(\underline{W})$, the dual is sometimes transformed to its logarithmic equivalent: $z(\underline{W}) = \ln[d(\underline{W})]$. This transformation is valid since the weights W_t lie between 0 and 1, and since the logarithmic function is monotonic in the dual variables (Beightler and Philips 1976). Hence, the objective of the dual function becomes

$$\text{maximize } z(\underline{W}) = \ln \prod_{t=1}^{T} \left(\frac{M_t}{W_t}\right)^{W_t} = -\sum_{t=1}^{T} W_t \ln\left(\frac{W_t}{M_t}\right) \tag{22}$$

The dual problem is therefore reduced to finding an optimum point for a concave objective, $z(\underline{W})$, subject to a set of convex constraints. From the principles of calculus, this function has a global maximum. Thus, the global minimum for the primal problem is equivalent to the global maximum of the dual function, subject to the normality and orthogonality equations.

Inequality Constraints

Linear or nonlinear constraints arise from physical limits, from economic and financial considerations, or from environmental concerns. These can be incorporated in the geometric programming solution provided that they are of the following form (Beightler and Philips 1976):

$$g_m(\underline{\mathbf{x}}) = \sum_{t=1}^{T_m} K_{mt} \prod_{n=1}^{N} x_n^{a_{mtn}} \leq 1, \quad 1, 2, \ldots, M \tag{23}$$

There are now $N + M$ posynomial expressions (N terms in the objective function and M terms in the inequality constraints). The addition of constraints to a given problem does not affect the number of simultaneous linear equations to be solved, since this is determined by the number of independent varia-

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bles ($N + 1$). However, the total number of unknowns will increase to $N + M$, since these are dependent on the number of posynomial terms in the mathematical statement of the problem. The number of terms in the normality condition (13) is not affected.

With the addition of constraints, the coefficient of each term is double subscripted (a first subscript of 0 indicates that the coefficient originates in the objective function, a value of 1 indicates that the coefficient originates in the first constraint, etc., and the second subscript refers to the posynomial term in question). The exponent is not triple subscripted: The first two subscripts are similar to those used for the coefficient, while the third subscript indicates the variable. The dual function now has the following form (Beightler and Philips 1976):

$$\text{maximize } d(\underline{W}) = \prod_{m=0}^{M} \prod_{t=1}^{T_m} \left(\frac{K_{mt} W_{mt}}{W_{mt}} \right)^{W_{mt}} \quad (24)$$

$$\text{subject to } \sum_{t=1}^{T_o} W_{ot} = 1 \quad (25)$$

$$\sum_{m=1}^{M} \sum_{t=1}^{T_m} a_{mtn} W_{mt} = 0, \quad n = 1, 2, \ldots, N \quad (26)$$

$$W_{mo} = \sum_{t=1}^{T_m} W_{mt}, \quad m = 1, 2, \ldots, N \quad (27)$$

The normality condition (25) is formulated with respect to the number of posynomial terms in the objective function (first subscript of zero). Eq. (26) indicates that the number of unknowns is equal to the number of posynomial terms in both the objective function and the constraint equations. On the other hand, (27) shows that for each constraint having more than one posynomial term, the total weight for each constraint W_{mo} must be included in the term within parentheses in (24). Once the normality and orthogonality equations are known, the solution for an engineering problem with inequality constraints is found as before. For a more detailed description of posynomial geometric programming, the reader is referred to standard references such as Beightler and Philips (1976).

COST OPTIMIZATION OF ARTESIAN LANDFILL SYSTEM

In designing an artesian landfill, the basic decisions are to select the upper and lower liner thicknesses, to choose the liner material, and to determine the operating hydraulic potential in the artesian layer. Initially, the geometric programming model is applied to the artesian landfill system with the upper and lower liner thicknesses and hydraulic conductivities as variables. A sensitivity analysis indicates the relative importance of each cost estimate. Subsequently, the upper and lower hydraulic potentials are added to the set of decision variables, which demonstrates the incorporation of inequality constraints into the optimization model. If several types of construction materials are available, iteration of the optimization procedure is necessary to identify the lowest cost scenario that maintains the required hydraulic conditions.

Variables: Liner Thicknesses and Hydraulic Conductivities

With the liner thicknesses and hydraulic conductivities as decision variables, the hydraulic potential differences across the liners are assumed to be known. The potentials are determined from the operating hydraulics of the landfill. Cost estimates for the liner materials, the collection and treatment of leachate, the recharging of water, and placing liners with specific hydraulic conductivities are assumed available for the site, as shown in Table 1. The upper and lower hydraulic potentials, H_u and H_l, are set at an average value of 0.5 and 1.0 m, respectively. The optimal design parameters are calculated by applying the geometric programming method to (9). The associated normality and orthogonality conditions from (13) and (17) are given by

TABLE 1. Economic Analysis Parameters

Item (1)	Value (2)
Cost of collecting and treating leachate, C_1	\$0.90/m^3
Cost of recharge water, C_2	\$0.25/m^3
Discount rate, r	0.045
Material cost, $C_c = M_1 = M_2$	\$2.0/m^3
Workmanship cost, $C_3 = M_3 = M_4$	\$2.0 s/m^4
Material exponential coefficient, b	2.0
Derived parameter: $M_5 = (C_1 + C_2)H_u[(1 + r)/r]$	\$13.35 s/m^4
Derived parameter: $M_6 = C_2 H_l[(1 + r)/r]$	\$5.81 s/m^4

$$W_1^* + W_2^* + W_3^* + W_4^* + W_5^* + W_6^* = 1 \quad (28)$$

$$W_1^* + W_3^* - W_5^* = 0 \quad (29)$$

$$W_2^* + W_4^* - W_6^* = 0 \quad (30)$$

$$-bW_3^* + W_5^* = 0 \quad (31)$$

$$-bW_4^* + W_6^* = 0 \quad (32)$$

There is one degree of freedom in this problem. Hence, these constraints of the dual problem can be solved explicitly in terms of one of the variables, say W_6^*:

$$W_1^* = \frac{1 - 2W_6^*}{2} \left(1 - \frac{1}{b}\right) \quad (33)$$

$$W_2^* = W_6^* \left(1 - \frac{1}{b}\right) \quad (34)$$

$$W_3^* = \frac{1 - 2W_6^*}{2b} \quad (35)$$

$$W_4^* = \frac{W_6^*}{b} \quad (36)$$

$$W_5^* = \frac{1 - 2W_6^*}{2} \quad (37)$$

The dual (20) can then be written as

$$d = \left(\frac{M_1}{W_1^*}\right)^{W_1^*} \left(\frac{M_2}{W_2^*}\right)^{W_2^*} \left(\frac{M_3}{W_3^*}\right)^{W_3^*} \left(\frac{M_4}{W_4^*}\right)^{W_4^*} \left(\frac{M_5}{W_5^*}\right)^{W_5^*} \left(\frac{M_6}{W_6^*}\right)^{W_6^*} \quad (38)$$

where $M_1 = M_2 = C_c$; $M_3 = M_4 = C_3$; $M_5 = (C_1 + C_2)H_u(1 + r)/r$; and $M_6 = C_2 H_l(1 + r)/r$. By using the expressions determined for the optimal weights [(33)–(37)] (38) can be transformed to its logarithmic form (22) to yield

$$z = -\left(\frac{1 - 2W_6^*}{2}\right)\left(1 - \frac{1}{b}\right) \ln \left[\frac{\frac{1 - 2W_6^*}{2}\left(1 - \frac{1}{b}\right)}{M_1}\right] - W_6^*\left(1 - \frac{1}{b}\right) \ln \left(\frac{W_6^*\left(1 - \frac{1}{b}\right)}{M_2}\right) - \left(\frac{1 - 2W_6^*}{2b}\right) \ln \left(\frac{\frac{1 - 2W_6^*}{2b}}{M_3}\right) - \frac{W_6^*}{b} \ln \left(\frac{\frac{W_6^*}{b}}{M_4}\right) - \left(\frac{1 - 2W_6^*}{2}\right) \ln \left(\frac{\frac{1 - 2W_6^*}{2}}{M_5}\right) - W_6^* \ln \left(\frac{W_6^*}{M_6}\right)$$

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The global optimum for this dual function can be found by setting the first derivative of (39) with respect to the variable W_6 to zero. Performing this operation and rearranging gives

$$2\ln\left(\frac{1-2W_6}{W_6}\right) = \left(1-\frac{1}{b}\right)\ln\left(\frac{2M_1}{M_2}\right) + \frac{1}{b}\ln\left(\frac{2M_3}{M_4}\right) + \ln\left(\frac{2M_5}{M_6}\right) \quad (40)$$

When the constants of the right-hand side of (40) are known, the weights are determined by first solving for W_6 and then by using this result in (33)–(37) to determine the other weights. The optimal variables (liner thicknesses and hydraulic conductivities) are evaluated using (12).

If economies of scale occur in the construction of the upper and lower liners, the first and third terms of (9) will be of the form $C_{cl}B_u^{1+\alpha}$, where the coefficient $(1 + \alpha)$ lies between zero and 1 (Forgie 1983). In this case, the term $(1 + \alpha)$ appears as the coefficient of W_1^* and W_3^* in (13) and (17), which are otherwise handled in the same manner as previously described.

The optimal weights of the artesian landfill cost equation are determined by substituting parameter values from Table 1 into (40) to produce $\ln((1 - 2W_6)/W_6) = 1.498$. Solving this equation gives $W_6^* = 0.155$. The other optimal weights follow from (33) to (37): $W_1^* = 0.115$, $W_2^* = 0.052$, $W_3^* = 0.230$, $W_4^* = 0.103$, and $W_5^* = 0.345$. Next, the optimal cost is obtained from (20) and the optimal liner thicknesses from (12).

The posynomial geometric programming technique offers several advantages for cost optimization. For instance, the calculated optimal weights reveal the distribution of capital to keep costs at a minimum. In the present example, the material cost of construction of the upper and lower liner should be 5.8 and 2.6% of the optimal total cost, respectively. The parameter values used yield optimal thicknesses of 1.104 and 0.494 m for the upper and lower liners, respectively; optimal hydraulic conductivities of 2×10^{-8} m/s for both upper and lower liners; and optimal total cost of \$19.18/m^2.

Sensitivity Analysis

Although preliminary cost parameters are often crudely estimated, sensitivity analysis may reveal which coefficients most significantly affect the optimal cost of the design and ultimately the thicknesses and hydraulic conductivities of the liners. With this information, a greater effort in the estimation of the significant cost parameters can be subsequently expended. A set of expressions used to achieve this task was originally developed by Theil (1972).

By comparing (9)–(11), the values of a_{tn} are given in matrix form as

$$\tilde{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \\ a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -b & 0 \\ 0 & 1 & 0 & -b \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad (41)$$

For convenience, a 6×6 diagonal matrix $\tilde{\mathbf{W}}^*$ is introduced with the known optimal weights (W_1^*, W_2^*, etc.) as the diagonal entries. An intermediate matrix $\tilde{\mathbf{S}}$ is then formed to find the change in W_t^* (Theil 1972) as follows:

$$\tilde{\mathbf{S}} = \tilde{\mathbf{W}}^*\tilde{\mathbf{A}}(\tilde{\mathbf{A}}'\tilde{\mathbf{W}}^*\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{A}}'\tilde{\mathbf{W}}^* \quad (42)$$

where $\tilde{\mathbf{A}}'$ = transpose of $\tilde{\mathbf{A}}$ and the minus one superscript indicates the inverse function. The change in W_t^* (i.e., dW_t^*) can now be found as

$$dW_t^* = W_t^* d[\ln \bar{M}_t] - \sum_{j=1}^{T} S_{tj} d[\ln M_j], \quad t = 1, \ldots, T \quad (43)$$

where

$$d[\ln \bar{M}_t] = d[\ln M_t] - \sum_{k=1}^{T} W_k d[\ln M_k], \quad t = 1, \ldots, T \quad (44)$$

The new values of W_t^* equal $W_t^* + dW_t^*$. By altering the values of the parameters listed in Table 1, different values of y^*, K_u^*, K_l^*, B_u^*, and B_l^* can be obtained. For example, by changing the value of C_1 from 0.9 to 1.8, the new set of solutions can be obtained immediately. Finally plotting these values allows the complete sensitivity analysis to be clearly displayed. For completeness, a thorough sensitivity analysis is performed by observing the effect of varying each coefficient of (9) on the optimal cost and liner thicknesses. However, most of the observed trends can also be inferred by comparing the relative weights of each term in (9).

Fig. 2 shows that an increase in the material cost of the liners, in the hydraulic potentials, in the cost of supplying the recharge water, in the cost of collecting and treating leachate, and in the cost associated with placing a liner with specific hydraulic conductivity is translated to an increase in the optimal cost of the artesian landfill system and vice versa. It is also revealed that changes in the parameters associated with the hydraulic potential difference across the upper liner have a greater effect on the optimal cost. Conversely, Fig. 2 shows that an increase in the discount rate decreases the optimal total cost of the landfill although it does not vary the weights of each posynomial term in the objective function. In fact, the discount rate appears in the objective function to give an equivalent present value to the operational costs associated with the supply of recharge water and the dilution of the leachate. Since a higher discount rate is equivalent to a lower present value for the operational costs, the optimal linear thicknesses decrease with an increase in the discount rate, as it is more economical to treat leachate and to supply more water in the future than to construct robust liners. Clearly the choice of the discount rate is crucial to the optimization model.

Figs. 3 and 4 indicate that the greater the material costs of the liners, the lesser their optimal thicknesses. On the other hand, an increase in the cost of supplying the recharge water, in the cost associated with the dilution of the leachate, and in the cost associated with placing a liner with specific hydraulic conductivity is translated to an increase in the optimal thicknesses. This was expected from the formulation of the objective function, which identified a trade-off between the material cost of construction the upper liner and the cost of treating additional leachate, between the material cost of constructing the liners and the cost of supplying recharge water, and between the material cost of constructing the liners and the cost of having a liner with lower hydraulic conductivity. Hence, for

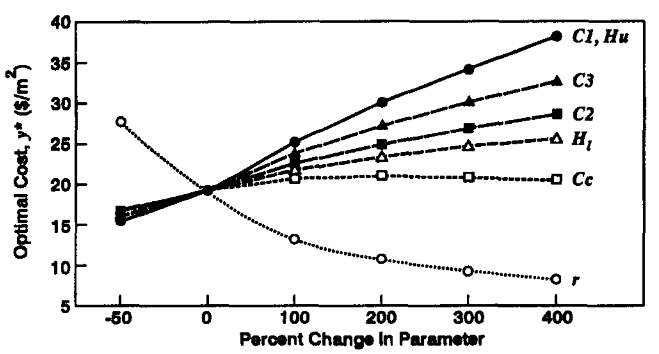

FIG. 2. Sensitivity Analysis of Optimal Cost (Unconstrained Problem)

a greater material cost, it is economically advantageous to construct thinner liners, thus allowing more recharge water to seep through the liners and requiring additional leachate to be collected and treated.

Figs. 5 and 6 indicate that the greater the material cost of the liners, the lesser the optimal hydraulic conductivities. Conversely, an increase in the cost of building a liner with low hydraulic conductivities leads to an increase in optimal hydraulic conductivities. Again, there is a trade-off between the hydraulic conductivities of the liners and their material cost.

The sensitivity analysis also gives an indication of design criteria for the hydraulic potentials. Once the material has been fixed, Fig. 4 reveals that the higher the hydraulic potential applied to the artesian layer, the greater is the optimal thickness of the upper liner; therefore, the greater is the optimal cost. This is not surprising, since a larger applied potential (for a constant thickness) would cause more water to leak through the liner and mix with the leachate. Although the design must account for fluctuations in the upper potential, hydraulic potentials should always be kept at their minima to minimize total cost.

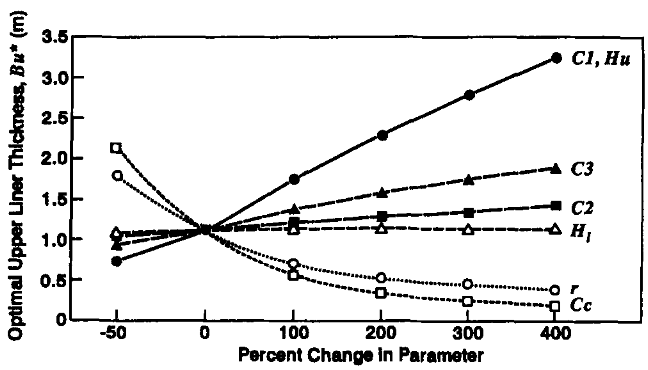

FIG. 3. Sensitivity Analysis of Optimal Upper Liner Thickness (Unconstrained Problem)

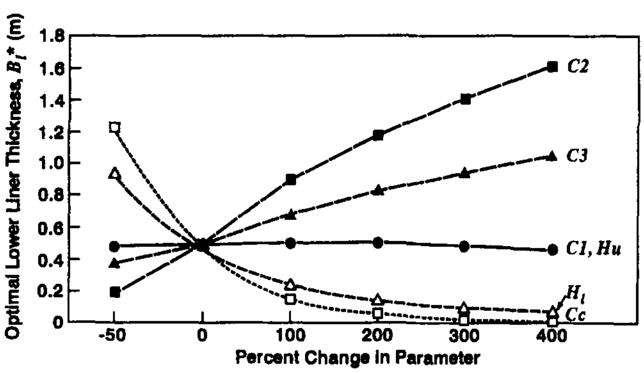

FIG. 4. Sensitivity Analysis of Optimal Lower Liner Thickness (Unconstrained Problem)

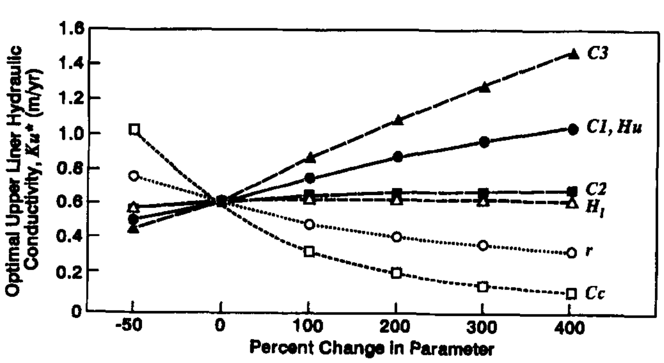

FIG. 5. Sensitivity Analysis of Optimal Hydraulic Conductivity of Upper Liner Thickness (Unconstrained Problem)

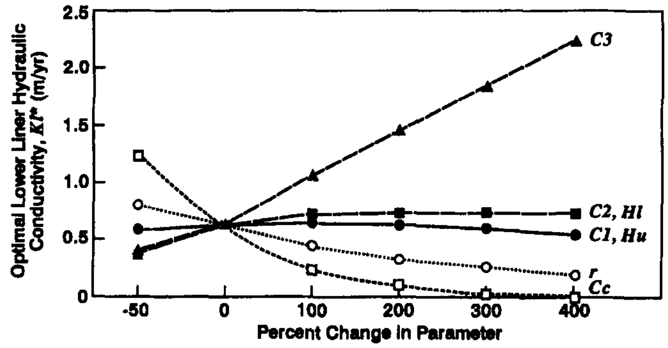

FIG. 6. Sensitivity Analysis of Optimal Hydraulic Conductivity of Lower Liner Thickness (Unconstrained Problem)

Variables: Liner Thickness, Hydraulic Conductivities, and Hydraulic Potentials

Posynomial geometric programming is capable of handling linear and nonlinear constraints. To demonstrate the treatment of inequality constraints, the decision variables in the objective function can be chosen as the liner thicknesses, the hydraulic conductivities, and the hydraulic potentials. These constraints can be written in the following form [from (23)]:

$$\frac{H_{u,\min}}{H_u} \le 1 \qquad (45)$$

$$\frac{H_{l,\min}}{H_l} \le 1 \qquad (46)$$

The number of unknowns in this case is equal to the number of posynomial terms in both the objective function and the constraint equations. The normality equation is formulated with respect to the number of posynomial terms in the objective function (25) as follows:

$$W_{01}^* + W_{02}^* + W_{03}^* + W_{04}^* + W_{05}^* + W_{06}^* = 1 \qquad (47)$$

Notice that this expression is identical to (28) except that the subscript changes from i to $0i$. Similarly, the first four orthogonality conditions can still be written as (29)–(32) by changing the subscripts from i to $0i$. The last two orthogonality conditions are

$$W_{05}^* - W_{11}^* = 0 \qquad (48)$$

$$W_{06}^* - W_{21}^* = 0 \qquad (49)$$

In total, the normality and orthogonality conditions provide seven equations and eight unknowns. All of the other optimal weights can be written in terms of W_{06}^*. The first five expressions are identical to (33)–(37) with $0i$ substituted in place of the subscript i; the final two expressions are given by

$$W_{11}^* = \frac{1 - 2W_{06}^*}{2} \qquad (50)$$

$$W_{21}^* = W_{06}^* \qquad (51)$$

By using (24)–(27), the dual function is formed. Again, by setting the first derivatives of the logarithm of the dual function to zero, the weight W_1^* can be found. If the cost estimates from Table 1 are used and the minimum hydraulic potential is equal to 0.3 m, the optimal thicknesses for the upper and lower liner are 0.855 and 0.270 m, respectively; optimal hydraulic conductivities for the upper and lower liner are both 2×10^{-8} m/s; the optimal cost is \$13.51/m^2, and the optimal hydraulic potential difference is equal to the minimum desirable values.

Sensitivity Analysis

Another set of procedures has been developed by Dinkel and Kochenberger (1974) to perform the sensitivity analysis

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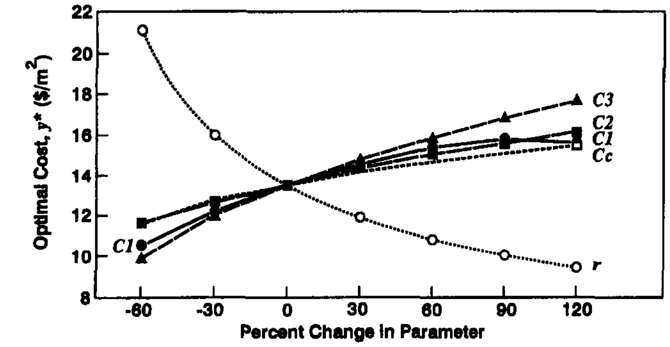

FIG. 7. Sensitivity Analysis of Optimal Cost (Constrained Problem)

on constrained problems. In this approach, developed by Theil (1972) for the unconstrained problem, the optimal weights, W^*_{mt}, for the initial values of the parameters are adjusted by changing the value of any one of the parameters. After this, new values of y^*, B^*_u, B^*_l, H^*_u, and H^*_l can be obtained. Results of the sensitivity analyses are summarized in Fig. 7 for optimal cost. The sensitivity analyses of other design parameters, B_u, B_l, K_u, and K_l, are given in Cormier (1990). However, the results are similar to those of the unconstrained problem except that they also reveal that the optimal hydraulic potentials are equal to the stipulated minima.

APPLICATION OF CONCEPTUAL COST MODEL

The preceding examples demonstrate the procedures of performing the optimization and sensitivity analysis techniques. During the discussions, the hydraulic potential differences across the upper and lower liner are assumed to be known and can be controlled by the rate of water supplied by the recharge well. The piezometric head along the artesian layer generated by the water supplied by the recharge well is called the operating head, H_o. The hydraulic potential differences across the upper and lower liner are functions of the operating head. Under normal operation, it is assumed that the piezometric head along the lower boundary of the lower liner and that along the upper boundary of the upper liner are close to atmospheric pressure. Referring to Fig. 1, the relationships between the operating head and the hydraulic potential differences are

$$H_u = H_o - (B_u + A + B_l) \tag{52}$$

$$H_l = H_o \tag{53}$$

in which H_u and H_l = hydraulic potential differences across the upper and lower liner, respectively; B_u and B_l = thicknesses of the upper and lower liner, respectively; H_o = operating head; and A = thickness of the artesian layer (A = 0.75 m here). Therefore, in applying the optimization technique, instead of having the hydraulic potential differences as the variables, the operating head is the variable. By specifying a proper value of the operating head, the optimal design is determined by combining the preceding optimization technique with (52) and (53). An example used to illustrate this procedure is discussed in Lai et al. (1995).

NUMERICAL ANALYSIS WITH FINITE-ELEMENT MODEL

The flow patterns in an artesian landfill system can often be complex—particularly under abnormal operating conditions such as those caused by a liner rupture or a clogged drain. In the design of an artesian system, it is important to know the behavior of the system under such situations and a numerical approach is developed for this purpose.

The present algorithm solves a heterogeneous anisotropic coupled saturated-unsaturated flow equation using Galerkin's finite-element method [e.g., Istok (1989)]. The finite-element code was verified with a block-centered finite-difference code for simple flow conditions and showed good agreement with analytical flow solutions (Seneviratne 1991). The two-dimensional equation governing flow in the landfill is given as follows (Bear 1972; Freeze and Cherry 1979):

$$[C(\psi) + \theta' S_0]\frac{\partial \psi}{\partial t} = \frac{\partial K_x(\psi)\dfrac{\partial \psi}{\partial x}}{\partial x} + \frac{\partial K_y(\psi)\dfrac{\partial \psi}{\partial y}}{\partial y} \tag{54}$$

in which $C(\psi)$ = specific moisture capacity; θ' = degree of saturation; S_0 = specific storage; $K(\psi)$ = unsaturated hydraulic conductivity; ψ = pressure head; x and y = space variables; and t = time. In the unsaturated zone, the compressibility effects of the soil matrix and the pore water are assumed insignificant; hence, $C(\psi)$ dominates the left-hand side of (54). In the saturated zone $C(\psi)$ = 0, and the specific storage term dominates while $K(\psi)$ becomes equal to the saturated hydraulic conductivity. The associated finite-element equations used to represent transient ground-water flow are given in Istok (1989) while Lai (1994) provides detailed descriptions of the application of the finite-element method to the current problem.

To simulate the flow within an artesian landfill system, it is necessary to define the boundary conditions and the geometry of the landfill. Fig. 8 depicts a symmetric half of the landfill domain. (For reference, the 240-element grid for the finite-element solution is depicted in Figs. 10 and 11.) In this case, the property of symmetry is used to reduce the number of elements and, hence, the computational time. However, such a simplification can slightly distort reality, particularly when failure conditions are analyzed, because it conservatively assumes that a given failure such as a rupture in the top liner has occurred on both sides symmetrically. The landfill domain is modeled as an initial-boundary value problem. The boundary conditions and system geometry are as follows: The bottom surface in contact with the soil is a Dirichlet boundary with a negative pressure of 0.002 m; the central boundary of symmetry is a no-flow boundary and the top surface is a Newmann boundary with an infiltration rate of 0.006 m/day; water is ponded at the top edge of the sloping artesian layer; the confining material is clay and the artesian material is sand; and the drain slope is 4% while the side slope is 45°.

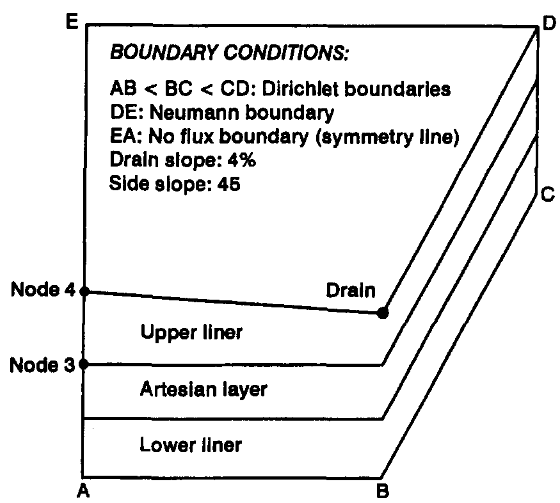

FIG. 8. Symmetric Half of Artesian Landfill: Boundary Conditions

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SYSTEM PERFORMANCE

For an artesian landfill to work properly, the potential difference across the upper liner must be maintained above a desired value. This can be controlled by altering the operating head. If the system fails due to the rupture of the top liner or the clogging of the drains, the failure can be detected immediately by knowing the hydraulic potential difference across the upper liner. Therefore, under normal and abnormal operating conditions, a simple monitoring program can be used to maintain automatically the potential difference across the upper liner within a desirable range by increasing the operating (or ponding) head when necessary. Similar adaptive strategies have been recommended by others for different applications [e.g., Jones (1992)].

The procedure for maintaining the artesian system is quite simple: The operating system monitors the head difference at critical nodes and determines whether the head difference is adequate; if the head difference is too small, the operating head is increased and vice versa. In all situations considered herein, the minimum hydraulic potential difference occurs on the far left-hand side of the flow system near the axis of symmetry. This is because the upper boundary of the upper liner is sloping downward towards the drain. Under normal operation, the pressure head along the upper boundary of the upper liner is close to atmospheric pressure. Therefore, the piezometric head is largest at node 4 (see Fig. 8). Moreover, when the water is flowing from the supply on the right-hand side through the artesian layer the piezometric head is smallest at node 3 since the head decreases in the flow direction. Hence, the upward hydraulic potential difference is smallest across nodes 3 and 4. By monitoring the hydraulic heads at nodes 3 and 4, the minimum desirable hydraulic potential difference across the upper liner can be maintained. A physically reasonable minimum value is assumed to be 0.3 m here. Before the optimum value of the operating head has been determined, the system is first simulated with an arbitrarily chosen value of the operating head. If, during the simulation, the operating head is found to be insufficient to provide the desired hydraulic potential difference across the upper liner, the operating head is increased by a predetermined amount (taken here as 0.04 m). This operation is illustrated in the following examples.

Normal Operating Condition

The boundary conditions and the geometry of the landfill have been described earlier. The desired average hydraulic potential difference across the upper liner is set to 0.30 m. The thickness of the artesian layer is assumed to be 0.75 m, and its hydraulic conductivity is 4.8×10^{-3} m/s. The thicknesses of the liners, their hydraulic conductivities, and the operating head can then be determined using the cost optimization technique described previously. The resulting optimal thicknesses for the upper and lower liners are 0.855 and 0.815 m, respectively. The optimal hydraulic conductivities are both 2×10^{-8} m/s, and the optimal operating head, as measured above the base of the lower line, is 2.72 m.

These design variables are average values only and are based on the steady-state optimization model. Thus, for example, the "optimal operating head" of 2.72 m refers to the average hydraulic potential difference acting on the upper liner and does not explicitly account for hydraulic losses in the artesian system. If this value is incorrectly interpreted as the applied operating head instead of its average net (after losses) value, the numerical model predicts a steady-state hydraulic potential across nodes 3 and 4 of only 0.13 m. To find an appropriate operating head, an initial estimate of 3.3 m was chosen. Simulation showed that although an operating head of

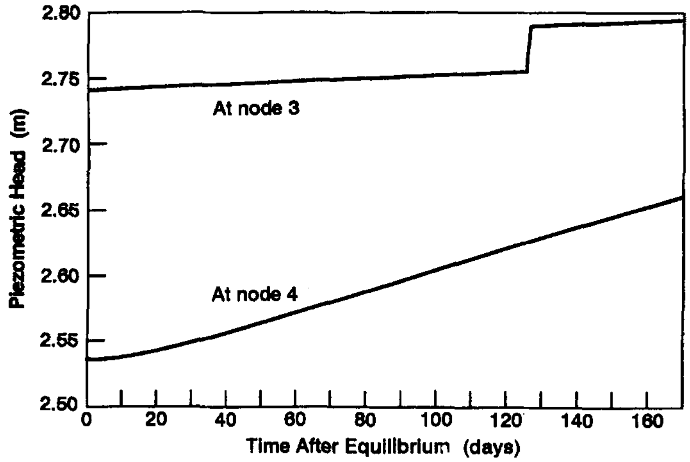

FIG. 9. Head versus Time under Normal Operation with Adaptive Control

3.3 m is sufficient to prevent downward flow for the first 170 days, a larger head value will be needed shortly. Thus, the simulation was repeated with adaptive control on the operating head, as shown in Fig. 9. The adaptive approach maintains the hydraulic potential difference across nodes 3 and 4 at all times above the desirable value. At a time equal to 170 days, the operating head needed is 3.46 m. Fig. 10 depicts the flow velocity field at 170 days and is typical of the artesian system in normal operation.

Of course, achieving a desirable flow distribution like that shown in Fig. 10 for normal operation is important, but it does not guarantee that other problems will not arise. In this paper, two kinds of "failures" are investigated: (1) The failure of the hydraulic integrity of the upper liner; and (2) the clogging of a drain pipe. These numerical studies demonstrate the robust nature of the artesian design relative to the conventional landfill system.

Rupture of Upper Liner

The top liner of an artesian landfill could conceivably rupture for different reasons. Thus, it is prudent to test the artesian system for its behavior when subject to this kind of failure. For this purpose, the finite-element simulation is run for a period of time to achieve steady-state operation, and then a significant rupture is initiated in the upper liner. Clearly, only a significant rupture need be considered since any small adjustment in hydraulic conductivity over a limited area has a negligible effect on the hydraulics of the artesian system.

Failure conditions are simulated by inserting elements into the top liner with a relatively higher hydraulic conductivity. In this case, four elements are ruptured starting with the third element from the right edge of the top clay liner (Fig. 11). The hydraulic conductivities of the ruptured elements are 1.2×10^{-6} m/s, an increase of two orders of magnitude from their previous values. The velocity distribution one day after failure is shown in Fig. 11, while Fig. 12 shows the heads at nodes 3 and 4 both with and without adaptive control. Clearly, these figures indicate that without adaptive control the system fails as early as 1 day after the rupture occurs. Fig. 12 shows that with adaptive control the system still fails after 1 day, but that the adaptive function is soon able to restore artesian operation. In fact, due to the loss of water through the failure, the pressure in artesian layer drops substantially, and it takes about 6 days to restore the system and correct the failure. The adaptive increase in operating head under such failing condition is not large enough to correct instantly for the loss in pressure. However, if the adaptive increase in operating head is doubled to 0.08 m, the system can be restored in 4 days. The final op-

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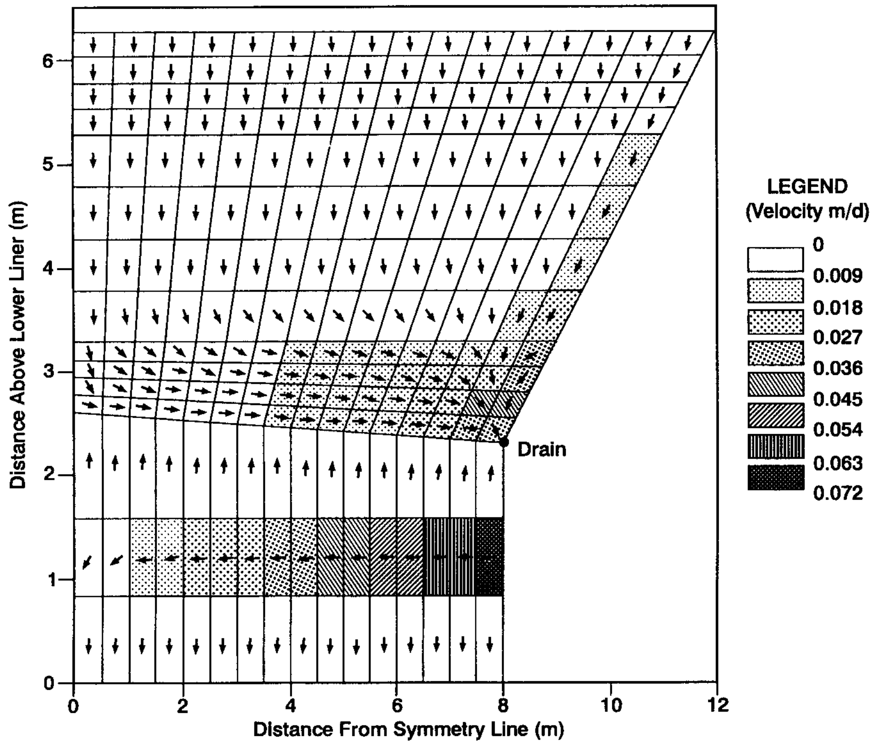

FIG. 10. Velocity Diagram at 170 days under Normal Operation

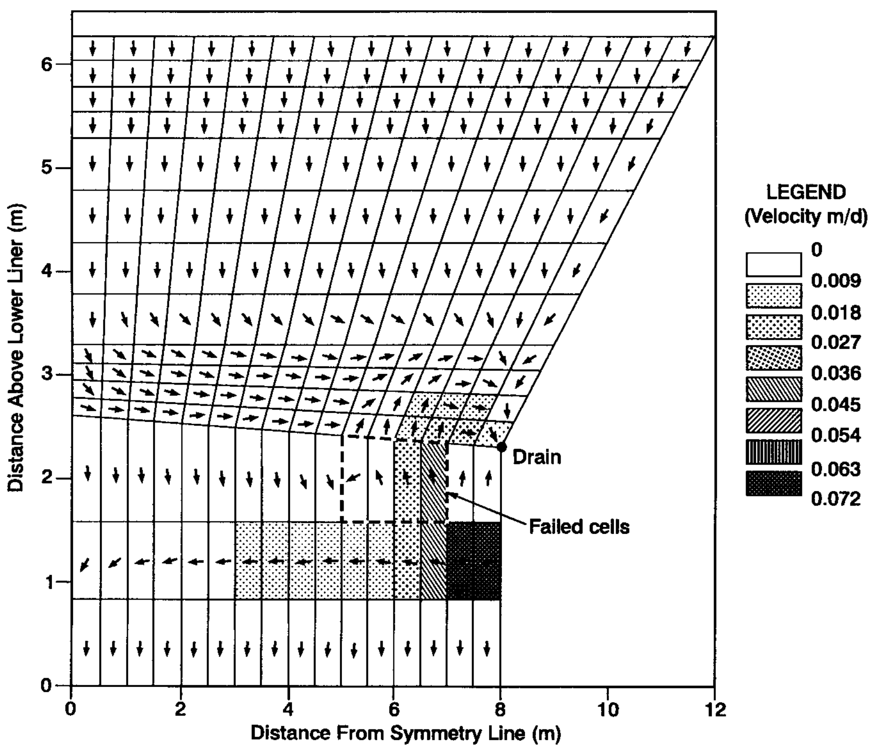

FIG. 11. Velocity Diagram at 1 day after Rupture

erating head needed at 170 days is 5.14 m, which produces a net operating head at node 3 of 3.15 m as shown in Fig. 12.

The rupture of the upper liner can impose a significant effect on the artesian design. If the liner failure is dramatic and covers a substantial area, the landfill will respond quickly to this kind of failure. However, the adaptive control system can restore the system to normal operating conditions within several days. The artesian landfill design currently allows a solution to this kind of failure that is not possible in a conventional landfill design.

Clogging of Drains

In the following analysis, a scenario is modeled in which the drains become clogged causing the hydraulic potential to build up on the upper liner. This is a failure mechanism that is particularly severe for a conventional landfill design because the rate of leachate loss tends to increase significantly as the hydraulic driving force increases. However, the artesian system behaves robustly for this kind of operational problem.

Under normal operating conditions, the drain is treated as a

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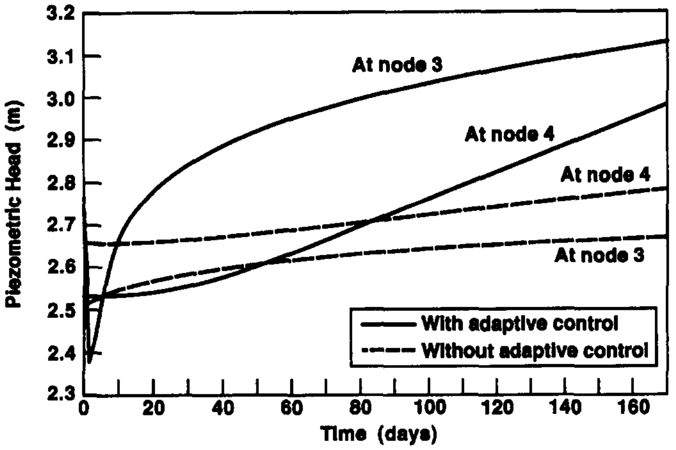

FIG. 12. Head versus Time with Upper Liner Rupture with and without Adaptive Control

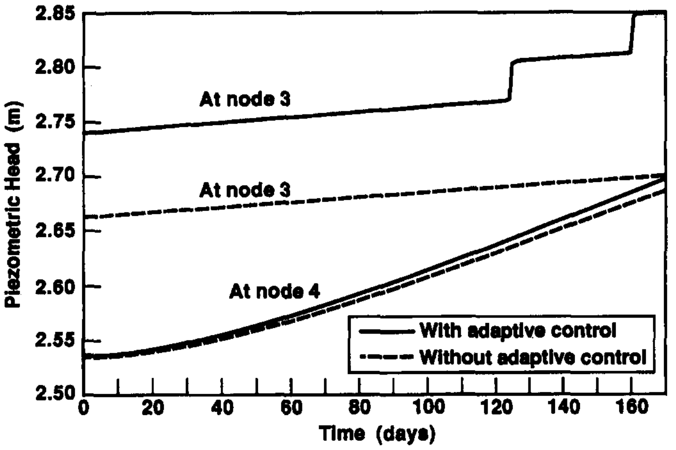

FIG. 13. Head versus Time in Clogging Scenario with and without Adaptive Control

Dirichlet boundary with atmospheric pressure being its magnitude. The clogging is simulated by removing this boundary condition and treating the "drain node" as a regular node. The procedure is to simulate the system under normal conditions until equilibrium is achieved and then to introduce the clogging event. This scenario was again simulated both with and without the adaptive operating head. With a constant operating head of 3.3 m, the system does not fail hydraulically at 170 days after clogging (Fig. 13). With the adaptive operating head, the hydraulic potential difference across nodes 3 and 4 is maintained above the desirable value (Fig. 13). The operating head needed to maintain the system at 170 days is 3.51 m. Thus, unlike the conventional system, the artesian landfill reacts slowly and controllably to this kind of failure.

SUMMARY AND CONCLUSIONS

The artesian landfill system differs from conventional sanitary landfill design in that it contains both a lower semipermeable liner and a granular recharge (artesian) layer. Its multilayered construction permits the creation of an artesian potential, controlled through the supply of recharge water. Hence, the hydraulic potential on its upper liner is reversed with respect to the conventional landfill: The flow is inward from the artesian layer through the upper liner. A hydraulic potential also moves water downward through the lower liner.

In presenting a conceptual model of the artesian landfill system, economic trade-offs were identified between the capital costs of constructing a robust upper liner and the operational costs associated with the collection and the treatment of a diluted leachate. Also, a trade-off exists between the capital costs of constructing robust upper and lower liners and the operational costs of supplying recharge water. The technique of posynomial geometric programming was employed for the cost minimization of the artesian landfill design. The values of the relative weights of the posynomial objective equation and the sensitivity analyses provide insight to the importance of the various parameters involved in the design of the landfill. The sensitivity analyses revealed that the choice of a discount rate and the specified hydraulic potential are crucial to a reasonable estimate of the optimal cost and the optimal liner thicknesses. Also from the sensitivity analyses, the material cost of constructing the liner is critical to the determination of optimal liner thicknesses.

A finite-element model of hydraulic performance provides an integrated understanding of the response of the saturated-unsaturated zones in the artesian landfill. Further, it demonstrates the significance of the transient conditions and exhibits the necessary changes to be made in the operating head with time. In general, the finite-element approach verifies that the artesian landfill system is a robust and safe-fail system. Two hypothetical failure modes were examined: (1) Rupture of the upper liner; and (2) clogging of the leachate drains. In both these failure situations, the artesian system provides a simple remediation—an increase in operating head. In fact, even for a significant failure of the upper liner, this simple stratagem was able to restore normal operations quickly. Moreover, unlike conventional landfills, the artesian system performs well if a clogged drain occurs and forces temporary operation with higher heads. In fact, it should be emphasized that although failure of the artesian landfill is possible under some circumstances, the artesian system inevitably performed better than the conventional system under similar conditions.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = artesian layer thickness;
$\tilde{\mathbf{A}}$ = matrix of exponents on decision variables;
a_{tk}, a_{tn}, a_{mtn} = exponents on decision variables;
B_u, B_l = upper and lower liner thicknesses, respectively;
b = hydraulic conductivity cost exponent;
$C(\psi)$ = specific moisture capacity;
C_c = construction cost of placing liner material per unit volume;
C_k = capital cost of placing liner with certain hydraulic conductivity per unit volume;
C_l = cost of collecting and treating additional leachate;
C_1, C_2, C_3 = constants converting flux rates to costs;
$d(W)$, d = dual function or dual objective function;
$g_m(\underline{\mathbf{x}})$ = mth constraint function;
$H_{l,\min}$, $H_{u,\min}$ = minimum hydraulic potential difference across lower and upper liners, respectively;
H_o = operating head;
H_u, H_l = hydraulic potential differences acting across upper and lower liners, respectively;
K, K_u, K_l = hydraulic conductivities (single subscript);
$K(\psi)$ = unsaturated hydraulic conductivity, where (ψ) is pressure head;
K_{mt} = cost coefficient on mth constraint;
M_t = cost coefficient on tth term of objective function;
N = number of component terms in objective function;
N_t = number of years of landfill operation;
P_l = total construction cost of placing upper and lower liner material per unit area;
$P_t(\underline{\mathbf{x}})$ = tth polynomial term of objective function;
Q_u, Q_l = specific discharge through upper and lower liners, respectively;
r = annual discount rate;
$\tilde{\mathbf{S}}$ = matrix used to determine change in weight terms in sensitivity analysis;
S_0 = specific storage;
$\tilde{\mathbf{W}}$ = matrix of weights of unconstrained objective function;
W_{mt} = weight in dual/constraint function and weight in unconstrained objective function ($m \neq 0$) and ($m = 0$), respectively;
W_t = weight of tth term of unconstrained objective function;
$\underline{\mathbf{x}}$ = vector of decision variables ($x_1, \ldots, x_n$);
x_n = nth decision variable;
$y(\underline{x})$, y = objective function (y = value);
y_1 = present value of upper liner, leachate, and treatment cost;
y_2 = present value of cost of supplying recharge water;
y_3 = present value of upper and lower liner per unit area;
$z(\underline{W})$, z = logarithm of dual function (z = value);
α = material cost exponent;
θ' = degree of saturation; and
∂t, $\partial\psi$ = time increment and pressure head increment, respectively.

Subscripts

k, n = indices for decision variables, k, $n = 1, \ldots, N$;
m = index for constraint function $m = 1, \ldots, M$; $m = 0$, for objective function;
min = index for minimum of constraint level;
t = index for polynomial terms $t = 1, \ldots, T$;
u, l = index for upper and lower liners, respectively; and
x, y = horizontal and vertical coordinate directions, respectively.

Superscripts

$*$ = optimal value of variable or function.