Does diversification contribute to the resiliency of a residential loans guarantee scheme?

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Abstract

To secure housing loans, French banks do not necessarily ask for a mortgage but rely mainly on the guarantee provided by a residential property loan guarantor. Using an economic capital approach and the data of a major French guarantor over the 1997-2013 periods, this paper illustrates the ability of a guarantee mutual fund to benefit from diversification effects coming from the heterogeneity across participating banks. Thanks to these large diversification benefits, the required deposits of the borrowers amount on average over the period to less than 0.10 percent of the total of their loans to insure the stability of the fund at the one year horizon. During the recent crisis, this percentage rises to 0.20, to be compared to the current charge of 1.5%, of which a part is usually refunded to borrowers when the loan is repaid. These results are evidence of the resilience of the French guarantee system in adverse scenarios. However, analyzing the impact of the 2008 crisis on the required economic capital of the fund also shows that the heterogeneity across banks still exposes the fund to concentration risk, i.e. the risk to observe extreme losses concentrated on a given lender.

Keywords: housing finance, mortgage insurance, credit guarantee funds, credit risk management.

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1. Introduction

As demonstrated by the recent crisis, residential loans are risky and lenders are looking for protection against tail risk. Moreover, the recent crisis has revealed that mortgage insurance could be subject to significant stress in the worst tail events, as emphasized by the BCBS Joint Forum (2013). To secure loans, French banks mainly rely on the guarantee provided by a residential property loan guarantor. They ask their borrowers to directly subscribe to a mutual guarantee fund managed by a guarantor. In case of default, the guarantor pays back the loan value (including accrued interests) to the bank and manages the resolution of the failure of the borrower on its own. Thus, instead to ask for a mortgage or to require the services of an insurer, the lenders rely on a guarantee scheme based on the mutualization of credit risk. The use of mutual guarantees has significantly increased in France since the beginning of the 1990s. A yearly survey of the French banking supervisor shows that more than 51% of outstanding housing loans were secured by a guarantee at end 2013, while only 36% were secured by a mortgage (ACPR, 2014). One can distinguish three reasons why credit guarantee schemes emerge (Honohan, 2010). First, informational advantages of the guarantor over the lender can help overcome information asymmetries and reduce borrowing costs, especially for borrowers considered as opaque or with insufficient track record (small firms, young households...). As underlined by Beck and al. (2010), quoting Ghatak and Guinnane (1999), this is the fundamental reason why cooperative banks emerged in Germany in the 19th century. Second, guarantee schemes can emerge to exploit regulatory arbitrage if the guarantor is not subject to the same regulatory requirements as the lender. And, third, guarantee schemes can help diversify risk across lenders. This paper focuses on this last argument by evaluating the diversification provided by the aggregation of several portfolios in one guarantee fund. Using data from a guarantor that aggregates owner-occupied housing loans from six French major banking groups, we show that the mutualization of portfolios from heterogeneous contributing banks offers a considerable potential of risk reduction. However, the analysis of the impact of the 2008 crisis shows, that if diversification benefits still exist, a mutual guarantee fund would still be exposed to the risk of concentrated losses at the contributing bank level.

In a guarantee scheme, the borrowers feed a reserve mechanism or a mutual fund, depending of the case, so that guarantors enable a mutualization of credit risks between borrowers. The guarantor manages the mutual fund and provides the mutualization services either to several independent banks or to only the lending entities of the banking group to which it belongs. In case of default, the guarantor adds the loan on its book and puts the guarantee into force. Then, payments are made to the guaranteed bank which receives around 100% of the outstanding loan. Consequently, contrary to a mortgage, the guarantee mechanism disconnects loans losses from the house prices from the lender's perspective. Thus, the lenders are protected against adverse changes in the residential property market Therefore, contrary to a standard mortgage; the French guarantee provides a mechanism for financial stability that disconnects the lender's loss given default from house prices. However, this feature calls for the supervision of the risks borne by the guarantors. In France, guarantors operate under a banking or an insurance license and they are subject to prudential regulations (in particular, capital requirements) that are comparable to the Basel II and III regulations for financial institutions and Solvency I and II for insurance companies, which limits the potential for regulatory arbitrage. This economic model of guarantee schemes has several advantages for the lenders and, in a more general perspective, for the stability and resiliency of the residential credit market. More specifically, these features obviate the need for banks to provision for defaulted guaranteed loans. Moreover, being specialized in the management of credit risk, guarantors provide a second screening of banks' credit files before loans are granted. They also allow for structural costs savings in managing the recovery of defaulted loans.

Guaranteed residential loans have experienced lower delinquency rate than the total French housing loan market in the 2000s[1]. In addition, the loss rate for guaranteed loans stayed in the range of 10% to 12%, lower than the average loss rate on housing loans (EBA, 2014). Therefore, due to the extensive risk coverage supplied by the guarantor, we observe a highly controlled loss rate for French residential property loans and a risk-weighting for guaranteed loans lower than that of mortgaged loans. The abovementioned "double screening" and the efficiency of guarantors in the recovery process certainly contribute to explain this feature[2].

Dealing with this diversification issue calls for an appropriate methodology. In the guarantee scheme,

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[1] The rate of doubtful residential loans for guaranteed loans was around one third of the same rate for the entire market over 2003–2012 (ACPR, 2014; EBA, 2014).

[2] This may also be the consequence of the French rule of "common pledge" (articles 2028 and 2029 of the French Civil Code) under which every lender is entitled to seek reimbursement of the debt by taking control of all assets or income sources of the borrower who defaults, what allows maintaining LGDs to relatively low levels. Indeed, ass long as the guarantor is not included in the lender's consolidation perimeter, its guarantee may count as non-financed credit protection in order to adjust the risk-weights computed on guaranteed portfolios. In that respect, it is worth reminding that banks can either consider that the guarantee lowers their loss given default (LGD) -just like for mortgage credits- or that they are directly exposed to the guarantor (that is, the bank applies the probability of default (PD) and LGD -thus the risk weight- attached to the guarantor instead of the borrowers' ones).

insolvency occurs if the total required reimbursements exceed the total capital invested in the mutual fund. Assuming a given level of capital, the more exposed are the borrowers to economic situations which create simultaneous defaults (potentially, a wave of defaults), the more likely is the insolvency of the fund. Hence, the capital of the mutual fund will protect the guarantee scheme only at a given confidence level, which is chosen by the mutual fund itself. Therefore, an economic capital approach is particularly suitable to deal with this issue. Economic capital is defined as an estimate of the worst possible decline in the fund's amount of capital at a specified level of confidence within a chosen time horizon. Thus, economic capital can be viewed as the amount of capital that is invested in the mutual fund as a direct function of the risks to which the fund is exposed.

An economic capital approach considers credit risk at the portfolio level. At the portfolio level, the main issue is to avoid an excessive concentration of credit risk on certain groups of borrowers. In mortgage markets, theoretical and empirical research has demonstrated that certain categories of borrowers, such as low income borrowers (Ergungor, 2010) or borrowers choosing adjustable rate mortgages (Campbell and Cocco, 2003, Koijen, Van Hemert and Van Niewerburg, 2009, Van Hemert, 2009), may exhibit high sensitivities to external shocks which may greatly increase their propensity to default on their loans. Thus, the lender should avoid a potential concentration of defaults in certain segments of borrowers sensitive to adverse conditions. Thus, this paper is related to the rich literature about mortgage default (Campbell and Cocco, 2014) and the housing boom and bust (Davis and Van Nieuwerburgh, 2014). But, to the best of our knowledge, no study before was devoted to the economy of the residential guarantee scheme. Most papers in the guarantee field are devoted to public credit guarantee schemes as mechanisms to ease the access to credit for SME (Beck, Klapper and Mendoza, 2010, Cowling, 2010, Columba, Gambacorta and Mistrulli, 2010, D'Ignazio and Menon, 2013).

This paper is focusing on the observation that all borrowers are not sensitive to the same common risk factors or are not equally exposed to the common risk factors. In a sense, borrowers are heterogeneous and this heterogeneity could potentially be a source of credit portfolio diversification if heterogeneous borrowers are exposed to risk factors which are weakly or negatively correlated. Thus, managing mutual fund risk implies to take into account the borrowers' heterogeneity[3] to try to extract diversification benefits as far as possible.

[3] In a portfolio of housing loans, borrowers' heterogeneity may be associated to loan characteristics. Indeed, different borrowers are making different choices of interest rate type, LTV and DSTI ratios, and the choice of these characteristics reflects the exposure of these borrowers to different risk factors (Dietsch and Welter-Nicol, 2014).

To implement the economic capital approach, this paper expands the Asymptotic Single Risk Factor (ASRF) model (Gordy, 2000) grounding on the Merton structural default framework to quantify the dependency structure across borrowers. The ASRF framework assumes perfect granularity, i.e. perfect diversification of specific risks, and a single source of systematic risk. Departures from these assumptions could result in substantial deviations of economic capital requirements. Indeed, because borrowers are not equally sensitive to systematic risk and because their financial heath is linked to multiple sources of credit risk, the standard single risk framework could induce a misrepresentation of the dependency structure across obligors as well as a misrepresentation of the concentration risk even in large portfolios of retail exposures. Then, to deal with this issue, we propose a multifactor extension of the ASRF model, in order to consider heterogeneity and the existence of multiple sources of credit risk as sources of benefits for the guarantee fund. Therefore, the multifactor framework allows quantifying the guarantor's ability to achieve the highest possible level of risk mutualization. More specifically, we consider the credit risk of each bank contributing to the guarantee fund as a systematic risk factor from the guarantor's perspective. This rests upon the assumption that each bank portfolio is granular, given the lender's origination policy and risk appetite. Given the share of each bank in the guarantor's portfolio (see section 3 below), this assumption can easily be considered as fulfilled.

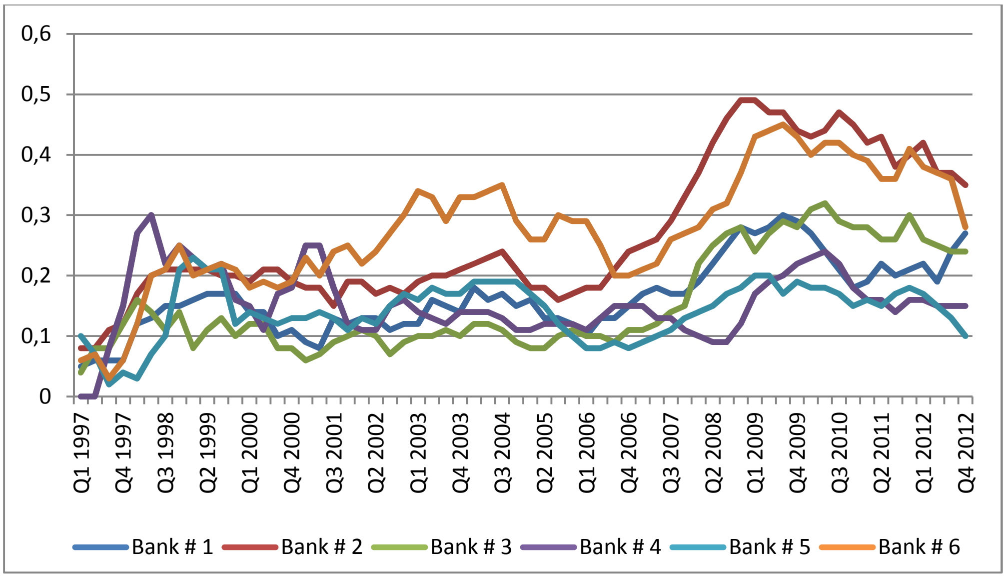
Section 2 presents descriptive statistics of the loans portfolios of the contributing banks in order to highlight their heterogeneity. Section 3 describes the multifactor credit risk model that expands the standard single factor model in order to capture the diversification potential across contributing banks. Section 4 presents the results in terms of economic capital requirements. Section 5 evaluates the resiliency of the guarantee fund by analyzing the effect of the crisis on the risk level of the fund and section 6 concludes.

2. Heterogeneity across contributing lenders: descriptive statistics.

As mentioned before, in France, the guarantors either provide the mutual fund services to several different banking groups, or they provide this service for only one banking group. In this paper, using the data of the major French guarantor which manages guarantees for several banks, and knowing that different banks are characterized by different clienteles and different levels of risk appetite, we are able to consider the capacity of the guarantor to extract diversification benefits from the contributing banks' heterogeneity. We will see in what follows that such diversification matters and is able to improve the efficiency of the mutual guarantee scheme.

The database supplied by a major French guarantor provides information about the quality of each bank which is at the origination of the loans and contribute to the guarantee mechanism (six different banks in total). So it is possible to compute risk parameters at the originating bank level as well as for the total portfolio of the guarantor company. Therefore, this part of the study will document the differences in sub-portfolios which reflect the difference in bank's clienteles, strategy and risk appetite. Here, these differences are observed at the end of the period under study, e.g. in the 2008-2013 period. Moreover, for consistency reasons, the analysis is restricted to loans which finance owner-occupied houses.

Figure 1: Average annual rates of default by contributing bank – 1997-2013 (in %).



Note: This figure shows the rate of default at the one year horizon computed for each quarter of the period. The rate of default is defined as the ratio of the total number of defaults during the year beginning with this quarter to the total number of safe borrowers at the end of the previous quarter.

Figure 1 shows that the annual average rate of default differs significantly from one bank to the other, reflecting differences in banks' clienteles or banks' risk aversion. However, for all contributing banks, we observe what seems to be a structural increasing in default rates from 2008. This reflects the systematic impact of the consequences of the US subprime crisis and the Lehman failure. The next step consists in analyzing the distribution of the bank portfolios according to the type of loan and by borrowers' profession. These features reflect the borrowers' wealth and income level. Five types of loans are distinguished: loans benefiting from a public financial assistance (AID in Table 1), such as social loans which are allocated to low income borrowers and zero interest rate loans which are conditioned by income ceilings, the Prêts d'Epargne Logement (PEL), which are conditioned to a prior savings effort, the standard loans (STL), the bridge loans (BRI), and the other loans (OTH).

Table 1: Distribution of borrowers by type of loans in contributing banks (in %) – 2008-2013

	AID	STL	PEL	BRI	OTH
bank # 1	8,99	79,23	3,32	7,55	0,92
bank # 2	12,02	78,04	0,61	7,22	2,12
bank # 3	7,62	76,97	1,93	12,69	0,78
bank # 4	1,17	65,74	29,11	3,84	0,14
bank # 5	3,53	89,3	1,69	4,81	0,67
bank # 6	14,34	75,21	6,89	2,89	0,67
Guarantor	8,37	78,59	4,58	7,42	1,05

Source: Guarantor data and authors' computation. See legend in the text.

Table 1 shows a strong differentiation of the banks' clienteles through this loan type criterion. Indeed, banks #1, #2, and #6 have a significantly higher proportion of loans which are chosen by low income borrowers, while bank # 4 lends more than the other banks to borrowers with prior savings effort, which could be associated to higher downside payments, hence lower risk. Moreover, bank #3 grants a higher share of bridge loans than its competitors. Their risk being directly linked to the value of the good to be sold, bridge loans are typically characterized by a higher risk level. Then, to illustrate the differentiation of clienteles in terms of income, we display in table 2 the distribution of borrowers by profession. One can see, for instance, that bank #2, bank #4, and bank #5 portfolios contain a higher proportion of low and middle income borrowers. Bank #4 portfolio contains a high proportion of retired people. Some banks have a higher proportion of independent professions and retailers and craftsmen.

Table 2: Distribution of borrowers by profession in contributing banks (in %) – 2008-2013

	managers	middle managers	workers and employees	retailers and craftsmen	independent professions	retired people	others
bank # 1	41,46	8,23	32,16	3,41	9,21	3,92	1,6
bank # 2	38,64	5,2	40,7	2,42	7,58	3,88	1,58
bank # 3	40,62	12,5	31,85	2,02	6,92	4,72	1,37
bank # 4	40,23	3,98	41,5	1,15	3,28	8,92	0,94
bank # 5	37,7	5,84	43,18	2,67	6,57	3,1	0,93
bank # 6	41,85	6,18	36,15	1,79	9,95	3,15	0,93
guarantor	39,93	7,52	37,07	2,41	7,45	4,3	1,32

Source: Guarantor data and authors' computation

The following criteria reflect the risk of the borrowers. First, we consider the distribution of borrowers by rating in each sub-portfolio. Table 3 shows large differences in terms of borrowers' probability of default. Banks #3 and #4 appear to hold less risky portfolios than the other banks.

Table 3: Distribution of borrowers by borrowers' ratings in contributing banks (in %) – 2008-2013

	A (lower risk)	B	C	D (higher risk)
bank # 1	48.45	33.54	10.34	7.67
bank # 2	49.78	33.75	9.69	6.78
bank # 3	55.17	31.32	8.08	5.44
bank # 4	76.53	18.38	3.28	1.82
bank # 5	48.01	36.60	9.57	5.83
bank # 6	51.84	33.31	8.81	6.04
Guarantor	52.83	32.32	8.83	6.03

Source: Guarantor data and authors' computation. See legend in the text.

Table 4: Quartile values of of Downside payment and DSTI ratio in contributing banks – 2008-2013

	Downside payment ratio			Debt service to income ratio		
	Q25	Median	Q75	Q25	Median	Q75
bank #1	0	0.02	0.26	0.07	0.13	0.20
bank #2	0	0.01	0.23	0.08	0.14	0.21
bank #3	0	0.02	0.24	0.08	0.15	0.24
bank #4	0.03	0.20	0.42	0.07	0.13	0.20
bank #5	0	0.04	0.25	0.08	0.13	0.21
bank #6	0	0.09	0.30	0.07	0.13	0.21
Guarantor	0	0.04	0.27	0.07	0.14	0.21

Source: Guarantor data and authors' computation.

As other criterions of risk, the median value of the LTV and DSTI ratios in each sub-portfolio are used. Table 4 shows the results. Again, banks accept very different level of the downside payment ratio, e.g. different levels of LTV. But, on average, they ask for very similar levels of the DSTI ratio, following the French common credit standard which is to condition the supply of loan to the acceptation by the borrower of a DSTI ratio cap.

3. Measuring credit risk and diversification in the guarantee fund: a multifactor framework.

The benchmark for credit risk modeling is the widely known ASRF model described by Gordy (2000) that also underlies the IRB risk weight functions for credit exposures in the banking book. The central parameter in the ASFR model is the correlation to a latent (unobservable) risk factor assumed to represent the "state of the economy", conventionally called asset correlation. Therefore, asset correlation quantifies the dependency across obligors within a portfolio. If the correlation is high, this sensitivity to the general systematic risk factor is high, and in the case where extreme values of the general factor occur, losses will rise to very high levels, due to correlated defaults. Asset correlation reflects the uncertainty associated to events which can produce numerous simultaneous defaults and generate, for this reason, extremes unexpected losses. In a credit risk model, correlations affect the credit portfolio Value-at-Risk (VaR) which determines the economic capital required to cover losses at a chosen quantile (confidence level). Thus, the modeling of individual asset correlations has a strong impact on economic capital for loans portfolios.

In the ASRF model, default is triggered if the ability-to-pay process Y_i of borrower i falls below an exogenous default threshold γ_i. Y_i is assumed to follow a standard normal distribution. It can be decomposed into the return of a systematic and unobservable factor X and an idiosyncratic firm-specific part ε_i:

$$Y_i = \sqrt{\rho_i}.X + \sqrt{1-\rho_i}.\varepsilon_i$$

X and ε_i are independent for every obligor i and follow a standard normal distribution. The factor loading $\sqrt{\rho_i}$ of the systematic risk factor can be interpreted either as the sensitivity to systematic risk or as the square root of the asset correlation ρ_i. For this analysis the common assumption of a constant ρ_i is applied. In this framework, the event of default is described by a binary variable L_i recording if a credit event has occurred during the considered horizon ($L_i = 1$) or not ($L_i = 0$).Default occurs when the ability-to-pay crosses a downside threshold γ_i. The unconditional default probability of obligor i is then defined by:

$$P(L_i = 1) = P(Y_i < \gamma_i) = \Phi(\gamma_i)$$

where Φ denotes the cumulative distribution function of a standard normal distribution. Since homogeneity in the obligor buckets is assumed, the index i for the default threshold of a given borrower is dropped.

As mentioned above taking the heterogeneity across lenders into account calls for an extension of the single risk factor framework to a multifactor one. The main benefit of using a multifactor framework is that taking additional risk factors into account allows detecting more precisely potential credit risk concentration due to correlated defaults or, on the contrary, assessing the potential diversification benefits in banks' loans portfolios. Thus, the measurement of credit risk within a portfolio requires methodologies that first allow for the computation of a portfolio-wide risk measure and subsequently permit risk to be allocated at the sub-portfolio level to establish the cartography of risk within the portfolio.

Here, the conceptual framework that we use is a multi-factor extension of the structural single factor model (Dietsch and Petey, 2015). As the ASRF model, the multifactor model belongs to the class of structural credit risk models. It is in fact an extended version of the standard asymptotic single risk factor ASRF model. The extension consists to introduce additional factors varying across groups of borrowers. Here, we have expanded the model by adding new latent factors of systematic risk that are linked to the existence of several contributing lenders to the guarantee fund. Such an extension to a multi-factor model improves substantially the computation of the dependency structure across exposures. Using this approach allows in particular comparing credit risk in the different lenders' sub-portfolios within the mutual fund.

This section presents the multi-factor extension of the structural single factor model. Then, this model is specified as a generalized linear mixed model (GLMM) to produce estimates of the credit-risk parameters that are required for the parameterization of the model. Finally, these estimated risk parameters are used to compute potential losses that may occur at the portfolio level and at the sub-portfolio levels, applying a multi-factor capital allocation procedure.

3.1 The asymptotic multifactor credit-risk framework

The multifactor model is in fact an extended version of the standard asymptotic single risk factor (ASRF) model devised by Merton. Losses at the portfolio level can be defined as the sum of individual losses on defaulting loans in the portfolio, adjusted for the severity of these losses. Thus, if u_i is defined as the loss given default (LGD) of an obligor i and if Y_i is defined as the default indicator variable of obligor i (Y_i takes the value of 1 if there is a default and 0 otherwise), then the total portfolio losses L may be computed as follows:

$$L = \sum_{i=1}^{n} u_i Y_i$$

In structural credit-risk models (Merton, 1974), default occurs if the situation of a borrower crosses an default threshold that is calibrated in accordance with the stationary (long-term) default probability $\overline{p_i}$ of obligor *i*.:

$$Y_i = 1 \Leftrightarrow U_i = w'_i s + \sqrt{1 - w'_i R w_i \varepsilon_i} < \Phi^{-1}(\bar{p}_i) \quad (1)$$

Here, the financial health of obligor *i* is represented by a latent (unobservable) variableU_i, and the level of U_i is determined by the realizations *s* of a set of *S*, w_i is the vector of sensitivities (or factor loadings) of the *i*-th borrower to the systematic factors and ε_i is a specific risk factor for borrower *i*. In the above equation, *R* is the correlation matrix of the risk factors, assuming that the risk factors are multivariate Gaussian. Φ is the standard normal cumulative distribution function. U_i is standard normal. Specific risk factors are assumed to be uncorrelated among obligors and independent from systematic factors.

Thus, given a realization s of the systematic risk factor, equation (1) can be rewritten such as a default occurs when:

$$\varepsilon_i < \frac{\Phi^{-1}(\bar{p}_i) - w'_i s}{\sqrt{1 - w'_i R w_i}}$$

As the borrower's specific risk factor is normally distributed, the default probability conditional to *s* follows the standard normal cumulative distribution function. Moreover, assuming that specific risk can be entirely diversified away, then losses can be approximated by their expected value conditional to *s* (Gordy, 2000). Conditional portfolio losses are then defined as follows:

$$L(s) \approx \sum_{i=1}^{n} u_i \, \Phi\left[\frac{\Phi^{-1}(\bar{p}_i) - w'_i s}{\sqrt{1 - w'_i R w_i}}\right] \qquad (2)$$

This framework is known as the asymptotic multi-factor framework of credit risk (e.g., Lucas et al., 2001). Such an extension to a multi-factor model improves substantially the computation of the dependency structure across exposures in a typical housing loans portfolio (Lucas et al., 2001).

Equation (2) assumes that each obligor can be characterized by his individual default threshold and factor sensitivities. However, in retail loan portfolios, default rates are generally computed based on rating classes, and sensitivities to risk factors cannot be computed on an individual basis. Thus, assumptions are required to reduce the number of parameters of the loss variable. A common assumption is that obligors who belong to the same rating notch *j* will share the same default threshold. Moreover, one could further assume that the vector of risk factor sensitivities is the same for obligors that share the same characteristic (or a set thereof). Hence, assuming the existence of a portfolio that is composed of *K* segments, losses can be rewritten as follows:

$$L(s) \approx \sum_{k=1}^{K} \sum_{i=1}^{n_k} u_i \, \Phi\left[\frac{\Phi^{-1}(\bar{p}_j) - w'_k s}{\sqrt{1 - w'_k R w_k}}\right] \qquad (3)$$

3.2 Econometric estimation of the portfolio's risk parameters

The default thresholds and factor sensitivities are estimated by using an econometric model that belongs to the class of generalized linear mixed models (GLMMs). This model combines fixed and random effects for observable and (latent) unobservable factors, respectively[4]. Indeed, one can establish a correspondence between the conditional default probability entailed in the loss variable as defined by equation (3) and the specification of a GLMM. First, the default threshold $\Phi^{-1}\left(\overline{p}_j\right)$ is taken as the fixed effect of the GLMM. Second, the systematic risk factors are supposed to be latent factors and they correspond to the random effects of the GLMM. However, this assumption constrains the meaning of the systematic risk factors which are considered. In GLMMs, random effects are linked to some observable characteristics of the subjects. In other words, the chosen random effects are defined by some segmentation of the portfolio. The segmentation considered here will be based on the fact that the mutual fund collects deposits from guarantees buyers who are customers of different independent lenders. Consequently, we can build a segmentation depending of the number of the lenders. Precisely, this approach implies that a borrower belonging to a given sub-portfolio (defined by its bank's affiliation) is exposed to a unique systematic risk factor and that the number of systematic factors equals the number of segments within the portfolio[5.]

Within the GLLM framework, the conditional default probability of equation (3) is defined as follows. Let Y_t be an (N × 1) vector of observed default data at time *t* and γ_t be the (K × 1) vector of random effects. The conditional expected default probability of obligor *i* at time *t* is then:

$$P(Y_{ti} = 1|\gamma_t) = \Phi(x'_{ti}\beta + z_i\gamma_t)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function, β denotes the vector of parameters associated with the fixed effect (the borrower's rating class) and z_i is the design matrix of the random effects, here an identity matrix with size the number of random effects. If the rating scale is properly built, we expect the β parameters which correspond to the default thresholds

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4 Presentations of the implementation of GLMM models in credit-risk modeling can be found in McNeil and Wendin (2007).

5 An alternative approach would be to identify explicit, mainly macroeconomic, risk factors such as GDP growth, unemployment rates or real estate prices either at the national or regional levels. However, we are less interested in identifying the systematic risk factors than capturing their aggregate effect at the portfolio level. Specific risk factors that might be related to health, changes in marital status, or any personal events not driven by general economic conditions, are not explicitly introduced in the GLMM as we assume that the lender holds a perfectly diversified (granular) portfolio where the aggregate effect of specific risks tends to zero.

associated to the ratings to be ordered and increasing as credit quality decreases. In the above equation, $x'_{ti} = [0, \dots, 1, \dots, 0]$ is a (1 × J) vector of dummies defining the rating of borrower *i* at time *t*. The random effects are assumed to follow a multivariate standard normal distribution with covariance matrix Σ and correlation matrix *R*. Because we assume that borrowers within segments are interchangeable, the estimations of Σ and β do not involve individual borrowers but instead use the quarterly default rates within segments resulting from the combination of the variables associated with the fixed and random effects. Assuming that defaults are independent conditional on random effects, the number of defaults in the portfolio is binomially distributed. The conditional probability of $Y_t = (Y_{t1} = 1, \dots, Y_{tn} = 1)$ is then

$$P(Y_t = y_t|\gamma_t) = \prod_{i=1}^{n_t} P(Y_{ti} = 1|\gamma_t)^{y_i}\left(1 - P(Y_{ti} = 1|\gamma_t)\right)^{1-y_i}, \forall\, y_i \in \{1, 0\}^{n_t}$$

Further assuming that the random effects are serially independent (but possibly cross-sectionally correlated in the case of multiple random effects), the unconditional probability of Y_t is as follows, defining θ as the parameter vector that comprises all unknowns in Σ, *R* and *g* as the multivariate Gaussian distribution:

$$f(y_t|\beta, \theta) = \int P(Y_t = y_t|\gamma_t)\, g(\gamma_t|\theta) d\gamma_t$$

The likelihood function with serially independent random effects is finally:

$$L(\beta, \theta|data) = \prod_{t=1}^{T} f(y_t|\beta, \theta)$$

3.3 Computation of marginal contributions of lenders' sub-portfolios

Once the credit-risk parameters are estimated, we can build the distribution of losses at the portfolio level by the Monte Carlo simulation of the risk factors, with each realization of risk factors being converted into a conditional default probability at the fixed/random effects sub-portfolio level as defined by equation (3), and lastly, conditional expected losses at the portfolio level. However, to assess the credit risk of a given type of borrower within the portfolio, we need to compute the economic capital contribution of each borrower type. This calculation requires the portfolio-wide economic capital to be allocated to sub-portfolios or individual assets. From the findings of Tasche (1999) and Gouriéroux et al. (2000), the marginal contributions to a portfolio value-at-risk (VaR) can be expressed as the expected loss on a given exposure, conditional on losses reaching this VaR:

$$RCVAR_i = E[L_i|L = VaR_\alpha(L)] = \frac{E\left[L_i \mathbf{1}_{VaR_\alpha(L)=L}\right]}{P[L = VaR_\alpha(L)]} \qquad (4)$$

Equation (4) indicates that if there is a positive probability for losses to reach a portfolio's VaR, then the computation of marginal contributions will rely heavily on the ability to estimate individual losses as aggregate losses approach this VaR. Thus, in the context of a Monte Carlo simulation, the conditional mean may be based only on a limited number of simulations, producing unreliable estimates. To improve the estimation procedures, some authors (Tasche, 2009, Glasserman and Li, 2005) have used importance sampling. Importance sampling consists of shifting the parameters of a distribution in ways that increase the likelihood of observing certain desired realizations of the variables. The main difficulty with respect to this approach relates to the choice of the alternative distribution *F**. In this study, we follow the methodology of Tasche (2009) and shift only the risk factor (*S*) means in the following manner:

$$S_i^* = S_i - E_F[S_i] + \mu_i \; with \; \mu_i = E[S_i | L = VaR_\alpha(L)]$$

The next step is the computation of the conditional expectation as defined by equation (4). Because the computation of VaR is accomplished through Monte Carlo simulations, both the realizations of the risk factors and the resulting credit losses are known. This information permits the utilization of the non-parametric Naradaya-Watson estimator for conditional expectations. If the standard normal density is used as the kernel and h is used to denote the bandwidth of the kernel, then the estimator of the conditional expectation for risk factor k may be defined as follows:

$$\hat{E}[S_k | L = VaR_\alpha(L)] = \frac{\sum_{t=1}^{T} S_k \Phi\left(\frac{VaR_\alpha(L) - L_t}{h}\right)}{\sum_{t=1}^{T} \Phi\left(\frac{VaR_\alpha(L) - L_t}{h}\right)} \qquad with \; h = 1.06\sigma_L T^{-1/5}$$

To limit the computational burden involved in the simulations of marginal contributions, we rely on the size of the portfolio under consideration and assume the complete diversification of idiosyncratic risk. This assumption allows for losses to be simulated using conditional probabilities instead of requiring the simulation of defaults (and their associated losses). Thus, given the assumed homogeneity of exposures within sub-portfolios, it is possible to compute a single marginal contribution based on the rating/segmentation variable combination rather than by proceeding at the asset level. For borrowers with rating j with characteristic k, losses are then approximated by the following expression:

$$L(s_k) \approx \sum_{j=1}^{n_k} u_j \, \Phi\left[\frac{\Phi^{-1}(\bar{p}_j) - w'_k s_k}{\sqrt{1 - w'_k R w_k}}\right]$$

Once the shifts in the means are computed for all of the risk factors, the next step in the analysis is to obtain realizations of the risk factors under the new distribution to once again compute the aggregate losses for the portfolio and the individual losses within each sub-segment and rating grade. Tasche (2009, proposition 4.2), establishes that conditional on VaR, the expected losses under the natural distribution can be defined as follows, with δ as the likelihood ratio between distributions F and F*:

$$E_F[L_i|L = VaR_\alpha(L)] = \frac{E_{F^*}[L_i\delta|L = VaR_\alpha(L)]}{E_{F^*}[\delta|L = VaR_\alpha(L)]}$$

As discussed above, these conditional expectations can be computed with the Naradaya-Watson estimator, and simulations of risk factors and losses can be obtained under the shifted distribution. Lastly, these expected losses can be aggregated across ratings for each modality of the segmentation variable to compute segment-wide economic capital requirements.

4. Estimation results

In the multifactor framework we consider all borrowers belonging to the sub-portfolio of each lender which participates to the mutual fund as one distinct group and we take into account diversification benefits coming from the interactions of borrowers belonging to the different contributing banks through imperfect correlations across contributing banks. As shown before, the characteristics of each bank sub-portfolio differ due to differences in clienteles, , in bank's strategy, and risk appetite. The computation of variance-covariance matrixes (of random effects) among lender related risk factors allows assessing the existence of concentration or diversification effects.

Table 5 shows the variances and the correlation matrix of random effects (provided by the multifactor GLMM estimation,) and illustrates the potential for risk diversification by associated to the composition of the mutual fund. Panel A of table 5 shows that there is a considerable heterogeneity across banks in their exposure to general economic conditions. Considering correlations across random effects (Panel B of table 5), these are high on average with values mostly comprised between 50% and 90%. However, there is one noticeable exception with bank #5 which appears to be relatively orthogonal to its competitors. These figures illustrate that when even considering concentrated retail markets as residential housing loans, there is still place for significant heterogeneity in origination policies and risk management practices as reflected by the estimated risk parameters. Thus, it appears that aggregating customers from different banks may bring diversification benefits in the mutual fund. In order to quantify these benefits, we use the results from the multifactor credit risk model to compute the marginal contributions of each bank's sub-portfolio to the mutual fund unexpected losses (measured at the 99.9% quantile of the distribution of the fund losses), using the risk parameters provided by the GLMM econometric model (variance, covariance, and default thresholds by rating). Technically, marginal contributions correspond to the derivative of the total credit VaR of the mutual fund relative to a unit (euro) of exposure related to each contributing bank to the mutual fund.

Table 5– Variances and correlation matrix of random effects

	bank #1	bank #2	bank #3	bank #4	bank #5	bank #6
Panel A Random effects variances						
	0.00804	0.06371	0.01807	0.01123	0.00541	0.05078
Panel B Random effects correlation matrix						
	bank #1	bank #2	bank #3	bank #4	bank #5	bank #6
bank #1	1					
bank #2	0.8011	1				
bank #3	0.8212	0.6262	1			
bank #4	0.7532	0.9000	0.5608	1		
bank #5	0.3583	-0.03823	0.3309	0.03953	1	
bank #6	0.7393	0.9688	0.5182	0.9375	0.04122	1

Source: Guarantor data and authors' computation. Tests (not shown here) show that variance and correlation values are statistically significant at usual levels.

The distribution of portfolio losses, the portfolio Value-at-Risk and marginal contributions are computed through a Monte Carlo simulation of random effects. Each contributing bank could contribute more or less to the total unexpected losses of the fund depending on the asset correlation within its sub-portfolio and the correlations between its sub-portfolio and the others. If diversification benefits actually result from these interactions, the mutual fund would benefit from lower capital requirements to cover unexpected losses. Consequently, if diversification benefits exist, the sum of marginal contributions could be lower than the sum of the capital requirements computed for each contributing bank on a stand-alone basis, .

Therefore, to assess the capacity of the mutualization mechanism to reduce the exposure of the fund to extreme losses, we simply compare for each contributing bank the economic capital requirements corresponding to the marginal contribution given by the multifactor model with the economic capital requirements given for the same bank on a stand-alone basis using a single factor model that does not take into account the dependence between bank portfolios. Table 6 presents the results under the form of a capital ratio. The first column relates the economic capital requirements (assuming a fixed LGD value of 15%) relative to corresponding total exposures for each sub-portfolio. The table presents also under the form of capital ratios the economic capital requirements measured by the regulatory capital requirements formulas given by the Basel 2 formulas in the IRB approach (second column). Here, the measures shown in the table are obtained by using data for the entire 1997 –

2013 period. Recall that they are relative to the housing loans financing own occupied houses[6].

Table 6: Economic capital ratios for each guaranteed bank and computation of total capital requirements needed to cover the guarantor's portfolio unexpected losses 1997 – 2013 (in%).

	Economic capital			
Contributing banks	Multifactor marginal contributions	Single factor	Regulatory capital ratio (IRB)	Share of portfolio exposure
Bank # 1	0,04	0,30	1,50	20,00
Bank # 2	0,15	0,60	1,39	29,40
Bank # 3	0,04	0,48	1,41	22,60
Bank # 4	0,04	0,32	1,22	7,50
Bank # 5	0,02	0,23	1,53	12,80
Bank # 6	0,13	0,44	1,52	7,50
Guarantor	0,08	0,43%	1,43	100,00

Source: Guarantor data and authors' computation

Results show that marginal contributions of each bank vary substantially, due to the differences in clienteles, selection process, risk appetite, and credit risk management across contributing banks. For all banks, the multifactor marginal capital ratio is lower than the single factor capital ratio. Four banks in particular show relatively low marginal contribution to the total losses. The comparison of capital ratio given by the multifactor and the single factor models show the existence of considerable diversification benefits. Indeed, the overall economic capital ratio amounts to 0.08% while the capital requirements provided by a single factor model computed over the complete portfolio amounts at 0.43%. Results show that the deposits of the borrowers who feed the mutual fund amount to less than 0.10 percent in order to insure the stability of the fund at the assumed one year horizon. This contribution again is well below the regulatory requirement which amounts at 1.43 percent.

5. The resiliency of the guarantee fund in adverse conditions

The results in the previous section were obtained using the total 1997-2013 period. However, all risk parameters – default thresholds (stationary default rates), asset correlations and correlation matrixes – can change over time and particularly in times of crisis. So, to evaluate the impact of the crisis on these parameters, we consider two additional exercises. The first one concerns the asset

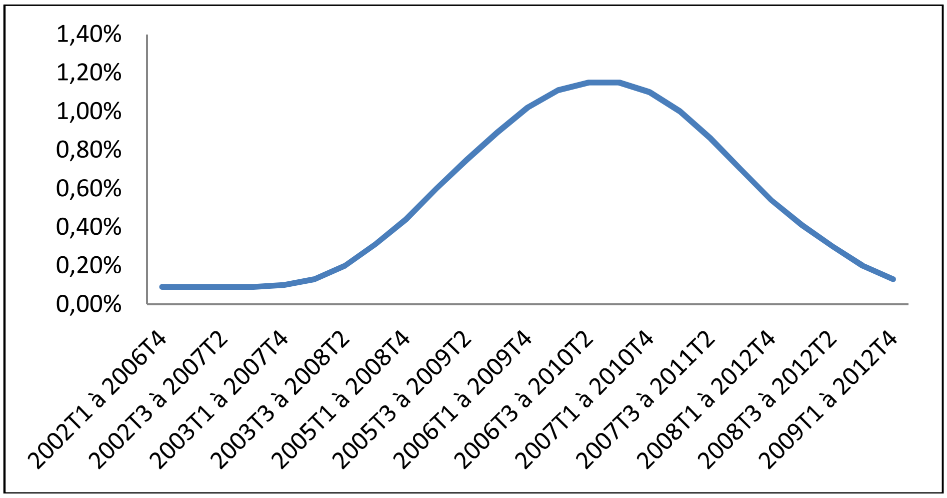
6 Results obtained on the all loans, including those which finance rental investments, show higher diversification benefits portfolios and lower economic capital requirements for the mutual fund.

correlations, the second one the economic capital requirements. In the third on

5.1. Stress-testing asset correlations

Here, we illustrate the impact of the macroeconomic environment of the financial crisis on asset correlations. To highlight the impact of the severe downturn of 2008-2009 on potential credit losses, we have adopted a rolling window approach for the estimation of the asset correlations. More precisely, we estimate the asset correlation for the entire guarantor portfolio using a single factor model the asset correlation are estimated over rolling windows of 5 years. Results are presented on figure 2.

Figure 2: Impact of the credit crisis on the portfolio asset correlation



Source: Guarantor data and authors' computation.

As expected, we observe an increase of the asset correlation starting in the first quarter of 2008 and culminating in 2010. But, after the crisis, the asset correlation decreased sharply, and this result might be the consequence of the reinforcement of credit standards by commercial banks in the period following the crisis.

5.2. The impact of the crisis on the diversification of risk within the guarantee fund

Here, we extend the logic of the preceding paragraph to the allocation of the economic capital at the fund level to contributing banks. Specifically, we estimate the impact of bad economic conditions by applying the multifactor model to successive periods. We start with the pre-crisis period from Q4-2003 to Q4-2007, then we include in the sample successively one year from 2008 to 2012. We run the estimation on six successive sub-periods in order to compare the levels of economic capital requirements associated to each sub-period.

Table 7 Variance and correlation matrices among guaranteed banks: impact of the crisis

Panel A Q4-2003/Q4-2007						
Random effects variances						
	bank #1	bank #2	bank #3	bank #4	bank #5	bank #6
	0.00268	0.01437	0.00863	0.00036	0.02125	0.03461
Random effects correlation matrix						
	bank #1	bank #2	bank #3	bank #4	bank #5	bank #6
bank #1	1					
bank #2	0.07831	1				
bank #3	0.49457	-0.46026	1			
bank #4	-0.06736	-0.16050	0.37871	1		
bank #5	0.59509	-0.10021	0.63188	-0.44653	1	
bank #6	-0.06065	0.71878	-0.15701	-0.47157	0.39836	1
Panel B Q4-2003/Q4-2008						
Random effects variances						
	bank #1	bank #2	bank #3	bank #4	bank #5	bank #6
	0.00647	0.05792	0.02434	0.00363	0.00907	0.04438
Random effects correlation matrix						
	bank #1	bank #2	bank #3	bank #4	bank #5	bank #6
bank #1	1					
bank #2	0.61102	1				
bank #3	0.84809	0.41371	1			
bank #4	0.15789	0.43668	-0.30137	1		
bank #5	0.09649	-0.40436	-0.08381	0.00999	1	
bank #6	0.19484	0.79090	-0.14512	0.76013	-0.16880	1

Source: Guarantor data and authors' computation.

First, estimation results show changes in the covariance matrixes of risk factors. Table 7 illustrates these changes by showing the random effects variances and correlation matrix for the Q4-2003 to Q4-2007 period, before the crisis, and the Q4-2003 to Q4-2008 period, which includes the first year of crisis. If we compare the results of the two matrixes, we do observe significant increases of most of the variance and covariance values when including the first year of the crisis. However, it is not necessarily the case for all contributing banks. In particular, in the sub-portfolio of bank # 5, the values of the risk parameters tend to decrease, what shows that the banks could react differently to the crisis. We both observe a lower exposure to systematic risk and lower correlation when the year 2008 is added in the estimation. This confirms the specificity of this bank observed in table 5. Borrowers in this portfolio seem to be resilient to the crisis, likely due to the banks' allocation strategies, while the other banks became more sensitive to the crisis.

As a consequence of these changes of risk parameters, the capital ratios changed during the crisis. Table 8 presents the economic capital ratios allocated from the multifactor credit risk model when introducing sequentially the years of crisis (2008 to 2010) and the following years. Results on the last line show an increase of the capital ratio for the guarantor in the years following the crisis. This is the direct consequence of the increases of the capital ratio in certain banks (bank # 2 and 5, in particular). However, diversification benefits still remain in the portfolio and the level of credit risk remains sustainable for the mutual fund. The deposits of the borrowers who feed the mutual fund climb to 0.20 percent (versus 0.08 in the entire period) of the total amount of guaranteed loans in order to insure the stability of the fund at the assumed one year horizon. Then, this percentage may decrease as a consequence of the changes in the bank lending policies after the crisis.

Table 8 Economic capital ratios (multifactor) for increasing estimation windows (in %)

	Q4-2003/ Q4-2007	Q4-2003/ Q4-2008	Q4-2003/ Q4-2009	Q4-2003/ Q4-2010	Q4-2003/ Q4-2011	Q4-2003/ Q4-2012
bank #1	0,03	0,03	0,04	0,02	0,03	0,03
bank #2	0,07	0,13	0,40	0,30	0,55	0,26
bank #3	0,02	0,03	0,06	0,05	0,05	0,06
bank #4	0,02	0,02	0,01	0,04	0,03	0,03
bank #5	0,03	0,02	0,01	0,01	0,00	0,01
bank #6	0,11	0,12	0,14	0,23	0,31	0,24
Guarantor	0,04	0,06	0,15	0,12	0,20	0,12

Source: Guarantor data and authors' computation

In order to assess the impact of the crisis on the diversification potential at the guarantor level, table 9 reproduces the same exercise considering the single factor model for each bank considered on a stand-alone basis.

Table 9 Economic capital ratios (single factor) for increasing estimation windows (in %)

	Q4-2003/ Q4-2007	Q4-2003/ Q4-2008	Q4-2003/ Q4-2009	Q4-2003/ Q4-2010	Q4-2003/ Q4-2011	Q4-2003/ Q4-2012
bank #1	0,22	0,32	0,34	0,32	0,32	0,36
bank #2	0,34	0,69	0,73	0,72	0,69	0,66
bank #3	0,16	0,44	0,55	0,57	0,58	0,55
bank #4	0,17	0,27	0,36	0,37	0,34	0,36
bank #5	0,24	0,25	0,26	0,25	0,24	0,23
bank #6	0,39	0,44	0,47	0,46	0,45	0,54

Source: Guarantor data and authors' computation

Tables 8 and 9 reflect similar reactions to the crisis with strong increases in economic capital levels.

However, comparing tables 8 and 9 suggests that while the whole guarantee fund still benefits from strong diversification effects when taking into account the heterogeneity across contributing banks, banks with the lower risk levels on a stand-alone basis mostly contribute to the reduction of the overall risk of the fund. In order to illustrate this observation, we compute in table10 the ratio between the multifactor and the single factor measures of economic capital.

Table 10 Comparison of economic capital ratios (multifactor over single factor) in %.

	Q4-2003/ Q4-2007	Q4-2003/ Q4-2008	Q4-2003/ Q4-2009	Q4-2003/ Q4-2010	Q4-2003/ Q4-2011	Q4-2003/ Q4-2012
bank #1	13,95	9,26	11,84	6,19	9,51	8,34
bank #2	20,37	18,82	54,92	41,78	79,20	39,23
bank #3	12,14	6,81	10,99	8,76	8,64	10,83
bank #4	11,77	7,53	2,75	10,93	8,76	8,38
bank #5	12,75	8,01	3,87	4,00	0,00	4,28
bank #6	28,36	27,02	29,77	50,37	68,79	44,58

Source: Guarantor data and authors' computation

From table 10, we see that bank #2 and bank #6, which have the higher economic capital levels on a stand-alone basis also show the highest increase in economic capital during the crisis, with multifactor economic capital levels reaching comparable levels with their single factor levels (resp. up to 80% and 70%). Thus, the mutualization of risk achieved by the fund through the aggregation of portfolios from heterogeneous contributing banks, while leading to sizeable diversification gains, nevertheless still leaves the fund exposed to the risk of concentration of defaults, here at the contributing bank level. From a methodological point of view, the multifactor specification of our credit risk model appears to be sufficiently flexible to reflect both the low risk levels of the pool of guaranteed residential loans in stable times and the steep risk increase during a crisis.

6. Concluding remarks

To secure loans, French banks do not necessarily ask for a mortgage but mostly rely on the guarantee provided by an internal or external residential property loan guarantor. Using an economic capital methodology that allows computing the total deposits needed to insure the stability of the guarantee scheme and applying this methodology to the data provided by the major French guarantor over the 1997-2013 time period, this paper presents the first assessment of the capacity of a mutual guarantee fund for residential loans to implement a resilient risk-pooling mechanism across borrowers and across lenders. The paper demonstrates the capacity of the fund to extract significant diversification benefits from the borrowers and lenders' heterogeneity.

In this paper, we have computed the total required amount the guarantor should hold in the mutual fund to cover extremes losses created by tail risk event. Thanks to large diversification benefits, the deposits of the borrowers who feed the mutual fund amount on average over the period to around 0.10 percent in order to insure the stability of the fund at the one year horizon. The paper verifies also the resiliency of the French guarantee system in adverse scenarios. In the recent crisis, the percentage climbed to 0.20 percent at the portfolio level, what verifies the strength of diversification benefits in adverse scenarios. However, the fund remains nevertheless exposed to the risk of concentrated losses, as shown by the strong increase of the economic capital of some contributing banks during the crisis. This reflects the reduction of diversification gains during adverse times.

References

ACPR, 2014, Housing Finance in France, Analyses et Synthèses, n°32, July, available at: http://acpr.banque-france.fr/fileadmin/user_upload/acp/publications/analyses-syntheses/20140901-Housing-finance-in-France-in-2013.pdf

BCBS, Joint Forum, 2013, Mortgage insurance: market structure, underwriting cycle and policy implications, Consultative document, BIS, February.

Beck, T., L. Klapper and J. Mendoza, 2010, The typology of partial credit guarantee funds around the world, Journal of Financial Stability 6(1), 10-25.

Campbell, J.Y, and J. Cocco, 2003, Household risk management and optimal mortgage choice, Quarterly Journal of Economics 118, 1449-1494.

Campbell, J.Y, and J. Cocco, 2014, A model of Mortgage Default, Journal of Finance, forthcoming..

Columba, F., L. Gambacorta and P. Mistrulli, 2010, Mutual guarantee institutions and small business finance, Journal of Financial Stability 6(1), 45-54..

Cowling M., 2010, The role of loan guarantee schemes in alleviating credit rationing in the UK, Journal of Financial Stability 6(1), 36-44..

Crédit Logement, 2014, The financing of residential property loans in France, February.

Davis M., S. Van Nieuwerburgh, 2014, Housing, Finance, and the Macroeconomy, NBER working paper, n° 20287, July

Dietsch M., J. Petey, 2015, The credit risk implications of home ownership promotion: The effects of public subsidies and adjustable-rate loans, Journal of Housing Economics 28, 103-120.

Dietsch M., C. Welter–Nicol, 2014, Do LTV and DSTI caps make banks more resilient? ACPR, Cahiers économiques et financiers, n°13, June.

D'Ignazio, A., C. Menon, 2013, The causal effect of credit guarantees for SMEs: evidence from Italy, working paper, Bank of Italy, May.

EBA, 2014, EBA report on EU covered bond frameworks and capital treatment, 17 June 2014.

Ergungor O., 2010, Bank Branch Presence and access to credit in low- to moderate-income neighborhoods, *Journal of Money, Credit and Banking* 42, 1321-1349.

Frey, R., A. McNeil, 2003, Dependent defaults in models of portfolio credit risk, Journal of Risk 6 (1), 59–92.

Ghatak, M., T. Guinnane, 1999, The economics of lending with joint liability: Theory and Practice, Journal of development Economics 60, 195-208.

Glasserman P., Li J., 2005, Importance sampling for portfolio credit risk, Management Science, 51(11), 1643–1656.

Gordy, M. B., 2000, A comparative anatomy of credit risk models. Journal of Banking & Finance 24 (1), 119-149.

Gordy, M. B., 2003, A risk-factor model foundation for ratings-based bank capital rules, Journal of Financial Intermediation 12 (3), 199-232.

Gordy M. B. and E. Heitfield, 2002, Estimating default correlations from short panels of credit ratings performance data, w.p., Board of Governor of the Federal Reserve System.

Gouriéroux C., Laurent J.P., Scaillet O., 2000. Sensitivity analysis of Values at Risk, Journal of Empirical Finance 7, 225-245.

Honohan, P., 2010, Partial Credit Guarantees: Principles and Practice, Journal of Financial Stability 6(1), 1-9.

IMF, 2013. Financial Sector Assessment Program Update: France, housing prices and financial stability, Technical Note, June.

Koijen,R., O. Van Hemert, and S. Van Nieuwerburgh, 2009, Mortgage Timing, Journal of Financial Economics 93(2),, 292-324.

Lucas A., Klaassen P., Spreij P., Straetmans S., 2001, An analytic approach to credit risk of large corporate bond and loan portfolios, Journal of Banking and Finance 25, 1635-1664.

McNeil A., Wendin J., 2007. Bayesian inference for generalized linear mixed models of portfolio credit risk, Journal of Empirical Finance 14, 131-149.

Tasche D., 1999, Risk contributions and performance measurement, Working paper, Technische Universität München.

Tasche D., 2009, Capital allocation for credit portfolios with kernel estimators, Quantitative Finance 9(5), 581-595.

Van Hemert O., 2009, Household interest rate risk management, working paper, Stern Business School.