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# A THEORETICAL ASSESSMENT OF THE INFLUENCE OF MYOSIN FILAMENT DISPERSION ON SMOOTH MUSCLE CONTRACTION

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## ABSTRACT

A new constitutive model for the biomechanical behavior of smooth muscle tissue is employed to investigate the influence of statistical dispersion in the orientation of myosin filaments. The number of activated cross-bridges between the actin and myosin filaments governs the contractile force generated by the muscle and also the contraction speed. A strain-energy function is used to describe the mechanical behavior of the smooth muscle tissue. The predictions from the constitutive model are compared to histological and isometric tensile test results for smooth muscle tissue from swine carotid artery. In order to be able to predict the active stress at different muscle lengths, a filament dispersion significantly larger than the one observed experimentally was required. Furthermore, a comparison of the predicted active stress for a case of uniaxially oriented myosin filaments and a case of filaments with a dispersion based on the experimental histological data shows that the difference in generated stress is noticeable but limited. Thus, the results suggest that myosin filament dispersion alone cannot explain the increase in active muscle stress with increasing muscle stretch.

## INTRODUCTION

Smooth muscle tissue is found in organs such as the stomach, the intestines, the urinary bladder, the airways, and blood vessels. The ability to maintain prolonged contractions without showing fatigue is a property of smooth muscle that sets it apart from skeletal and cardiac muscle. The reason is that the energetics of smooth muscle working at constant length is characterized by low energy consumption. The physiology of smooth muscle contraction is a complex interaction between electrical, biochemical and mechanical processes. Smooth muscle is normally organized in thin layers or sheets made up of spindle-shaped cells with a single nuclei. The contractile apparatus in smooth muscle cells consists mainly of thick filaments (myosin) and thin filaments (actin), see Fig. 1. It appears that the myosin/actin complexes in the contractile apparatus have a preferred direction, coinciding with the long axis of the cell (Bitar, 2003; Herrera et al., 2005; Seow and Par, 2007). Some dispersion around this direction does, however, exist (Walmsley and Murphy, 1987). A theoretical model for the constitutive behavior of smooth muscle tissue, undergoing large deformations, is proposed. The approach is based on a previous model (Kroon, 2010) to which a statistical dispersion in myosin filament orientation has been added.

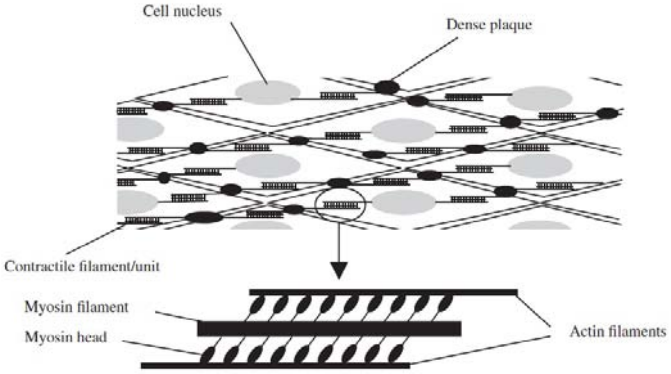


**Fig. 1. Structure of smooth muscle.**

## BIOMECHANICAL MODEL FOR SMOOTH MUSCLE

The constitutive behavior of the smooth muscle is described by use of a strain-energy function $\Psi$, which is split into an active and a passive part $\Psi_a$ and $\Psi_p$, respectively:

$$\Psi = \Psi_a + \Psi_p - \left(p_a + p_p\right)\left(J-1\right).$$

Two Lagrangian multipliers $p_a$ and $p_p$ have also been introduced to model incompressibility, and $J$ is the Jacobian. The active part is defined as

$$\Psi_a = \frac{\mu_a}{4}\left(n_3+n_4\right)\int_S \rho\left(\boldsymbol{M}\right)\left(\frac{\boldsymbol{MCM}}{\lambda_{fc}^2\left(\boldsymbol{M}\right)}-1\right)^2 dS,$$

where $\mu_a$ is a material parameter, $n_3$+$n_4$ is the fraction of force-generating cross-bridges, $\rho$ is a density function for the distribution of myosin filaments, $\boldsymbol{M}$ is an orientation vector, $\boldsymbol{C}$ is the right Cauchy-Green deformation tensor, $\lambda_{fc}$ is a state variable accounting for anisotropic muscle contraction, and $S$ denotes the unit sphere, see Fig. 2.

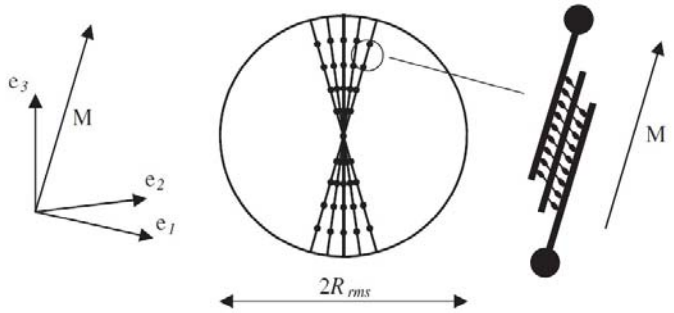


**Fig. 2. Representative micro-sphere used to describe the constitutive behavior of smooth muscle.**

## RESULTS

An axisymmetric distribution on the form

$$\rho = c\left(a^2 - \theta^2\right)$$

is adopted, where $\theta$ is the angle between a filament and the average filament direction, and $c$ and $a$ are constants. The isometric stress $P_{33}$ is predicted for a few different values of $a$, and the outcome is shown in Fig. 3, where experimental results are also enclosed. The model parameters were fitted for the isometric test with a constant stretch of 1.17 (dashed line), and then the isometric curve for a test with a stretch of 1.67 was predicted (solid lines). Fig. 3 shows that the steady-state stress increases with increasing filament dispersion. Interestingly, the curve for a=0.44, which provided the best prediction for the experimental histological data, severely underestimates the stress curve for the stretch 1.67. Two reasons for this discrepancy may be offered: the myosin filament dispersion may not be perfectly axisymmetric (as was assumed in the present analysis), and there may be additional mechanical processes involved, such as a varying overlap between myosin and actin filaments.

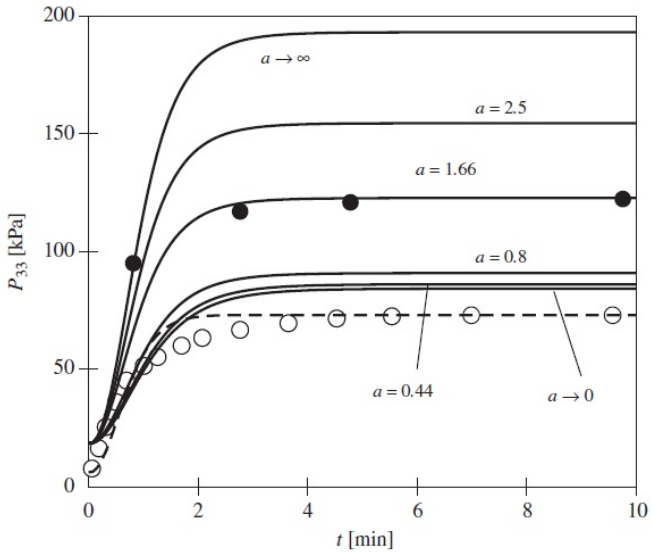


**Fig. 3. Predicted isometric stress for a few different values of the dispersion parameter *a* together with experimental results. Open symbols pertain to a test with constant stretch of 1.17 and the filled symbols pertain to a test with constant stretch 1.67. Solid lines are model predictions for the stretch 1.67.**

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