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GROWTH BEHAVIOR OF TWO INTERACTING SURFACE CRACKS OF DISSIMILAR SIZE

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ABSTRACT

When multiple cracks approach to each other, the stress intensity factor is likely to change due to the interaction of the stress field. This causes change in the growth rate and the shape of cracks. Because of the orientation with respect to the loading, and existing of K_{II} and K_{III} which can develop and increase with interaction, the shape of cracks becomes non-planar. In this study, the complex growth of interacting cracks is evaluated by using the S-Version finite element method, in which local detailed finite element mesh (local mesh) is superposed on coarse finite element model (global mesh) representing the global structure. The effect of relative size and spacing of cracks on the growth behavior is investigated. It is shown that the smallest crack stops growing due to the interaction when the difference in size of two cracks is large enough.

INTRODUCTION

Fatigue fracture of mechanical components sometimes causes a catastrophic accident. It is very important to predict the fatigue crack growth. Fracture mechanics can predict the crack growth rate by using Paris' law. Under mixed mode loading condition, crack growth direction can also be estimated well. For these predictions, precise numerical analysis is needed. Presently, FEM is generally used for this purpose. But re-meshing of crack model for new crack configurations is a bottleneck of FEM analyses. To solve this problem, many studies have been done and new methods were proposed for easy re-modeling of the problem. These are the Element Free Galerkin Method [1], Free Mesh Method [2], X-FEM [3] and S-version FEM [4].

In this paper, interaction of two surface cracks problem is solved. By changing size of one crack and distances between two cracks, parametric studies are done. For this purpose, S-version FEM (S-FEM) is employed by combining auto-mesh generation technique, and fully automatic fatigue crack growth simulation system is developed by one of authors [5]. By using this system, parametric studies become easy even for 3-dimensional problems. In the followings, effects of relative size and spacing of two cracks on the interaction behavior and fatigue life are studied and discussed. And a criterion to evaluate interaction between two cracks is proposed. The detailed description of S-FEM is skipped here because it is described by one of authors' previous paper [6].

CRACK GROWTH CRITERION FOR 3-D. PROBLEM

In the mixed mode loading condition, crack growth direction changes in a complicated manner. In the 2-dimensional problem, MTS (Maximum Tangential Stress) criterion is widely used to determine the crack growth direction, and it is verified that it gives correct prediction. In the 3-dimensional problem, several criteria have been proposed [7]-[10]. But there is no commonly accepted criterion for crack growth direction.

To predict the crack growth rate, widely known Paris' law can be used in 3-dimensional case as well as in 2-dimensional case.

$$da/dN = C(\Delta K_{eq})^n \quad (1)$$

In the 3-d.case, K_{III} component should be included for the evaluation of effective Stress Intensity Factor, K_{eff} . For the definition of K_{eff} , several equations have been proposed[7]-[10].

Through careful studies, criteria proposed by Richard et al. [10] are employed in this study. They are shown in the following equations. The crack growth angle is determined by the next equation. Fig.1 shows the direction of crack growth.

$$\varphi_0 = \mp \left[140^\circ \frac{|K_{II}|}{K_I + |K_{II}| + |K_{III}|} - 70^\circ \left(\frac{|K_{II}|}{K_I + |K_{II}| + |K_{III}|} \right)^2 \right] \quad (2)$$

where $\varphi_0 < 0^\circ$ for $K_{II} > 0$ and $\varphi_0 > 0^\circ$ for $K_{II} < 0$.

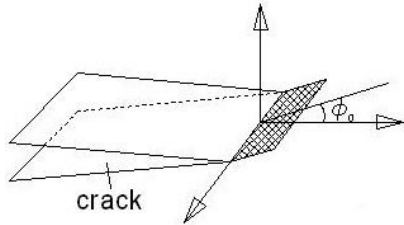


Fig.1 Crack growth direction.

Equivalent stress intensity factor is defined by the following equation.

$$\Delta K_{eq} = \frac{\Delta K_I}{2} + \frac{1}{2} \sqrt{\Delta K_I^2 + 4(1.115\Delta K_{II})^2 + 4(\Delta K_{III})^2} \quad (3)$$

EVALUATION OF INTERACTION EFFECT ON FATIGUE LIFE.

It is necessary to evaluate the interaction effect on fatigue life of the structure. For this purpose, an evaluation method is proposed in this study. In Fig.2 (a), two cracks exist on different levels. In this case, virtual crack is assumed and the length of this virtual crack is defined as R_x , which is the distance of outer crack tips, as shown in the figure. In Fig.2 (b), only one crack of these two cracks is assumed to exist, and this crack length is r_x . Two sets of numerical simulations are carried out for this problem. One is fatigue crack growth simulation of two parallel cracks, and another one is the same simulation but only considering a single crack in Fig.2 (b).

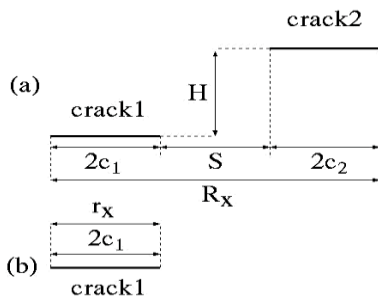


Fig.2 Two simulation models and definitions of R_x and r_x .

The first simulation is conducted until interaction effect saturates to some state, and final R_x value is obtained. The second simulation is conducted until this crack length, r_x , becomes the same R_x value, obtained in the first simulation. Through both simulations, numbers of loading cycles until the final states are counted, and they are called N_R for Fig.2 (a) and N_r for Fig.2 (b), respectively. If N_R is smaller than N_r , it means that fatigue life becomes shorter by the interaction effect between two cracks. On the other hand, if N_R is nearly same as N_r , it means that interaction effect which crack1 received from crack2 is negligibly small. In this study, by comparing N_R with N_r interaction effect which crack1 received from crack2 is evaluated.

SIMULATION OF TWO PARALLEL SURFACE CRACKS ON DIFFERENT LEVELS.

As shown in Fig.3, two surface cracks exist on a plate under cyclic tension stress. The stress ratio is, $R=0.1$. Two cracks, crack 1 and crack 2, are parallel to each other, and are located on different levels. The initial crack size is, $2c_1=2c_2=10\text{mm}$, and aspect ratio, a/c , of these cracks is 0.8, where a is crack depth and c is half crack length at the surface. The distances between these cracks are : along horizontal line, $S=10\text{mm}$, and along vertical line, $H=10\text{mm}$. The sizes of plate are: the thickness $T=300\text{mm}$, the height $h=500\text{mm}$, the width $W=500\text{mm}$. Material is assumed to be A533B steel, and C and n values of Paris' law are: $C=1.67 \times 10^{-12} [(\text{m}/\text{Cycle})/(\text{MPa m}^{1/2})^n]$ and $n=3.23$.

Fig.4(a) and (b) show the global and the local mesh of this problem. It is not necessary to consider the connectivity between global and local mesh, it is easy to set initial S and H values arbitrary.

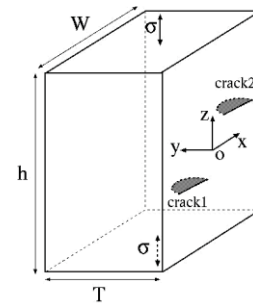
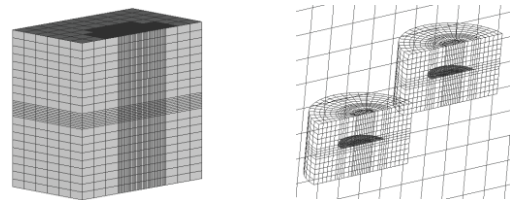


Fig.3 Two parallel surface cracks.



(a) Global mesh (b) Local mesh
Fig.4 Global mesh and local mesh.

Before fatigue crack growth simulation, static analysis is carried out and stress intensity factor distributions along crack front are obtained. As the crack tip is not in pure mode I stress state, K_I , K_{II} and K_{III} are evaluated. Stress intensity factor is calculated from energy release rate along crack front, which is evaluated by VCCM (virtual crack closure method) [11]-[12]. Ratios of K_{II} and K_{III} values with respect to K_I value are shown in Fig. 5. K_{II} and K_{III} components are very small comparing with K_I value. In general, it is known that fatigue crack growth occurs mainly under pure mode I stress state.

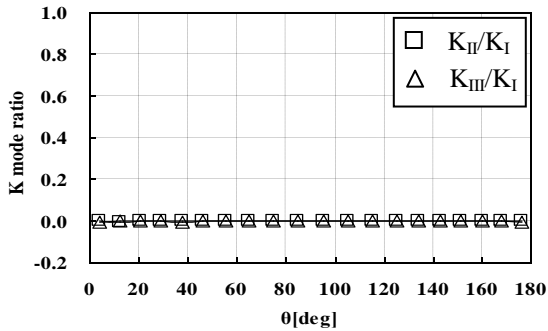


Fig.5 K_{II}/K_I and K_{III}/K_I values along initial crack front.

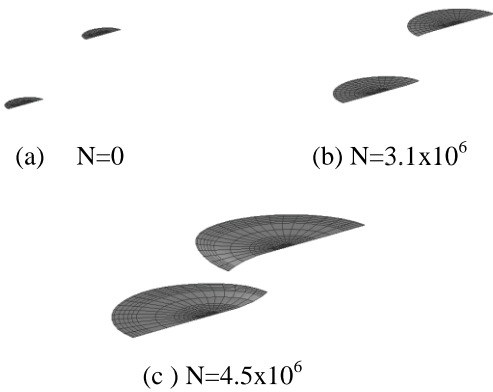


Fig.6 Changes of crack shapes.

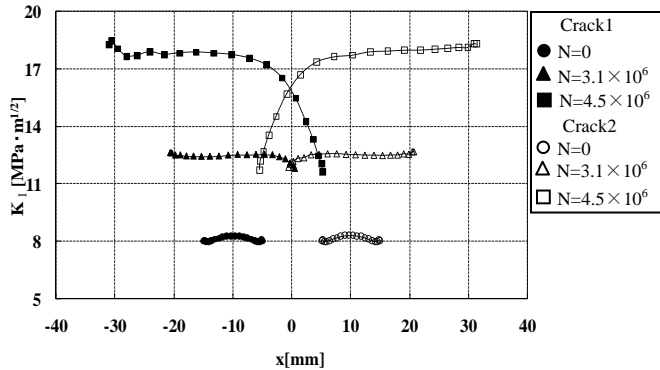


Fig.7 Changes of K_I distributions.

Fig.6 (a)-(c) show shapes of two surface cracks during fatigue growth process. First figure is the original shape, second one shows crack shape when Number of loading cycles, N , is 3.1×10^6 , and third one is for $N=4.5 \times 10^6$. After overlapping of inner crack tips, two cracks change the growing direction, and grow to be nearer to each other.

Fig.7 shows changes of stress intensity factor of two cracks. At first, K_I is the largest at the bottom of crack (deepest point), and by fatigue crack growth, it becomes nearly constant along crack front, and finally, K_I decreases largely by overlapping.

As the next problem, the size of crack 2 is changed to $2c_2=5\text{mm}$, and other parameters are kept the same. Crack shapes and stress intensity factors distributions are shown in Fig.8 and Fig.9, respectively. It is noticed from Fig.8 that crack growth occurs mainly in larger crack, crack 1. Smaller crack, crack 2, also grows larger, but after overlapped with crack 1, crack 2 stops growing. It is clear from K_I distributions. K_I value of crack 2 increases until overlapping occurs. After overlapping, K_I of crack 2 decreases a lot. And K_I of crack 1 keeps increasing, and distribution of it along the crack front is nearly constant. By the observations of crack growth behaviors and changes of K_I distributions, shown from Fig.6 to Fig.9,

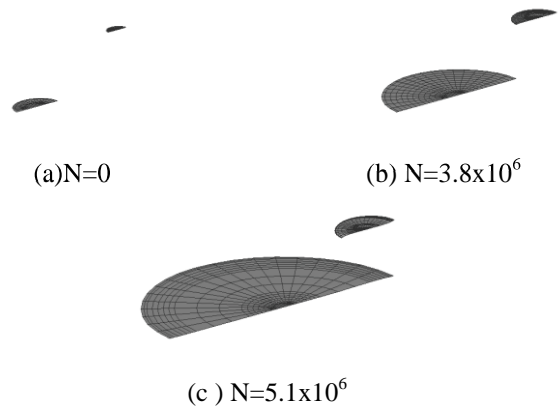


Fig.8 Changes of crack shapes.

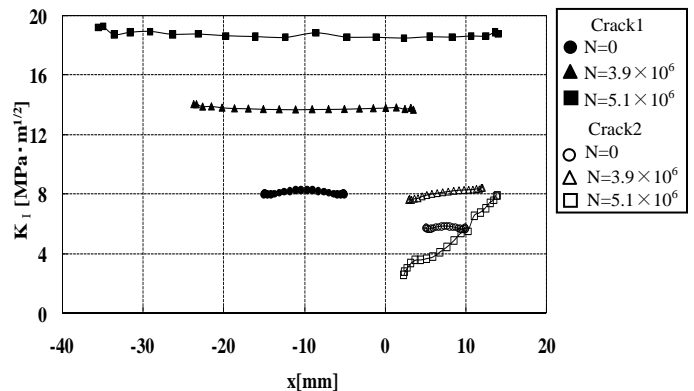


Fig.9 Changes of K_I distributions.

interaction patterns of two cracks are classified into two cases, as shown in Fig.10. Case A suggests that crack 2, smaller crack, continues to grow. And case B suggests that smaller crack, crack 2, stops growing any more (or growth of crack2 is negligibly small) after overlapped with crack 1, then crack 1 continues to grow and becomes large as a single crack. In case $2c_1=2c_2=10\text{mm}$ (Fig.6 and Fig.7), interaction patterns are classified as case A. In case $2c_1=10\text{mm}, 2c_2=5\text{mm}$ (Fig.8 and Fig.9), interaction patterns are classified as case B. So, it is necessary to determine how to judge which case to be classified.

It is possible to judge it by comparing K_{Ic1} with K_{Ic2} . K_{Ic1} and K_{Ic2} are defined in Fig.10. Stress Intensity Factor of inner crack tip of crack 1 is called K_{Ic1} , and K_I of outer crack tip of crack 2 is called K_{Ic2} . By the fatigue crack growth, both crack overlap, and K_I value decreases. But if K_{Ic2} is larger than K_{Ic1} , interaction patterns are classified case A. When K_{Ic2} becomes smaller than K_{Ic1} , interaction patterns are classified case B. Fig.7 and Fig.9 agree with this rules.

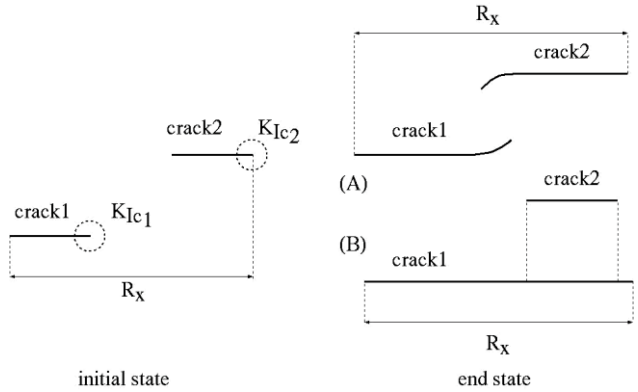


Fig.10 Difference of crack propagation.

At this moment, when comparing N_R with N_r about case $2c_1=2c_2=10\text{mm}$ (case A) and $2c_1=10\text{mm}, 2c_2=5\text{mm}$ (case B), in case $2c_1=2c_2=10\text{mm}$, $N_R/N_r=0.88$. And in case $2c_1=10\text{mm}, 2c_2=5\text{mm}$, $N_R/N_r=1.00$. This results suggest that in case A, fatigue life N_R is smaller than N_r , which means interaction effect can't be neglected. And in case B, N_R is nearly the same as N_r , which means interaction effect which crack1 received from crack2 is negligibly small. This means interaction effect which crack1 received from crack2 can be evaluated by observing that smaller crack stops growing or not.

To examine this assumption, besides four problems were solved. In these problems size of crack 2 is changed to $2c_2=9\text{mm}, 8\text{mm}, 7\text{mm}, 6\text{mm}$. These results agree with the assumption judging from K_{Ic2}, K_{Ic1} and N_R/N_r .

Fig.11 shows how interaction effect changes when size of crack 2 is changed($2c_2=10\text{mm}, 9\text{mm}, 8\text{mm}, 7\text{mm}, 6\text{mm}, 5\text{mm}$). In this figure, ordinate is N_R value, and abscissa is c_2 value. N_R is normalized by N_r . c_2 is normalized by c_1 . In this case if c_2/c_1 is less than 0.6 N_R is nearly same as N_r , which means

interaction effect which crack1 received from crack2 is negligibly small.

Then distances between two cracks is changed. Sized of crack 1 and crack 2 are assumed to be same as the previous case ($2c_1=10\text{mm}, 2c_2=5\text{mm}$), and S_0 is changed to 15mm and H_0 is assumed to be 5mm. Results are shown in Fig.12 and Fig.13. From Fig.12, it is shown that crack 2, smaller crack, does not stop growing after overlapping, and inside crack tip of crack 1, larger crack, stops growing. It means that the interaction effect can't be neglected until final state, and interaction effect should be considered. K_I distribution shows this result more clearly. At the final state, K_I value of outer crack tip of crack 2 is larger than that at inner crack tip of crack 1. K_I of crack 1 decreases largely and inside crack tip of crack 1 stops growing. Difference of this model from the previous model is the change of distances between two cracks.

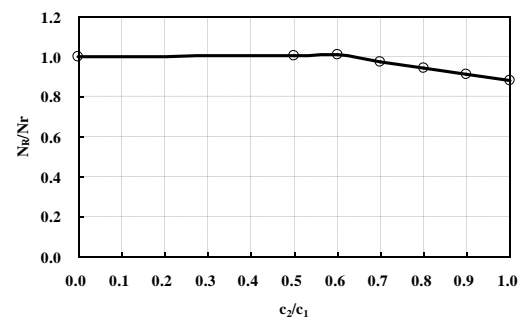


Fig.11 Changes of interaction effect.

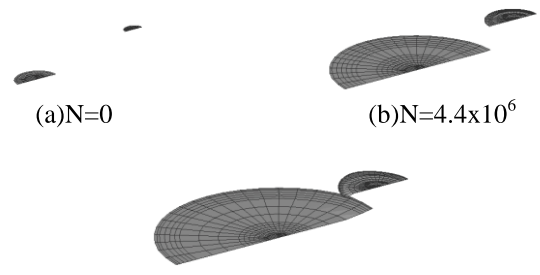


Fig.12 Changes of crack shapes.

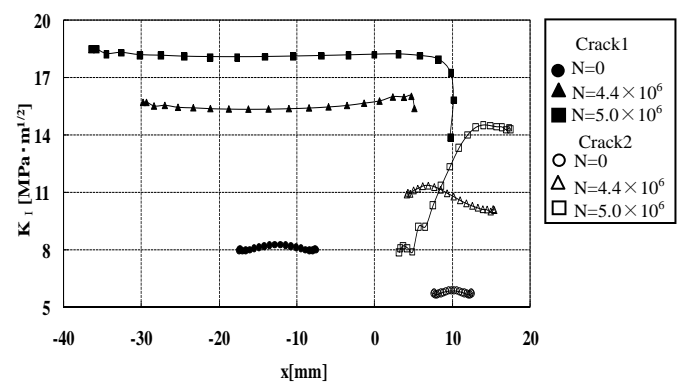


Fig.13 Changes of K_I distributions.

Based on the assumption that the interaction that effect which crack1 receives from crack2 can be evaluated by observing that smaller crack stops growing or not, parametric studies are carried out by changing distances between the two cracks in many cases. In the parametric study, crack lengths of crack 1 and crack 2 are fixed as $2c_1=10\text{mm}$ and $2c_2=5\text{mm}$. S and H values are changed from c_1 to $3c_1$ in three cases, totally 9 cases are simulated. By the behaviors of crack 2, interaction of two cracks is evaluated. Results are summarized in Fig.14. In this figure, abscissa is S value, and ordinate is H value. Both are normalized by c_1 . Cross mark means crack 2 does not stop, and interaction is observed clearly. Circle mark shows that crack 2 stopped growing, and interaction effect which crack1 received from crack2 is negligibly small. By this figure, it is concluded that H value is important parameter to determine interactions between two surface cracks.

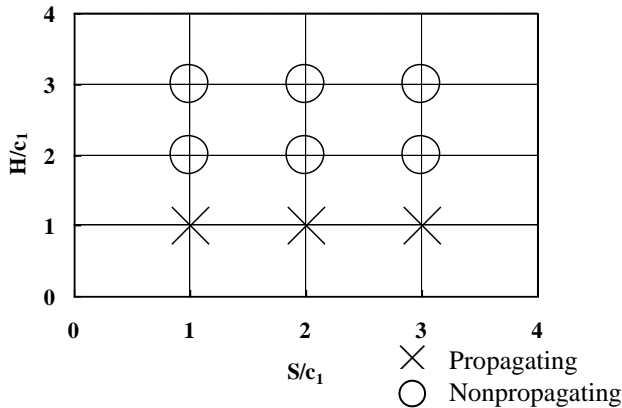


Fig.14 Result of parametric studies.

SUMMARY

Using fully automatic fatigue crack growth simulation system, interaction effect of two surface cracks is evaluated. And parametric studies are carried out for parallel surface cracks with different crack size. By changing distances between two cracks, effect of S and H values on the interaction behavior is studied. Based on parametric studies, new criterion to evaluate interaction between two surface cracks is proposed. This criterion suggests that interaction effect which larger crack, crack1 received from smaller crack, crack2 is negligibly small when crack2 stops growing (or growth of smaller crack is negligibly small).

Based on this study, it is concluded that not only the relative size but also the distance H is an important factor to determine the interaction.

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