Directivity of Frequency Sweeping Kinetic Instabilities

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I. Introduction Asymmetric frequency sweeping of Alfvénic eigenmodes has been reported in many experiments. Observations include down-sweeping TAE avalanches on NSTX [1], hooked electrostatic modes on JET [2] and extended sweeping of hot electron interchange modes on CTX [3]. Frequency sweeping is explained by the creation and evolution of holes and clumps in the fast particle distribution function. These may form when eigenmodes are driven near the instability threshold [4]. The holes and clumps move in phase space as they seek lower energy states to balance the dissipation in the background plasma. This motion corresponds to a frequency sweeping of up- and downshifted sidebands in Fourierspace. Asymmetric frequency sweeping was previously attributed to the effects of fast particle collisions, cf. [5].

In this contribution, we employ a 1D bump-on-tail model to investigate the effect of the global shape of the fast particle distribution function relative to the location of the initial resonance. First, we consider the limit of small frequency shift as described in [5]. Second, we study long range sweeping of single events by means of an *adiabatic* model [6]. Finally, we discuss the connection to 3D tokamak plasmas by calculating the radial motion of frequency sweeping holes and clumps in the idealized limit of a large aspect ratio tokamak with a circular cross section.

II. Numerical Modeling The model for short range sweeping preserves the initial, linear mode structure and models the fast particle distribution function, F_0, as a sinusoidal "bump-on-tail". We consider pure diffusion collisions with the result depicted in Figures 1 and 2. For small frequency shifts, the particle-to-wave energy transfer is

$$\frac{dW_E}{dt} \propto \frac{dF_0}{dv} \ . \tag{1}$$

Therefore, Figures 1 and 2 are upside/down mirror images. The results indicate that the sweeping occurs preferentially towards larger values of dW_E/dt.

The model for long range frequency sweeping allows for a general shape of the fast particle distribution function. We consider a Gaussian distribution on the form

$$F_0(v) \propto \exp\left[-\frac{(v-v_r)^2}{2w^2}\right] \ , \tag{2}$$

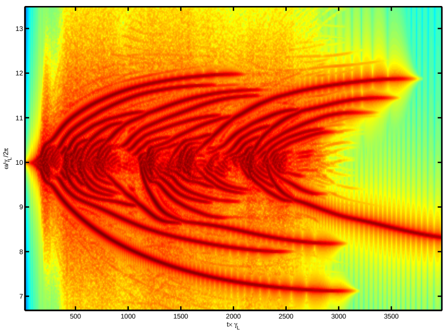

Figure 1: Frequency evolution for pure diffusion and dW_E/dt increasing toward lower frequencies.

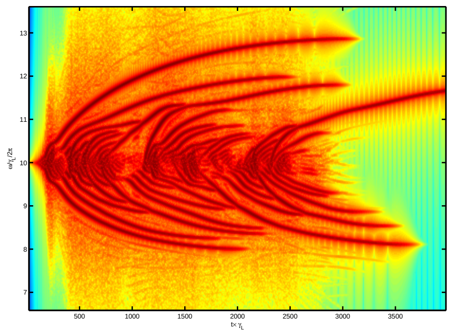

Figure 2: Frequency evolution for pure diffusion and dW_E/dt increasing toward higher frequencies.

where v_r is the initial resonant velocity. Figures 3 and 4 display the effect of varying v_r. We observe that when long range effects are taken into account there is a clear preferred sweeping directivity towards larger values of the particle-to-wave energy transfer

$$\frac{dW_E}{dt} \propto v \frac{dF_0}{dv} \ , \tag{3}$$

depicted in Figure 5.

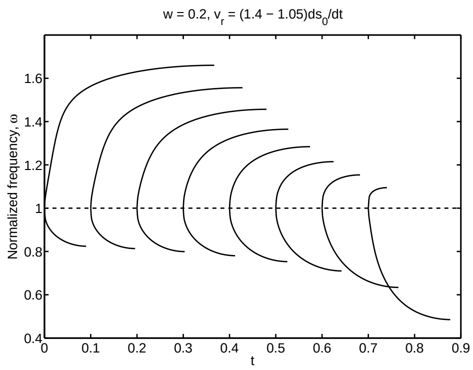

Figure 3: Frequency sweeping of holes and clumps with decreasing values of v_r.

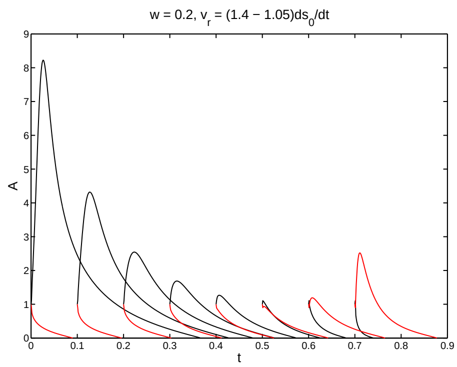

Figure 4: Amplitude of holes (black) and clumps (red) with decreasing values of v_r.

III. Generalization to Tokamaks The motion of fast particles in the equilibrium magnetic field of an axisymmetric tokamak is integrable and can be characterized by the following three constants of motion: The particle kinetic energy $E = Mv^2/2$, the toroidal angular momentum p_ζ and the magnetic moment $\mu = Mv_L^2/2B$. The conservation of E and p_ζ is exact, whereas μ is an adiabatic invariant. However, in the presence of electromagnetic wave fields with which particles resonate, these constants of motion are no longer preserved.

Nevertheless, wave-particle interaction in the case of Alfvénic eigenmodes still pre-

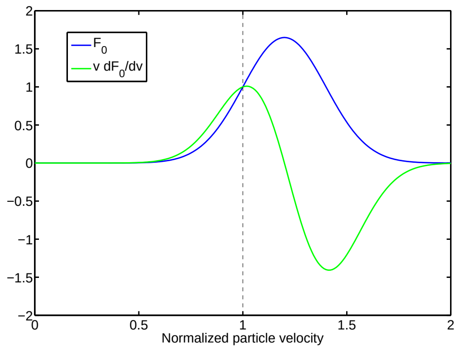

Figure 5: Gaussian distribution function and dW_E/dt, normalized to their values at the resonance.

serves two constants of motion, and is therefore effecively one-dimensional. Due to their low oscillation frequencies, shear-Alfvén instabilities do not resonate with the Larmor gyration. As a result, μ remains invariant, $\dot{\mu} = 0$, and the resonant particles lock with modes merely through their guiding center motion,

$$\omega = n \langle \omega_\zeta \rangle - m \langle \omega_\theta \rangle \ . \tag{4}$$

Here, ω is the mode frequency, while the orbital frequencies ω_ζ and ω_θ are those of the toroidal and poloidal transits. The integers n and m are toroidal and poloidal mode numbers, and the notation $\langle \ldots \rangle$ represents an average over one poloidal transit. Moreover, when the resonances do not overlap, the perturbed Hamiltonian (that governs the wave-particle interaction) can be shown to preserve the combination

$$n\dot{E} = \omega \dot{p}_\zeta \ . \tag{5}$$

Equation (5) holds as long as the mode structure evolves slowly as compared to the wave oscillations. Equations (4) and (5) determine the variations $\dot{E}$ and $\dot{p}_\zeta$ for the particles locked into the holes and clumps.

We now consider the idealized cases of deeply trapped and well passing particles in a large aspect ratio, low-β tokamak. We assume that the magnetic flux surfaces are circular, concentric tubes, we neglect finite orbit width effects and we carry our calculations to lowest order in the inverse aspect ratio.

For the description of well passing particles we use the following approximations,

$$E \approx \frac{1}{2} M v_\parallel^2 \ , \quad p_\zeta \approx M \left[R_A v_\parallel - \omega_{cA} \int_0^r \frac{r' dr'}{q\left(r'\right)} \right] \ , \tag{6}$$

$$\omega \approx k_\parallel v_\parallel \ , \quad k_\parallel = \frac{nq - m}{q R_A} \ . \tag{7}$$

Quantities denoted with the subscript A are to be evaluated at the magnetic axis and functions of r not labeled with A are to be evaluated at the magnetic flux surface at radius r. Differentiating the resonance condition (4) with respect to time together with equations (5) - (7) then results in

$$\frac{\dot{r}}{r} = -\frac{m}{r^2 k_\parallel^2 \omega_{cA}} \left[1 - \frac{m^2 S}{k_\parallel^3 r^2 q R_A} \frac{\omega}{\omega_{cA}} \right]^{-1} \dot{\omega} \ , \tag{8}$$

where $S \equiv (r/q)(dq/dr)$ is the so called *magnetic shear*.

For the deeply trapped particles we set

$$E \approx \frac{1}{2} M v_\perp^2 = \mu B \ , \quad p_\zeta \approx -M\omega_{cA} \int_0^r \frac{r' dr'}{q(r')} \ , \tag{9}$$

and transit frequencies for deeply trapped particles

$$\langle \omega_\zeta \rangle \approx \frac{qE}{r\omega_{cA} M R_A} \ , \quad \langle \omega_\theta \rangle \approx \frac{v_\perp}{q R_A} \sqrt{\frac{r}{2R_A}} \ . \tag{10}$$

Differentiation of (4), (9) and (10) with respect to time together with (5) results in

$$\frac{\dot{r}}{r} = -\left[(1-S)\,\omega + m\left(\frac{3}{2} - 2S\right)\omega_\theta \right]^{-1} \dot{\omega} \ . \tag{11}$$

A general expression for the particle-to-wave energy transfer in a tokamak needs to be computed numerically. However, within the idealized approach taken above, an analytic expression is available for well passing particles [7],

$$\frac{dW_E}{dt} \propto \frac{r^2}{q^3 R} \frac{\partial f}{\partial r} \ , \tag{12}$$

where f is the unperturbed fast particle distribution function.

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