

A Theory of Energy Use*

Raouf Boucekkine^a, Antonia Díaz^b and Luis A. Puch^{c†}

^aCORE and IRES, Université Catholique de Louvain

^bUniversidad Carlos III de Madrid

^cUniversidad Complutense, FEDEA and ICAE

February 2011

Very preliminary and incomplete. Do not quote. Please do not distribute without authors approval.

Abstract

The evidence shows that the short run elasticity of energy use is smaller than its long run elasticity. The recent evidence on energy use and energy prices suggests, though, that the short run response of energy use to energy prices has changed over time. Existing theories of energy use, namely, complementarity between capital and energy at the aggregate level, or putty-clay models of energy use, cannot account for this change in the short run elasticity of energy use. Here we propose a theory where, as in the data, the short run elasticity of energy use is smaller than the long run elasticity but it also may change depending on the rate of embodied technological progress, accounting for its increase in the recent years.

Keywords: Energy use, vintage capital, energy price shocks, investment-specific technology shocks
JEL Classification: E22, E23

*We thank Omar Licandro, Ramón Marimón and Franck Portier for helpful comments, and Alberto Sánchez-Martín and Guzmán González-Torres for excellent research assistance. Díaz, and Boucekkine and Puch, thank respectively the Dirección General de Investigación, projects ECO2010-20614 and SEJ2007-65552, for financial support. Boucekkine also thanks the Belgian Research Programmes PAI P4/01 and ARC 03/08-302. Puch also thanks Fundación FocusAbengoa, and the EUI for its hospitality while part of this research.

†Corresponding Author: Luis A. Puch, FEDEA, Jorge Juan 46, 28001 Madrid, Spain; E-mail: lpuch@fedea.es

1 Introduction

Energy economists have identified two salient features of data on energy use and energy prices. On the one hand, in time series data, energy use is not very responsive to energy price changes whereas energy expenditure varies very much (e.g. Berndt and Wood (1975)). On the other hand, in cross-section data across countries, energy use is very responsive to international differences in energy prices (e.g., Griffin and Gregory 1976; Pindyck 1991). These features of the data have been recently revised for instance in Kilian (2008), together with newer pieces of relevant evidence.

Part of this recent evidence suggests that the short run response of energy use to energy prices may have changed and, in fact, increased. Metcalf (2008) finds that more than two-thirds of the decline in energy intensity in the U.S. economy comes from improvements in energy efficiency vis-a-vis changes in sectoral composition since 1970. Edelstein and Kilian (2007) find that the energy price elasticity of electricity demand has increased up to 2006 compared to previous estimates. Finally, further work at a more disaggregate level in Steinbuks and Neuhoff (2010) shows that between 1990 and 2005 the energy efficiency of physical capital has increased in all manufacturing sectors in a sample of 19 OECD countries.

There are several theories that account for the basic features of the data with various degrees of success. The pioneer were Pindyck and Rotemberg (1983) who built a theory whose main feature is complementary of capital and energy at the aggregate level coupled with adjustment costs in capital. Atkeson and Kehoe (1999) consider a putty-clay model where capital can be combined with energy at different intensities but there is no ex-post substitution. Thus, Atkeson and Kehoe (1999) predict a more sluggish response of capital to permanent changes in energy prices than Pindyck and Rotemberg (1983). Nevertheless, Atkeson and Kehoe (1999) predict a too sluggish time series movement in the capital-energy ratio in response to changes in energy prices. Finally, Díaz, Puch, and Guilló (2004) build a theory in which production takes place at individual plants and capital can be used either to produce output or to reduce the amount of energy required to run a plant, and reallocating capital from one use to another is costly. This turns out to be crucial for the quantitative properties of the model to be in conformity with the low short-run and high long-run elasticities of energy use seen in data.

Nevertheless, existing theories do not account for the observed changes in energy demand relative to energy productivity, and they are silent about the sources of those changes. In this paper

we build upon Atkeson and Kehoe (1999) to provide a theoretical foundation for the role of investment decisions as a key channel in the response to changes in the relative price of capital with respect to energy. As these authors, we explicitly model heterogeneity in the energy efficiency of the capital stock. Differently from them, we incorporate this heterogeneity assumption into a vintage capital framework. Further, a competitive equilibrium of such a decentralized vintage economy is described, and the equilibrium price of capital is fully characterized. A key feature of our model economy is that utilization decreases with age and depreciation depends on the obsolescence rate as in Boucekkine, del Río, and Martínez (2009). In such a framework, the combined impact of embodied technical change and its importance relative to that of energy price shocks is reinterpreted then in terms of obsolescence. An acceleration in the rate of decline of the quality adjusted relative price of capital equipment induces investment in newer capital vintages and thus a more efficient use of energy inputs. Therefore, rising prices of energy transmit into changes in energy use through investment choices.

Next, some alternative modeling assumptions corresponding to complementarity between capital and energy or putty-clay models of energy use are embedded in our decentralized environment. We show that abstracting from vintage capital while modeling differences in energy requirements implies that the quality-adjusted relative price of capital increases with its efficiency. Thus, the higher the average efficiency of capital in the economy, the higher the quality-adjusted relative price of capital. This implication is at odds with the estimates of Gordon (1990) or Cummins and Violante (2002), among others.

To illustrate on the quantitative importance of our theory we proceed with an empirical implementation of the vintage capital model subject to stationary both energy price and investment shocks. We show that technological frictions operating through investment-specific technical progress are the key to the transmission of energy price shocks in the model. Thus, according to our theory, the short run elasticity of energy use is smaller than the long run elasticity as in the data. However, the short run elasticity may change in our framework depending on the rate of embodied technological progress, accounting for its increase in the recent years.

The organization of the paper is as follows. Section 2 describes the benchmark model economy, defines a competitive equilibrium of such an economy and its properties in a vintage framework and proceeds to find the equilibrium aggregates and the planner's problem. It also shows that when capital equipment can be scrapped, the benchmark economy embeds a putty-putty economy with

complementarity between capital and energy and adjustment costs to improvements in efficiency. Section 3 describes a version of a putty-clay model of energy in our decentralized environment. Section 4 presents a quantitative assessment of our theory under energy price and investment shocks. To this purpose, the combined impact of embodied and disembodied technical change and its importance relative to that of energy price shocks and other influences is evaluated. The last section concludes.

2 The benchmark model economy

We will assume that energy is entirely bought in an international market at an exogenously given price p_t . Therefore, from the point of view of the economic agents, the energy price follows a stochastic process. We assume that there is no international borrowing and lending. In absence of an international credit market we can think of the price of energy as given by nature. This implies that, under market completeness, the second welfare theorem applies and, therefore, we can restrict our attention to efficient allocations.

2.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)), \quad \beta \in (0, 1), \quad \alpha > 0, \quad (2.1)$$

where c_t is consumption and ℓ_t is leisure t . Each household is endowed with \bar{h} units of time and, therefore, works $\bar{h} - \ell_t$ hours every period.

2.2 Technology

Production of the unique final good is carried out at a continuum of autonomous plants which are indexed by their vintage z and their type v . The vintage is given by the age of the unit of capital installed at the time of being created.¹ In each plant output is produced with capital, labor, one

¹In general, output of a plant of age τ is described by $y_t(\tau)$, $y_t : T \rightarrow [0, \infty)$. We further restrict this assumption for aggregation purposes [cf. Benhabib and Rustichini (1991)].

unit of capital and energy, according to the technology

$$y_t(z, v) = A(1 + \gamma)^t \kappa_t(z, v)^\alpha h_t(z, v)^{1-\alpha}, \quad (2.2)$$

with $0 < \alpha < 1$, where $y_t(z, v)$ is the output of the plant at time t , Γ_t is the disembodied technological change factor which may change stochastically over time according to a process that we will specify in Section 4. The variable $\kappa_t(z, v)$ is the amount of services provided the the unit of capital installed, whereas $h_t(z, v)$ is the amount of labor services employed in the plant.

The amount of capital services, $\kappa_t(z, v)$, depends on the amount of energy used in the plant, $e_t(z, v)$, and its type v according to the technology,

$$\kappa_t(z, v) = (1 + \lambda)^z (1 + \gamma^e)^z v^{1-\mu} \min \{ e_t(z, v), B(1 + \gamma^e)^{-z} v^\mu \}, \quad (2.3)$$

where $\mu > 1$. λ is the embodied technological change growth rate, whereas γ^e is the growth rate of the level of energy efficiency. Thus, embodied technological progress brings not only higher productivity but also lower energy requirement. Notice that the energy requirement, $(1 + \gamma^e)^{-z} v^\mu$ is a convex function of v . Notice that if the amount of energy used is below the requirement $B(1 + \gamma^e)^{-z} v^\mu$, capital services fall with type, but if $e_t(z, v) = B(1 + \gamma^e)^{-z} v^\mu$, capital services do increase with type but energy use rises in a higher proportion with v . Thus, v allows for capital of the same vintage to have different energy efficiency.

Capital is irreversible; that is, capital of vintage z_1 cannot be converted in capital of vintage z_2 . Likewise, capital is irreversible across types. Capital of type v_1 cannot be converted into type v_2 . Finally, at the end of the period, once production has taken place, the plant faces a positive probability of death, $\omega \in [0, 1]$, which is i.i.d. across plants. This death implies the destruction of the unit of capital. This death probability plays the role of physical depreciation of capital. At any period t , final good can be transformed into capital with a technology that transforms one unit of final good in Θ_t , which may vary stochastically over time, of capital of vintage $t + 1$. This capital is installed in a new plant which starts producing output in period $t + 1$.

2.3 Market arrangements

Households are the owners of the plants and, therefore, of the capital installed. There is a market for plants that opens at the end of the period, once profits have been realized. Notice, though, that capital is not traded since it is already installed in a plant and it cannot be reallocated. Since there is a one to one correspondence between plants and units of capital, the price of a plant is also equal to the price of the unit of capital installed, $q_t(z, v)$, where $q_t(z, v)$ is the price of one unit of capital of vintage z and type v at the end of period t in units of consumption good at time t . We have already said that capital is irreversible. That is, since capital cannot be scrapped, plants cannot be scrapped either. They, however, can be left idle by setting $e_t(z, v) = 0$. We further assume that all households start out with the same amount of capital and shares of the plants installed. Additionally, we assume that households trade a one risk free bond which is in zero net supply.

The timing is the following: At the end of period $t - 1$ any prospective plant must install one unit of capital before the energy price is known. After this decision has been made, at the beginning of period t the uncertainty is resolved: agents learn the disembodied technological progress Γ_t , as well as the productivity of the investment technology Θ_t . The energy price is realized. Then, they decide the amount of energy used, $e_t(z, s)$, and the number of workers hired, $h_t(z, s)$. Households consume and save. A fraction ω of plants die.

2.4 The plant's problem

$$\begin{aligned}
 & \max_{\substack{y_t(z,v) \geq 0, h_t(z,v) \geq 0, \\ e_t(z,s) \geq 0}} \pi_t(z, v) = y_t(z, v) - w_t h_t(z, v) - p_t e_t(z, v) \\
 \text{s. t.} \quad & y_t(z, v) \leq A(1 + \gamma)^t \kappa_t(z, v)^\alpha h_t(z, v)^{1-\alpha}, \\
 & \kappa_t(z, v) = (1 + \lambda)^z (1 + \gamma^e)^z v^{1-\mu} \min \{e_t(z, v), B (1 + \gamma^e)^{-z} v^\mu\}.
 \end{aligned} \tag{2.4}$$

2.5 Household's problem

Plants of any vintage and type can be traded at the individual level. New investment, however, comes in new vintage—it is a technological restriction, as in the one sector model TFP grows exogenously, we cannot help to be more productive. Agents can, though, choose the type of the new capital units to be installed. The household's problem can be written in the following way:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + x_t + \sum_{z=-\infty}^{t+1} \int_0^{\infty} q_t(z, v) m_{t+1}(z, v) dv + b_{t+1} \leq w_t (\bar{h} - \ell_t) \\
& \tilde{q}_t \Theta_t x_t + \sum_{z=-\infty}^t \int_0^{\infty} [(1 - \omega) q_t(z, v) + \pi_t(z, v)] m_t(z, v) dv + (1 + r_t^b) b_t, \tag{2.5} \\
& x_t \geq 0, m_{t+1}(z, v) \geq 0, \text{ for all } z \leq t + 1, v > 0, b_{t+1} \geq \underline{b}, \\
& m_0(z, v), b_0 \text{ and energy prices given.}
\end{aligned}$$

Notice the difference between \tilde{q}_t , which is the price at which the new capital units are sold and $q_t(z, v)$, which is the price of capital of vintage z and type v .

2.6 Definition of equilibrium

An equilibrium for this economy, given the sequence of energy prices, $\{p_t\}_{t=0}^{\infty}$, is a sequence of prices $\{\{\tilde{q}_t, q_t(z, v)\}_{z=-\infty}^{t+1}, w_t, r_t^b\}_{t=0}^{\infty}$, an allocation for each consumer, $\{c_t, \ell_t, \{m_t(z, v)\}_{z=-\infty}^{t+1}, x_t, b_{t+1}\}$, and allocation for each plant of vintage z , type v , $\{y_t(z, v), h_t(z, v), \kappa_t(z, v), e_t(z, v)\}_{z=-\infty}^t$, such that:

1. $\{c_t, \ell_t, \{m_t(z, v)\}_{z=-\infty}^{t+1}, x_t, b_{t+1}\}$ solves the household's problem shown in (2.5) given the sequence of prices,
2. $\{y_t(z, v), h_t(z, v), e_t(z, v)\}_{z=-\infty}^t$ solves the plant's problem given the sequence of prices,
3. the relative price of the latest vintage is $q_t(t + 1, v) = \tilde{q}_t = \Theta_t^{-1}$, for any v ,
4. markets clear,
 - (a) the bond is in zero net supply, $b_{t+1} = 0$,
 - (b) the amount of plants of vintage z and type v traded must be equal to the amount of existing plants, $m_t(z, v) = k_t(z, v)$, for all $z \leq t, v > 0$,
 - (c) $\bar{h} - \ell_t = \sum_{z=-\infty}^t \int_0^{\infty} m_t(z, v) h_t(z, v) dv$,
 - (d) the final good market satisfies $c_t + x_t = \sum_{z=-\infty}^t \int_0^{\infty} m_t(z, v) y_t(z, v) dv$,
5. the law of motion of capital of vintage z is $\int_{v=0}^{\infty} k_t(z, v) dv = (1 - \omega)^{t-z} \Theta_{z-1} x_{z-1}$, for all $t \geq z$.

2.7 Properties of equilibrium

Here we develop some properties of equilibrium that will be useful to build the aggregate representation of the economy.

2.7.1 Production and profits

The operating profits of a plant of vintage z type v at time t are

$$\pi_t(z, v) = y_t(z, v) - w_t h_t(z, v) - p_t e_t(z, v), \quad (2.6)$$

The plant chooses $h_t(z, v)$, and $e_t(z, v)$ to maximize profits. Notice that if the plant chooses $e_t(z, v) < B(1 + \gamma^e)^{-z} v^\mu$, profits monotonically decrease with v . Thus, it must be the case that, in equilibrium, $e_t(z, v) = B(1 + \gamma^e)^{-z} v^\mu$, which implies that capital services satisfy

$$\kappa_t(z, v) = (1 + \lambda)^z v, \quad (2.7)$$

and output at the plant level is

$$y_t(z, v) = A(1 + \gamma)^t (1 + \lambda)^{z\alpha} v^\alpha h_t(z, v)^{1-\alpha}. \quad (2.8)$$

Since labor productivity must be equal across all plants and vintages, the labor demand of a plant of vintage z and type v_z and the amount of output produced satisfy

$$\frac{h_{t+i}(z, v_z)}{h_{t+i}(t, v_t)} = \frac{y_{t+i}(z, v_z)}{y_{t+i}(t, v_t)} = (1 + \lambda)^{z-t} \frac{v_z}{v_t}, \text{ for all } i \geq 0, \quad (2.9)$$

where $h_{t+i}(z, v_z)$ and $h_{t+i}(t, v_t)$ are, respectively, the amount of labor hired and output produced by the plant created at time t and whose type is v_t . The profit of the plant of vintage z and type v_z is

$$\pi_t(z, v_z) = \alpha \left(A B^\alpha (1 + \gamma)^t \right)^{\frac{1}{\alpha}} \left(\frac{1 - \alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}} (1 + \lambda)^z v_z - p_t B (1 + \gamma^e)^{-z} v_z^\mu, \quad (2.10)$$

which increases with z . Notice, though, is a strictly concave function of the type v . As a matter of fact, there exists a type that, ceteris paribus, yields maximum profits. For the plant to be operated,

i.e., to be assigned labor and energy, it must be the case that profits are non negative. The following Proposition establishes which plants will be operated in equilibrium:

Proposition 1. *For each vintage z only installed capital of types $v \geq \underline{v}_{z,t}$ are utilized in equilibrium, where $\underline{v}_{z,t}$ is defined as*

$$\alpha (A(1 + \gamma)^t)^{\frac{1}{\alpha}} \left(\frac{1 - \alpha}{w_t} \right)^{\frac{1 - \alpha}{\alpha}} (1 + \lambda)^z (1 + \gamma^e)^z \underline{v}_{z,t}^{1 - \mu} = p_t. \quad (2.11)$$

The type $\underline{v}_{z,t}$ decreases with p_t and increases with z .

Thus, more technological advanced capital (higher vintages) can be less efficient in their energy use, given the energy price. Thus, in a way, embodied technological change is an energy saving device: it increases profits and output given the energy price.

2.7.2 The equilibrium price of capital

Since the number of plants that use capital of vintage z and type v is equal to the amount of vintage z capital, $m_t(z, v) = k_t(z, v)$, for all $z \leq t + 1$, it follows that the price of a plant must be equal to the price of a unit of capital. Let us turn first to the household's investment decision. Inspecting (2.5) we find that the price at time t of a unit of capital of vintage $t + 1$, regardless its type v should be

$$q_t(t + 1, v) = \Theta_t^{-1}, \text{ for all } v > 0. \quad (2.12)$$

The following proposition establishes the optimal investment policy:

Proposition 2. *Suppose that in equilibrium all installed capital is utilized, $e_t(z, v) > 0$, for all t . Then, investment is positive for at most one type of capital v_{t+1} , $k_{t+1}(t + 1, v_{t+1}) > 0$.*

Proof. The first order condition with respect to the latest vintage and type v , $k_{t+1}(t + 1, v)$, satisfies

$$\chi_t(t + 1, v) - \varsigma_t \Theta_t^{-1} + E_t \sum_{i=1}^{\infty} ((1 - \omega)^{i-1} \varsigma_{t+i} \pi_{t+i}(t + 1, v)) = 0. \quad (2.13)$$

Investment in type v is positive only if $\chi_t(t + 1, v) = 0$. This multiplier is non negative so zero is its minimum value. Thus, we have to show that $\chi_t(t + 1, v)$ has a unique minimum. Equivalently,

$E_t \sum_{i=1}^{\infty} \left((1 - \omega)^{i-1} \varsigma_{t+i} \pi_{t+i}(t+1, v) \right)$ has a unique maximum. The derivative of the latter expression with respect to v is

$$E_t \sum_{i=1}^{\infty} \left((1 - \omega)^{i-1} \frac{\varsigma_{t+i}}{\varsigma_t} \frac{\partial \pi_{t+i}(t+1, v)}{\partial v} \right). \quad (2.14)$$

Inspection of (2.10) tells us that there is a unique v for which (2.14) is zero. Thus, $\chi_t(t+1, v)$ has a unique minimum. Thus, there must exist a unique value $v_{t+1} > 0$ for which $\chi_t(t+1, v) = 0$ and receives positive investment. \square

This proposition ensures that, if all installed capital is used in equilibrium, there is only one type per vintage. Thus, existing capital satisfies

$$k_t(z, v_z) = (1 - \omega)^{t-z} \Theta_z x_z, \text{ for all } z \leq t. \quad (2.15)$$

In equilibrium, the price of one plant of vintage $z \leq t+1$ —given the corresponding transversality conditions—satisfies:

$$q_t(z, v) = E_t \sum_{i=1}^{\infty} \left((1 - \omega)^{i-1} \frac{\varsigma_{t+i}}{\varsigma_t} \pi_{t+i}(z, v) \right). \quad (2.16)$$

which, given the expression for profits shown in (2.10), implies that $q_t(z, v)$ satisfies

$$q_t(z, v) = \Theta_t^{-1} (1 + \lambda)^{z-(t+1)} \frac{v_z}{v_{t+1}} + \Psi(z, t) E_t \sum_{i=1}^{\infty} \frac{\varsigma_{t+i}}{\varsigma_t} p_{t+i}, \text{ for all } z \leq t+1, \text{ where} \quad (2.17)$$

$$\Psi(z, t) = B (1 + \gamma^e)^{-z} v_z^\mu \left[(1 + \lambda)^{z-(t+1)} (1 + \gamma^e)^{z-(t+1)} \left(\frac{v_{t+1}}{v_z} \right)^{\mu-1} - 1 \right]. \quad (2.18)$$

2.8 Aggregation

Here we proceed to find the equilibrium aggregates in this economy and the planner's problem that is consistent with the microeconomic structure described.

2.8.1 Aggregate capital and its relative price

Remark 1. *In order to aggregate capital we are going to use the following approximation to the relative price of capital of vintage z and type v shown in (2.17),*

$$\widehat{q}_t(z, v) = \Theta_t^{-1}(1 + \lambda)^{z-(t+1)} \frac{v_z}{v_{t+1}}, \text{ for all } z \leq t + 1, v \in \mathbf{R}_{++} \quad (2.19)$$

In the dynamic economy the factor $\Psi(z, t)$, shown in (2.18) is a negligible part of the relative price. This will help us to find a suitable aggregation in our economy.

Let us define as k_t the aggregate volume of capital, in per capita terms, in units of the latest vintage. Thus,

$$k_t = \sum_{z=-\infty}^t \frac{\widehat{q}_t(z, v_z)}{\widehat{q}_t(t, v_t)} k_t(z, v_z). \quad (2.20)$$

The average relative price of capital in units of consumption good is, by definition of k_t , equal to the relative price of vintage t and type v_t .

$$q_t = \sum_{z=-\infty}^t \widehat{q}_t(z, v_z) \frac{k_t(z, v_z)}{k_t} = (1 + \lambda)^{-1} \frac{v_t}{v_{t+1}} \Theta_t^{-1}. \quad (2.21)$$

We can also find the aggregate amount of capital services which can be found using (2.7) and (2.19),

$$\kappa_t = (1 + \lambda)^t v_t \sum_{z=-\infty}^t \widehat{q}_t(z, v_z) k_t(z, v_z) = (1 + \lambda)^t v_t k_t. \quad (2.22)$$

The average relative price of capital services in units of final good would be

$$q_t^\kappa = \sum_{z=-\infty}^t \left[\frac{\widehat{q}_t(z, v_z)}{\kappa_t(z, v_z)} \right] \frac{\widehat{q}_t(z, v_z) k_t(z, v_z)}{q_t k_t}, \quad (2.23)$$

which collapses to

$$q_t^\kappa = \Theta_t^{-1}(1 + \lambda)^{-t} v_t^{-1}. \quad (2.24)$$

This price falls over time, consistently with the findings by Gordon (1990) and Cummins and

Violante (2002).

2.8.2 Output and hours worked and energy use

Production of all plants of vintage t at time t is the expected output of a plant of vintage z which, using (2.9), can be written as

$$y_t = \sum_{z=-\infty}^t (1 + \lambda)^{z-t} \frac{v_z}{v_{t+1}} k_t(z, v_z) y_t(t, v_t). \quad (2.25)$$

Likewise, using (2.9), and (2.19) we can write aggregate labor as

$$h_t = \sum_{z=-\infty}^t (1 + \lambda)^{z-t} \frac{v_z}{v_{t+1}} k_t(z, v_z) h_t(t, v_t) = k_t h_t(t, v_t). \quad (2.26)$$

Aggregate gross output is

$$y_t = A, (1 + \gamma)^t (1 + \lambda)^{\alpha t} (v_t k_t)^\alpha h_t^{1-\alpha}. \quad (2.27)$$

Likewise, the aggregate use of energy is

$$e_t = \sum_{z=-\infty}^t B (1 + \gamma^e)^{-z} v_z^\mu k_t(z, v_z). \quad (2.28)$$

2.8.3 The aggregated economy

The law of motion of capital is

$$k_{t+1} = \Theta_t x_t + \frac{1 - \omega}{1 + \lambda} \frac{v_t}{v_{t+1}} k_t. \quad (2.29)$$

If all plants are utilized in equilibrium, the law of motion of energy use is

$$e_{t+1} = B (1 + \gamma^e)^{-(t+1)} v_{t+1}^\mu \Theta_t x_t + (1 - \omega) e_t. \quad (2.30)$$

The quasi-social planner's problem as

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\bar{h} - h_t)) \\
\text{s. t.} \quad & c_t + x_t \leq A(1 + \gamma)^t (1 + \lambda)^{\alpha t} (v_t k_t)^\alpha h_t^{1-\alpha} - p_t e_t, \\
& e_{t+1} \leq B(1 + \gamma^e)^{-(t+1)} v_{t+1}^\mu \Theta_t x_t + (1 - \omega)e_t, \\
& k_{t+1} \leq \Theta_t x_t + \frac{1-\omega}{1+\lambda} \frac{v_t}{v_{t+1}} k_t, \\
& k_0 \text{ given, } v_t \geq 0, x_t \geq 0.
\end{aligned} \tag{2.31}$$

It will be useful to write the quasi-social planner's problem in terms of capital services, instead of capital, using (2.22):

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\bar{h} - h_t)) \\
\text{s. t.} \quad & c_t + x_t \leq A(1 + \gamma)^t \kappa_t^\alpha h_t^{1-\alpha} - p_t e_t, \\
& e_{t+1} \leq B(1 + \gamma^e)^{-(t+1)} v_{t+1}^\mu \Theta_t x_t + (1 - \omega)e_t, \\
& \kappa_{t+1} \leq v_{t+1}(1 + \lambda)^{t+1} \Theta_t x_t + (1 - \omega)\kappa_t, \\
& \kappa_0 \text{ given, } x_t \geq 0.
\end{aligned} \tag{2.32}$$

Finally we need to add the condition that ensures that this aggregation works: all installed capital is used in equilibrium. Using (2.11) with strict inequality, we can write the following assumption:

Assumption 1. *The energy price is never too large,*

$$\alpha A(1 + \gamma)^t \kappa_t^{\alpha-1} h_t^{1-\alpha} > p_t B^{1-\alpha} (1 + \gamma^e)^{-z} (1 + \lambda)^{-z} v_{t+1}^{\mu-1}. \tag{2.33}$$

3 The role of embodied technological progress: comparing alternative specifications

In this section we want to highlight the role of embodied technological progress. To this purpose, we want to compare our theory with two existing theories of energy use those by Atkeson and Kehoe (1999) and Díaz, Puch, and Guilló (2004). To simplify the discussion, let us assume away uncertainty.

3.1 Embodied technological change and the cost of saving energy

Notice that in our economy there are two types of investment specific technological change, embodied technological change, which affects to the amount of services provided by a unit of capital, λ , and disembodied technological change, which determines the cost of producing a new unit of capital, θ_t . Both affect in the same way the accumulation of capital services or, in other words, the accumulation of quality adjusted capital, $\kappa_{t+1} \leq v_{t+1}(1 + \lambda)^{t+1}\Theta_t x_t + (1 - \omega)\kappa_t$. If we call $\tilde{\Theta}_t = (1 + \lambda)^{t+1}\Theta_t$ the law that governs quality adjusted capital and energy use can be written as

$$\kappa_{t+1} = v_{t+1}\tilde{\Theta}_t x_t + (1 - \omega)\kappa_t, \quad (3.1)$$

$$e_{t+1} = B(1 + \gamma^e)^{-(t+1)}(1 + \lambda)^{-(t+1)}v_{t+1}^\mu \tilde{\Theta}_t x_t + (1 - \omega)e_t. \quad (3.2)$$

Notice that embodied technological change, as opposed to disembodied technological progress, increases productivity only if investment is positive. Thus, any shock to the economy that lowers investment will propagate over time through lower technological level. This suggests that an unexpected energy price increase will have more persistent effects in an economy with embodied technological change than in an economy where all technological progress is disembodied. Embodiment also implies that technological progress *is* a sort of energy saving technology, as shown in the expression of energy use, (3.2), which is not necessarily true in the case of disembodied technological progress. To illustrate this point lets turn to the theories by Atkeson and Kehoe (1999) and Díaz, Puch, and Guilló (2004).

3.2 The cost of saving energy in a putty-clay model economy

Let us turn now to the economy proposed by Atkeson and Kehoe (1999). They abstract from embodied technological progress but retain the assumption about efficiency types and capital irreversibility. In particular, the amount of capital services, $\kappa_t(v)$, depends on the amount of energy used in the plant, $e_t(v)$, and an index $u_t(v) \in \{0, 1\}$, which measures the utilization of the unit of capital, according to the technology,

$$\kappa_t(v) = f(v) \min \left\{ e_t(v), \frac{1}{v} u_t(v) \right\}, \quad (3.3)$$

where $f(v)$ is a strictly increasing function of v , where $f'(v) \geq 0$, and $f''(v) < 0$. In this framework, the production of one new unit of capital always takes one unit of output, which is equivalent to assuming in our framework that $\Theta_t = 1$, for all t . This economy aggregates in a very similar way to ours, as shown in Appendix A. The associated quasi-social planner's problem is the following:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^{t-} (\log(c_t) + \varphi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + x_t \leq A(1 + \phi)^t \kappa_t^\alpha h_t^{1-\alpha} - p_t e_t, \\
& c_t \geq 0, \ell_t \leq \bar{h} - h_t, \kappa_{t+1} \leq (1 - \omega)\kappa_t + \frac{f(v_{t+1})}{v_{t+1}} x_t, e_{t+1} \geq (1 - \omega)e_t + \frac{1}{v_{t+1}} x_t, \\
& x_t \geq 0, v_{t+1} \in \mathbf{R}_+, \\
& \kappa_0, \text{ and energy prices given, } t \geq 0.
\end{aligned} \tag{3.4}$$

In this economy, the efficiency type v_t plays the same role that v_t^{-1} in our model economy. In this case, however, since all technological progress is disembodied, only specific energy saving technological progress helps to save energy.

3.3 The cost of saving energy in a putty-putty model economy with costly capital reallocation

Let us now turn to the specification by Díaz, Puch, and Guilló (2004). This framework is a bit different, though, because the energy efficiency v is capital itself, energy saving capital, which can be accumulated independently from working capital. Plants' managers hire energy saving capital every period. This assumption may suggest that the response of energy use to prices may be very swift, but it is not because accumulating energy saving capital is subject to adjustment costs. Appendix B describes this economy and its associated quasi-social planner's problem is

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^{t-} (\log(c_t) + \varphi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + x_t^k + x_t^v \leq n_t^\alpha (\theta + \sigma(1 - n_t))^\alpha (1 + \xi)^t \zeta^\alpha k_t^\alpha h_t^{1-\alpha} - p_t e_t, \\
& c_t \geq 0, \ell_t \leq \bar{h} - h_t, n_t \in [0, 1], e_t \geq n_t \frac{\zeta k_t^2}{\mathbf{v}_t}, \\
& k_{t+1} \leq x_t^k + (1 - \omega)k_t, \mathbf{v}_{t+1} \leq x_t^v - \psi(x_t^v, \mathbf{v}_t), \\
& k_0, \mathbf{v}_0, \text{ and energy prices given, } t \geq 0.
\end{aligned} \tag{3.5}$$

Here we can see that accumulating energy saving capital is costly in terms of output but reduces energy use.

4 A quantitative assessment of our theory

The calibration of the model is relatively standard, and closely follows the methods discussed in Atkeson and Kehoe (1999) and Díaz, Puch, and Guilló (2004). Parameter choices are updated to a sample of macroeconomic and energy aggregates over the period 1970-2008 for the US economy. The share of capital in value added is $\alpha = 0.4$ and the share of energy expenditures is $s_e \simeq 0.05$. The parameter μ associated to the utilization of installed capital is 1.5.

Figure 1 depicts the more salient features of the aforementioned energy aggregates. We simulate the aggregate model with capacity utilization and embodied technical progress with two shocks. One shock are the innovations to realized energy price (see again Figure 1) shock process according to

$$\log p_{t+1} = (1 - \rho) \log \bar{p} + \rho \log p_t + \phi \epsilon_t + \epsilon_{t+1}, \quad (4.1)$$

The estimates for this process are given by $\rho \simeq 0.9$, and $\phi \simeq 0.3$ over the period 1970-2008. The other shock is an investment-specific technology shock which is identified with the relative price of investment. The relative price corresponds to the ratio of the chain weighted NIPA deflators for durable consumption and private investment over non-durable consumption. Our baseline estimates are based upon the innovations to the realized growth rate of relative price (ν_t) according to

$$\log \nu_{t+1} = (1 - \rho_\nu) \log \bar{\nu} + \rho_\nu \log \nu_t + \eta \epsilon_t + \epsilon_{t+1}, \quad (4.2)$$

The estimates for this process are given by $\rho_\nu \simeq 0.3$, and $\eta \simeq 0.9$ for 1990-08 in $d \log q$ (Figure 2). For newer vintages, for a given size of the energy price shock, aggregate capacity utilization together with an investment-specific technology shock act through the model so as to amplify actual energy price shocks. Figure 3 illustrates on this quantitative result of the vintage capital model of energy use.

[TO BE COMPLETED]

Appendix

A A Putty-Clay model of energy

This is the version of Atkeson and Kehoe (1999) in our decentralized environment.

A.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)), \quad \beta \in (0, 1), \quad \varphi > 0, \quad (\text{A.1})$$

where c_t is consumption and ℓ_t is leisure t . Each household is endowed with \bar{h} units of time and, therefore, works $\bar{h} - \ell_t$ hours every period.

A.2 Technology

Production of the unique final good is carried out at a continuum of autonomous plants which are indexed by its type v . The type is given by the efficiency in energy use. In each plant output is produced with labor, energy and the unit of capital installed, according to the technology

$$y_t(v) = A (1 + \phi)^t \kappa_t(v)^\alpha h_t(v)^{1-\alpha}, \quad (\text{A.2})$$

with $0 < \alpha < 1$, where $y_t(v)$ is the output of the plant at time t , $\phi \geq 0$ is the growth rate of the disembodied technological knowledge, and $\kappa_t(v)$ is the amount of services provided the the unit of capital installed, whereas $h_t(v)$ is the amount of labor services employed in the plant.

The amount of capital services, $\kappa_t(v)$, depend on the amount of energy used in the plant, $e_t(v)$, and an index $u_t(v) \in \{0, 1\}$, which measures the utilization of the unit of capital, according to the technology,

$$\kappa_t(v) = f(v) \min \left\{ e_t(v), \frac{1}{v} u_t(v) \right\}, \quad (\text{A.3})$$

where $f(v)$ is a strictly increasing function of v , where $f'(v) \geq 0$, and $f''(v) < 0$.

Each period households have the possibility of saving in the form of new units of capital. One unit of output can be transformed in one unit of consumption or in one unit of capital of type $v \in \mathbf{R}_+$. All types of capital are always available. Nevertheless, capital is irreversible; that is, capital of type v_1 cannot be converted in capital of type v_2 . Finally, at the end of the period, once production has taken place, the unit of capital installed has a positive probability of death, $\omega \in [0, 1]$, which is i.i.d. across types and plants. This death probability plays the role of physical depreciation of capital.

A.3 Market arrangements and timing

At the end of period $t - 1$, any prospective plant must install one unit of capital before the energy price is known. After this decision has been made, the energy price is known. Then, they decide the utilization of their installed capital, $u_t(v)$, the amount of energy used, $e_t(v)$, and the number of workers hired, $h_t(v)$.

The energy price p_t is observed. Plants decide the intensity of capital and energy use, as well the number of hours employed, to produce output. The stand-in household consumes and saves. A fraction ω of plants die.

Households are the owners of the capital as well as the plants. There is a market for plants that opens at the end of the period, once profits have been realized. Since there is a one to one correspondence between plants and units of capital, the price of a plant is also equal to the price of the unit of capital installed, which is equal to one in this case. We have already said that capital is irreversible. That is, since capital cannot be scrapped, plants cannot be scrapped either. They, though can be left idle by setting use $u_t(v) = 0$. We further assume that all households start out with the same amount of capital and shares of the plants installed. Additionally, we assume that households trade a one risk free bond which is in zero net supply.

A.4 Household's problem

In order to determine the usage of capital, we need first to solve the household's problem. It can be written as follows:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + \int_{v=0}^{\infty} m_{t+1}(v) dv + b_{t+1} \leq w_t(\bar{h} - \ell_t) + \int_{v=0}^{\infty} [(1 - \omega) + \pi_t(v)] m_t(v) dv + (1 + r_t^b) b_t. \\
& m_{t+1}(v) \geq 0, \text{ for all } v \in \mathbf{R}_+, b_{t+1} \geq \underline{b}, \\
& m_0(v), \text{ for all } v \in \mathbf{R}_+, b_0 \text{ and energy prices given.}
\end{aligned} \tag{A.4}$$

Notice that at the individual level, plants of any type can be traded.

A.5 Production and profits

The operating profits of a plant of type v at time t are

$$\pi_t(v) = y_t(v) - w_t h_t(v) - p_t e_t(v). \tag{A.5}$$

The plant chooses $h_t(v)$, $e_t(v)$, and $u_t(v)$ to maximize profits. Notice that, if the plant is utilized in equilibrium, $u_t(v) = 1$, the amount of energy used is $e_t(v) = \frac{1}{v}$, which implies that capital services satisfy

$$\kappa_t(v) = \frac{f(v)}{v} u_t(v). \tag{A.6}$$

The amount of labor demanded is

$$h_t(v) = \left(\frac{(1-\alpha)A(1+\phi)^t}{w_t} \right)^{\frac{1}{\alpha}} \frac{f(v)}{v} u_t(v). \quad (\text{A.7})$$

Thus, given $u_t(v)$, profits satisfy

$$\pi_t(v) = \alpha \left(\frac{(1-\alpha)^{1-\alpha} A(1+\phi)^t}{w_t^{1-\alpha}} \right)^{\frac{1}{\alpha}} \frac{f(v)}{v} u_t(v) - \frac{1}{v} p_t u_t(v) \geq 0. \quad (\text{A.8})$$

Proposition 3. *Only installed capital of types $v \geq \underline{v}_t$ are utilized in equilibrium, $u_t(v) = 1$, where \underline{v}_t is defined as*

$$\alpha \left(\frac{(1-\alpha)^{1-\alpha} A(1+\phi)^t}{w_t^{1-\alpha}} \right)^{\frac{1}{\alpha}} f(\underline{v}_t) = p_t. \quad (\text{A.9})$$

Notice that for all $v \geq \underline{v}_t$,

$$\frac{h_t(v)}{h_t(\underline{v}_t)} = \frac{y_t(v)}{y_t(\underline{v}_t)} = \frac{f(v)/v}{f(\underline{v}_t)/\underline{v}_t} \quad (\text{A.10})$$

Since $f''(v) < 0$, this implies that $f(v)/v < f(\underline{v}_t)/\underline{v}_t$. That is, production and employment decreases with v , meaning that production is lower in more energy efficient types. Nevertheless, profit grows with v .

A.6 Some properties of equilibrium

In equilibrium it must be the case that the aggregate amount of plants is

$$m_t(v) = k_t(v), \text{ for all } v \in \mathbf{R}_+, \quad (\text{A.11})$$

That is, the amount of plants of type v is equal to the amount of capital of type v . Moreover, notice that, aggregating the stock of capital of type v satisfies

$$k_t(v) \geq (1-\omega) k_t(v), \text{ for all } v \in \mathbf{R}_+, t \geq 0. \quad (\text{A.12})$$

Thus, solving the problem (A.4) is tantamount to solving the following problem

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\ \text{s. t.} \quad & c_t + \int_{v=0}^{\infty} k_{t+1}(v) dv + b_{t+1} \leq w_t(\bar{h} - \ell_t) + \int_{v=0}^{\infty} [(1-\omega) + \pi_t(z)] k_t(v) dv + (1+r_t)b_t. \\ & k_{t+1}(v) \geq (1-\omega) k_t(v), \text{ for all } v \in \mathbf{R}_+, b_{t+1} \geq \underline{b}, \\ & k_0(v), \text{ for all } v \in \mathbf{R}_+, b_0 \text{ and energy prices given.} \end{aligned} \quad (\text{A.13})$$

Proposition 4. *In equilibrium, investment is positive for at most one type of capital \bar{v}_{t+1} , $k_{t+1}(\bar{v}_{t+1}) >$*

$$(1 - \omega) k_t (\bar{v}_{t+1}).$$

Proof. The first order condition with respect to $k_{t+1}(v)$ is

$$-\varphi_t + E_t[\varphi_{t+1}((1 - \omega) + \pi_{t+1}(v))] \leq 0. \quad (\text{A.14})$$

Since $\pi_{t+1}(v)$ monotonically increases with v , there must exist a unique value $\bar{v}_t > 0$ for which the first order condition holds with strict equality. \square

A.7 Aggregation

Proposition 4 implies that the number of types that receive positive investment is countable. Without loss of generality, we are going to assume that at time zero only one type of capital was installed.

Assumption 2. $k_0(v) = 0$ for all $v \neq v_0$, where $v_0 > 0$, and $k_0(v_0) > 0$.

Thus, we can define as v_τ the type of technology that received positive investment at time τ . Notice that aggregate production at time t is

$$y_t = \sum_{\tau=0}^t k_t(v_\tau) y_t(v_\tau) = \sum_{\tau=0}^t k_t(v_\tau) \frac{f(v_\tau)/v_\tau}{f(v_0)/v_0} y_t(v_0). \quad (\text{A.15})$$

Likewise, labor satisfies

$$h_t = \sum_{\tau=0}^t k_t(v_\tau) h_t(v_\tau) = \sum_{\tau=0}^t k_t(v_\tau) \frac{f(v_\tau)/v_\tau}{f(v_0)/v_0} h_t(v_0). \quad (\text{A.16})$$

Aggregate capital is the depreciated value of the sum of all past investments,

$$k_t = \sum_{\tau=0}^t k_t(v_\tau) = \sum_{\tau=0}^t (1 - \omega)^{t-\tau} x_{\tau-1}, \quad (\text{A.17})$$

whereas aggregate capital services are

$$\kappa_t = \sum_{\tau=0}^t k_t(v_\tau) \frac{f(v_\tau)}{v_\tau} = \sum_{\tau=0}^t (1 - \omega)^{t-\tau} x_{\tau-1} \frac{f(v_\tau)}{v_\tau}. \quad (\text{A.18})$$

It is easy to check that aggregate output can be written as

$$y_t = A(1 + \phi)^t \kappa_t^\alpha h_t^{1-\alpha}. \quad (\text{A.19})$$

Now we are going to find the condition under which all types of installed capital are used in equilibrium, as we have guessed,

Conjecture 1. $\underline{v}_t = \underline{v} \equiv \min\{v_0, \dots, v_t\}$, for all t .

Since profits strictly increase with v , $v \in \mathbf{R}_+$, this conjecture amounts to guessing that for \underline{v}

$$\alpha \left(\frac{(1-\alpha)^{1-\alpha} A(1+\phi)^t}{w_t^{1-\alpha}} \right)^{\frac{1}{\alpha}} f(\underline{v}) > p_t, \text{ for all } t \geq \tau, \quad (\text{A.20})$$

which can be written as

$$\alpha A(1+\phi)^t \left(\frac{\kappa_t}{h_t} \right)^{\alpha-1} f(\underline{v}) > p_t, \text{ for all } t. \quad (\text{A.21})$$

If Conjecture 1 is satisfied, then, the evolution of capital services is

$$\kappa_{t+1} = (1-\omega)\kappa_t + \frac{f(v_{t+1})}{v_{t+1}} x_t, \quad (\text{A.22})$$

where x_t is investment at time t in units of consumption good. Likewise, the amount of energy used in equilibrium satisfies

$$e_{t+1} = \sum_{\tau=0}^t \frac{1}{v_\tau} k_t(v_\tau) u_t(v_\tau)^\mu = (1-\omega)e_t + \frac{1}{v_{t+1}} x_t. \quad (\text{A.23})$$

Since this economy is efficient, the equilibrium allocation must solve the following quasi-planner's problem:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\ \text{s. t.} \quad & c_t + \int_{v=0}^{\infty} k_{t+1}(v) dv \leq (1-\omega) \int_{v=0}^{\infty} k_t(v) dv + \\ & A(1+\phi)^t \int_{v=0}^{\infty} \left(\left(\frac{f(v)}{v} \right) k_t(v) u_t(v) \right)^\alpha h_t(v)^{1-\alpha} dv - p_t \int_{v=0}^{\infty} \frac{1}{v} k_t(v) u_t(v)^\mu dv, \\ & c_t \geq 0, h_t(v) \geq 0, u_t(v) \in \{0, 1\}, \text{ for all } v \in \mathbf{R}_+, \\ & \ell_t \leq \bar{h} - \int_{v=0}^{\infty} h_t(v) dv, \\ & k_{t+1}(v) \geq (1-\omega) k_t(v), \text{ for all } v \in \mathbf{R}_+, \\ & k_0(v) = 0 \text{ for all } v \neq v_0, k_0(v_0) > 0, \\ & b_0 = 0, \text{ and energy prices given, } t \geq 0. \end{aligned} \quad (\text{A.24})$$

Now we can state the following planner's problem:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^{t-} (\log(c_t) + \varphi \log(\ell_t)) \\ \text{s. t.} \quad & c_t + x_t \leq A(1+\phi)^t \kappa_t^\alpha h_t^{1-\alpha} - p_t e_t, \\ & c_t \geq 0, \ell_t \leq \bar{h} - h_t, \kappa_{t+1} \leq (1-\omega)\kappa_t + \frac{f(v_{t+1})}{v_{t+1}} x_t, e_{t+1} \geq (1-\omega)e_t + \frac{1}{v_{t+1}} x_t, \\ & x_t \geq 0, v_{t+1} \in \mathbf{R}_+, \\ & \kappa_0, \text{ and energy prices given, } t \geq 0. \end{aligned} \quad (\text{A.25})$$

Proposition 5. *Suppose that Conjecture 1 is satisfied in equilibrium. Then, both planners, whose problems are shown in (A.24) and (A.25), choose the same sequence $\{c_t, \ell_t, \kappa_t, e_t\}_{t=0}^{\infty}$.*

Proof. The proof of this proposition follows from properties of the utility function and the technology. It is easy to check that $v_{t+1} \geq \underline{v} \equiv \min \{v_0, \dots, v_t\}$ for all t . \square

Notice that Conjecture 1 implies that, in equilibrium, the interest rate must be bounded below. In an economy without labor choice, the condition (A.21) poses an upper bound on the energy price, or, likewise, a lower bound on \underline{v} .

A.8 The quality-adjusted relative price of capital

In this economy, as in our benchmark economy, the relative price of capital in units of the final good is one. Let us see the quality-adjusted price of capital,

$$q_t^\kappa = \sum_{\tau=0}^t \frac{1}{\kappa_t(v_\tau)} \frac{k_t(v_\tau)}{k_t}, \quad (\text{A.26})$$

which collapses to

$$q_t^\kappa = \sum_{\tau=0}^t \frac{v_\tau}{f(v_\tau)} \frac{(1-\omega)^{t-\tau} x_{\tau-1}}{\sum_{i=0}^t (1-\omega)^{t-i} x_{i-1}}. \quad (\text{A.27})$$

The quality-adjusted relative price of capital, though, increases with the type v . Thus, the higher the average efficiency of capital in the economy, the higher the quality-adjusted relative price of capital. This implication is at odds with the estimates of Gordon (1990), or Cummins and Violante (2002), for instance.

B A Putty-Putty model of energy with costly capital reallocation

This is the version of Díaz, Puch, and Guilló (2004) in our decentralized environment.

B.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)), \quad \beta \in (0, 1), \quad \varphi > 0, \quad (\text{B.1})$$

where c_t is consumption and ℓ_t is leisure t . Each household is endowed with \bar{h} units of time and, therefore, works $\bar{h} - \ell_t$ hours every period.

B.2 Technology

Production of the unique final good is carried out at a continuum of autonomous plants which are indexed by the amount of energy-saving capital used, v_t , as well as an idiosyncratic productivity index, s , which is uniformly distributed in $[-\sigma, \sigma]$. In each plant output is produced with labor, energy and the unit of capital installed, according to the technology

$$y_t(v_t, s) = (\theta + s)^\alpha (1 + \xi)^t \kappa_t(z_t, s)^\alpha h_t(v_t, s)^{1-\alpha}, \quad (\text{B.2})$$

with $0 < \alpha < 1$, and $\theta > 0$, where $y_t(v_t, s)$ is the output of the plan at time t , $\xi \geq 0$ is the growth rate of the disembodied technological knowledge, and $\kappa_t(v_t, s)$ is the amount of services provided the the unit of capital installed, whereas $h_t(v_t, s)$ is the amount of labor services employed in the plant.

The amount of capital services, $\kappa_t(v_t, s)$, depend on the amount of energy used in the plant, $e_t(v_t, s)$, and an index $u_t(v_t, s) \in \{0, 1\}$, which measures the utilization of the unit of capital, according to the technology,

$$\kappa_t(v_t, s) = v_t u_t(v_t, s)^{1-\mu} \min \left\{ e_t(v_t, s), \frac{\zeta}{v_t} u_t(v_t, s)^\mu \right\}, \quad (\text{B.3})$$

where $\mu > 1$.

Each period households have the possibility of saving in the form of new units of capital. Additionally, households have a technology that transforms final good into energy-saving capital, \mathbf{v}_t . This capital is rented to plants in period $t - 1$ to be used in period t . Plants can be scrapped at no cost. Finally, at the end of the period, once production has taken place, the unit of capital installed has a positive probability of death, $\omega \in [0, 1]$, which is i.i.d. across types and plants. This death probability plays the role of physical depreciation of capital.

B.3 Market arrangements and timing

At the end of period $t - 1$, any prospective plant must install one unit of capital before the energy price is known. After this decision has been made, the energy price is known. Then, they decide the utilization of their installed capital, $u_t(v_t, s)$, the amount of energy used, $e_t(v_t, s)$, and the number of workers hired, $h_t(v_t, s)$.

The energy price p_t is observed. Plants decide the intensity of capital and energy use, as well the number of hours employed, to produce output. The stand-in household consumes and saves. A fraction ω of plants die.

Households are the owners of the capital as well as the plants. There is a market for plants that opens at the end of the period, once profits have been realized. We further assume that all households start out with the same amount of capital and shares of the plants installed. Additionally, we assume that households trade a one risk free bond which is in zero net supply.

B.4 Household's problem

Notice that since plants can be scrapped at no cost, and the amount of energy-saving capital can be changed every period, all plants are ex-ante identical at all periods. Moreover, the total number of plants is always equal to the amount of physical capital, k_t . Thus, the problem of a household is

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + (1 + r_t^v v_{t+1}) k_{t+1} + \mathbf{v}_{t+1} + \psi(\mathbf{v}_{t+1}, \mathbf{v}_t) + b_{t+1} \leq \\
& w_t(\bar{h} - \ell_t) + \left((1 - \omega) + \int_{\underline{s}}^{\bar{s}} \pi_t(v_t, s) ds \right) k_t + (1 - \omega) \mathbf{v}_t + r_t^v \mathbf{v}_{t+1} + (1 + r_t^b) b_t. \\
& k_{t+1} \geq 0, \mathbf{v}_{t+1} \geq 0, v_{t+1} \geq 0, b_{t+1} \geq \underline{b}, \\
& k_0(v), b_0 \text{ and energy prices given.}
\end{aligned} \tag{B.4}$$

B.5 Properties of equilibrium

Since energy saving capital has to be installed before the idiosyncratic productivity shock is realized and the shocks is i.i.d., then the amount of energy saving capital is the same in all plants, and we can drop the v_{t+1} from the index of the plant. In any given plant that is used, $u_t(s) = 1$, the amount of energy used satisfies $e_t(s) = \frac{\zeta}{v_t}$. Thus, capital services are just $\kappa_t(s) = \zeta$. The level of output at the plant level is

$$y_t(s) = (\theta + s)^\alpha (1 + \xi)^t \zeta h_t(s)^{1-\alpha}. \tag{B.5}$$

The amount of labor hired in a plant where $u_t(s) = 1$ satisfies

$$\frac{h_t(s)}{h_t(\sigma)} = \frac{\theta + s}{\theta + \sigma}. \tag{B.6}$$

The operating profit in the plant must satisfy

$$\pi_t(s) = \alpha y_t(s) - p_t \frac{\zeta}{v_t} \geq 0. \tag{B.7}$$

Thus, there exists $\underline{s}_t \geq -\sigma$ such that for all $s \geq \underline{s}_t$ the plant is operated. We are going to examine equilibria where $\underline{s}_t \in (-\sigma, \sigma)$. Thus, we can define as n_t the fraction of plants that are operated,

$$n_t = \int_{\underline{s}_t}^{\sigma} \frac{1}{2\sigma} ds = \frac{\sigma - \underline{s}_t}{2\sigma}. \tag{B.8}$$

Output is proportional to idiosyncratic productivity,

$$\frac{y_t(s)}{y_t(\sigma)} = \frac{\theta + s}{\theta + \sigma}. \tag{B.9}$$

Aggregate output is

$$y_t = k_t y_t(\sigma) \int_{\underline{s}_t}^{\sigma} \frac{\theta + s}{\theta + \sigma} \frac{1}{2\sigma} ds. \tag{B.10}$$

Likewise, aggregate labor satisfies

$$h_t = k_t h_t(\sigma) \int_{\xi_t}^{\sigma} \frac{\theta + s}{\theta + \sigma} \frac{1}{2\sigma} ds. \quad (\text{B.11})$$

Then, aggregate output is equal to

$$y_t = \left[\int_{\xi_t}^{\sigma} \frac{\theta + s}{2\sigma} ds \right]^{\alpha} (1 + \xi)^t \zeta^{\alpha} k_t^{\alpha} h_t^{1-\alpha} = n_t^{\alpha} (\theta + \sigma (1 - n_t))^{\alpha} (1 + \xi)^t \zeta^{\alpha} k_t^{\alpha} h_t^{1-\alpha}. \quad (\text{B.12})$$

The aggregate energy used is

$$e_t = n_t \frac{\zeta k_t^2}{\mathbf{v}_t}, \text{ where } \mathbf{v}_t = k_t v_t. \quad (\text{B.13})$$

Now we can turn to the household's problem. Notice that the rental price of energy saving capital satisfies

$$r_t^v = 1 + \psi_1(\mathbf{v}_{t+1}, \mathbf{v}_t) + E_t \left[\frac{\varsigma_{t+1}}{\varsigma_t} \psi_2(\mathbf{v}_{t+2}, \mathbf{v}_{t+1}) \right], \quad (\text{B.14})$$

where ς_t is the Lagrange multiplier associated to the period t budget constraint. Thus, the rental price of energy saving capital depends on the adjustment cost $\psi(\cdot, \cdot)$. Moreover, since setting a plant requires jointly a unit of operating capital and v_t units of energy saving capital we find that the price of one unit of capital services is

$$q_t^k = 1 + r_t^v v_{t+1}. \quad (\text{B.15})$$

Since the economy is efficient, the competitive equilibrium allocation is the solution to the following quasi-social planner problem:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^{t-} (\log(c_t) + \varphi \log(\ell_t)) \\ \text{s. t.} \quad & c_t + x_t^k + x_t^v \leq n_t^{\alpha} (\theta + \sigma (1 - n_t))^{\alpha} (1 + \xi)^t \zeta^{\alpha} k_t^{\alpha} h_t^{1-\alpha} - p_t e_t, \\ & c_t \geq 0, \ell_t \leq \bar{h} - h_t, n_t \in [0, 1], e_t \geq n_t \frac{\zeta k_t^2}{\mathbf{v}_t}, \\ & k_{t+1} \leq x_t^k + (1 - \omega)k_t, \mathbf{v}_{t+1} \leq x_t^v - \psi(x_t^v, \mathbf{v}_t), \\ & k_0, \mathbf{v}_0, \text{ and energy prices given, } t \geq 0. \end{aligned} \quad (\text{B.16})$$

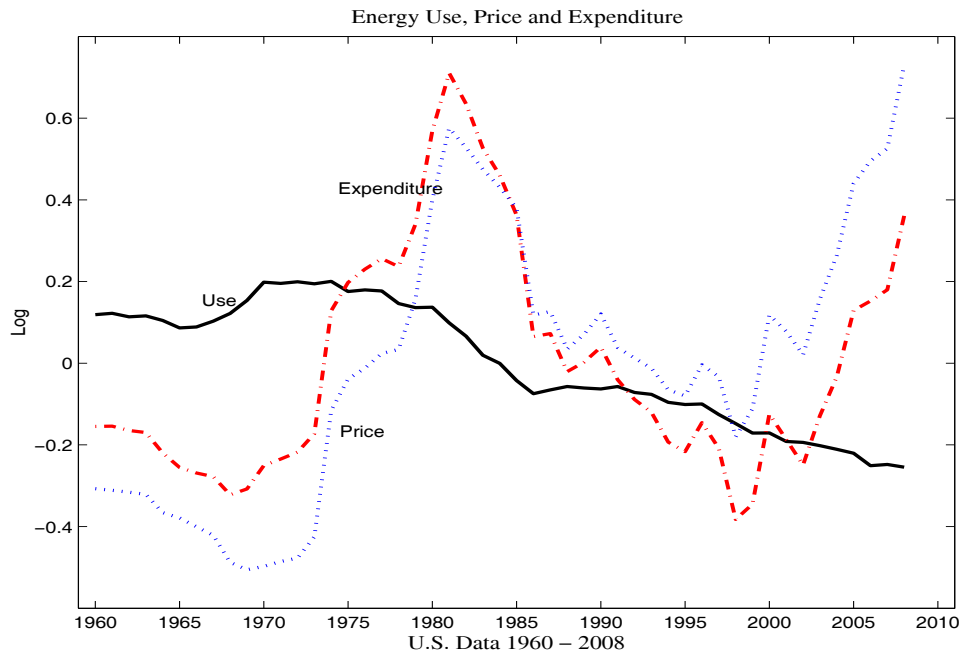


Figure 1: Energy price, energy use and energy expenditure.

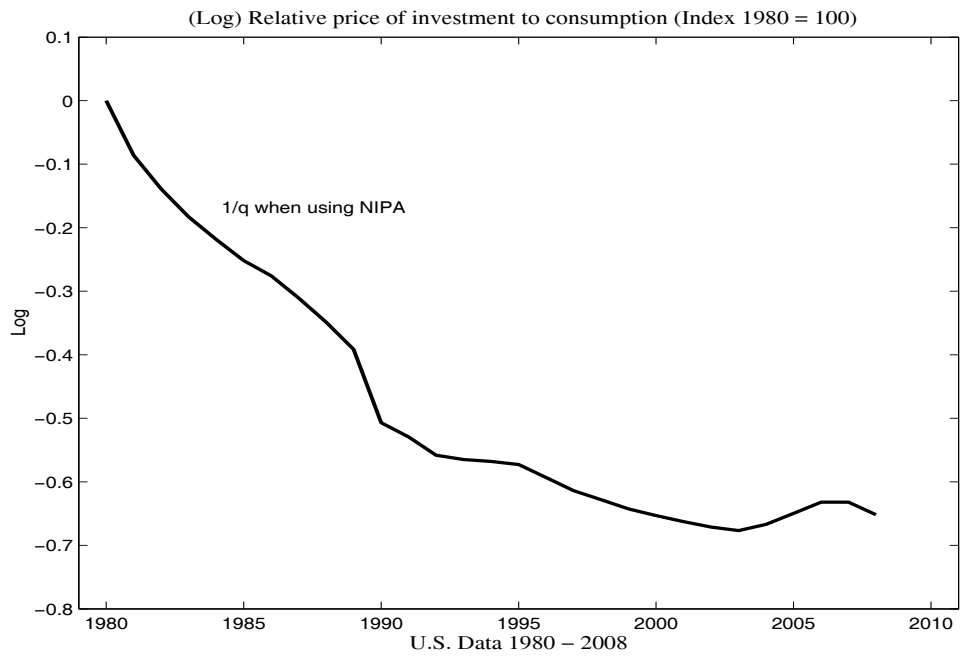


Figure 2: The relative price of investment.

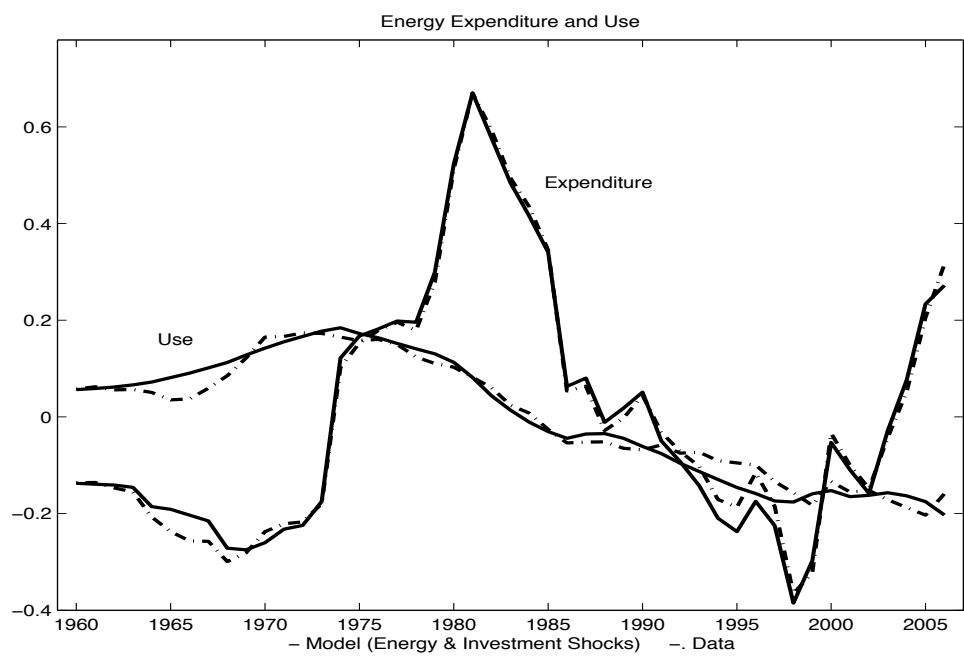


Figure 3: Accounting for energy use and expenditure (vintage model) .

References

- Atkeson, A. and P. J. Kehoe (1999). Models of energy use: Putty-putty versus putty-clay. *American Economic Review* 89(4), 1028–43.
- Benhabib, J. and A. Rustichini (1991). Vintage capital, investment, and growth. *Journal of Economic Theory* 55(2), 323–339.
- Berndt, E. R. and D. O. Wood (1975). Technology, prices and the derived demand for energy. *Review of Economics and Statistics* 57(3), 259–68.
- Boucekkine, R., F. del Río, and B. Martínez (2009). Technological progress, obsolescence, and depreciation. *Oxford Economic Papers* 61(3), 440–466.
- Cummins, J. G. and G. L. Violante (2002). Investment-specific technical change in the us (1947–2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics* 5(2), 243–284.
- Díaz, A., L. A. Puch, and M. D. Guilló (2004). Costly capital reallocation and energy use. *Review of Economic Dynamics* 7(2), 494.
- Edelstein, P. and L. Kilian (2007). The response of business fixed investment to changes in energy prices: A test of some hypotheses about the transmission of energy price shocks. *The B.E. Journal of Macroeconomics* 7(1).
- Gordon, R. J. (1990). *The Measurement of Durable Goods Prices*. Chicago: University of Chicago Press: National Bureau of Economic Research Monograph Series.
- Griffin, J. M. and P. R. Gregory (1976). An intercountry translog model of energy substitution responses. *American Economic Review* 66(5), 845–57.
- Kilian, L. (2008). The economic effects of energy price shocks. *Journal of Economic Literature* 46(4), 871–909.
- Metcalf, G. E. (2008). An empirical analysis of energy intensity and its determinants at the state level. *The Energy Journal* 29(3), 1–26.
- Pindyck, R. S. (1991). Interfuel substitution and the industrial demand for energy: An international comparison. *Review of Economics and Statistics* 29(2), 173–89.
- Pindyck, R. S. and J. J. Rotemberg (1983). Dynamic factor demands and the effects of energy price shocks. *American Economic Review* 73(5), 1066–79.
- Steinbuks, J. and K. Neuhoff (2010). Operational and investment response to energy prices in the oecd manufacturing sector. Cambridge Working Papers in Economics 1015, Faculty of Economics, University of Cambridge.