R. E. Mayle[1]
Associate Professor.
Department of Mechanical Engineering,
Rensselaer Polytechnic Institute, Troy, NY 12181
Mem. ASME

M. F. Blair
Research Engineer,
United Technologies Research Center,
East Hartford, Conn.
Mem. ASME

F. C. Kopper
Research Engineer,
Pratt and Whitney Aircraft
East Hartford, Conn.
Mem. ASME

Turbulent Boundary Layer Heat Transfer on Curved Surfaces

Heat transfer measurements for a turbulent boundary layer on a convex and concave, constant-temperature surface are presented. The heat transferred on the convex surface was found to be less than that for a flat surface, while the heat transferred to the boundary layer on the concave surface was greater. It was also found that the heat transferred on the convex surface could be determined by using an existing two-dimensional finite difference boundary layer program modified to take into account the effect of streamline curvature on the turbulent shear stress and heat flux, but that the heat transferred on the concave surface could not be calculated. The latter result is attributed to the transition from a two-dimensional flow to one which contained streamwise, Taylor-Görtler type vortices.

Introduction

The prediction of turbine blade heat loads is critical to a successful gas turbine design. The prediction process, however, is hampered by insufficient information on a number of "special effects" which significantly influence the viscous and thermal boundary layer development along the blades. The term special effects is used in the same context as Bradshaw's [1] and refers to the effects on a turbulent boundary layer caused by three-dimensionality, streamline curvature, Reynolds number and free-stream turbulence and unsteadiness. For turbine blades these effects commonly occur simultaneously and in regions of large (relaminarizing) streamwise pressure gradients, which generally give the designer an uneasy feeling about the accuracy of his heat load prediction and, hence, his blade life estimate. In an effort to understand one of these special effects, so that eventually a designer may routinely take it into account, the present work on the influence of curvature on heat transfer in a turbulent boundary layer was undertaken.

The recent upsurge in experiments on curved surfaces [2–6] has provided much information on the effect of streamline curvature on turbulent boundary layer growth, Reynolds stresses and wall shear stress. In general, the results show that the Reynolds stresses and wall shear stress are reduced on the convex surface and increased on the concave when compared to those for flow on a flat surface. In one case [2] the convex curvature was sufficiently strong to reduce the turbulent shear stress to zero. And in each case, the effect of streamline curvature on the Reynolds stresses was at least an order of magnitude greater than that expressed explicitly in the turbulent stress transport equations. The measurements also indicate that the flow on the concave surface may not be two dimensional, but exhibit steady (in the time-averaged sense) Taylor-Gortler type vortices within the boundary layer. Tani [7] was the first to study this phenomenon and since then additional information has been provided by Meroney [3] and So and Mellor [4]. For an extensive review of the work done previous to 1973 on turbulent boundary layers with streamline curvature see Bradshaw [8].

There is no reason to expect that the turbulent heat flux in curved flows should behave qualitatively different than the turbulent shear stress. Thomann [9] inferred this from his measurements of surface heat flux which showed a decrease on the convex surface and an increase on the concave when compared to the heat flux from a flat plate. This trend was also shown to be unaffected by the heat flux direction and streamwise pressure gradients. However, his experiments were conducted with a free-stream Mach number of 2.5 and, as pointed out by Bradshaw (either [8] or [10]), curvature effects can be much larger at high supersonic velocities. The present work was performed at a low speed so that the curvature effect unaugmented by compressibility effects could be evaluated.

If the boundary layer thickness is very much less than the radius of curvature, i.e., $\delta/R \ll 1$, as in the turbine blade application, the curvature terms in the equations of motion and energy can be neglected compared to the usual convective terms and the only way curvature can affect the mean motion and thermal field is through the turbulent shear stress and heat flux. An examination of the Reynolds stress transport equations for $\delta/R \ll 1$ reveal that only the production of $(\overline{v'^2})$ and $(-\overline{uv})$ are explicitly affected by curvature. If the free-stream and surface temperature difference is small compared to the free-stream temperature and again $\delta/R \ll 1$, the transport equation for the turbulent heat flux may be written (in the usual notation) as

$$D(\overline{v'T'})/Dt = -\overline{v'^2}\frac{\partial \overline{T}}{\partial y} - \frac{1}{\rho}\overline{T'\frac{\partial p'}{\partial y}} - \frac{\partial \overline{v'^2T'}}{\partial y} + \alpha\overline{v'\frac{\partial^2 T'}{\partial y^2}} + \nu\overline{T'\frac{\partial^2 v'}{\partial y^2}}$$

which is identical to that without curvature. Thus, the advection, production, diffusion and dissipation of turbulent heat flux are not explicitly dependent on mean-streamline curvature and, in particular, the production depends only on the effect curvature has on the Reynolds stress $\overline{v'^2}$ and mean temperature gradient.

This set of circumstances has made the calculation of curvature effects for turbulent boundary layer flow difficult and at best correlative in nature. The favored method at present is that proposed by Bradshaw [10] where a curvature correction is applied to either the dissipation or mixing length, and hence the local turbulent shear stress, in a Spalding-Patankar type calculation procedure. The correction generally has the form

$$\ell/\ell_0 = 1 - \beta\,\mathrm{Ri} = 1 - \beta\left(2\frac{u}{R}\right)\Big/\left(\frac{\partial u}{\partial y}\right)$$

where ℓ_0 is the length scale without curvature, Ri is the Richardson number, u is the mean velocity, R is the surface radius taken positive for a convex surface and negative for a concave, and β is a correlation constant. The Richardson number has been defined for incompressible flow and represents the ratio of centrifugal to inertia forces. Values for β of 7 to 10 for a convex surface and 4.5 for a concave have been suggested by Bradshaw [10] while Eide and Johnston [11] propose a value of 6 for both surfaces. In order to calculate the mean temperature distribution, the Prandtl number is assumed unaffected by curvature. This assumption, which was shown to be good at least in the outer region of the boundary layer by Mayle, et al. [12], and the length correction above are the basic assumptions used in the boundary layer calculations presented herein for comparison with the measurements.

Description of Experiment

The experiment was conducted in a closed-circuit wind tunnel with the curved surface test section forming one of the bends in the circuit. Air from a centrifugal blower passed through a honeycomb section,

[1] Consultant, Pratt and Whitney Aircraft, East Hartford, Conn.

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screens and a nozzle into a 51 cm by 20 cm by 74 cm long approach duct which directly preceded the curved surface test section. The flow from the test section was then diffused and entered a return duct which contained a heat exchanger that was used to maintain a steady thermal testing condition. Make-up air to compensate for air leakage and bleeds was drawn in upstream of the blower and mixed with the return flow.

The nozzle and approach duct boundary layers were bled off before the flow entered the test section. The boundary layers on the upper and lower walls were bled through adjustable scoop-like gaps as shown in Fig. 1. The sidewall boundary layers were bled through a series of adjustable flush slots in the approach duct sidewalls. This bleeding was necessary to reduce the secondary flow in the curved test section which normally migrates along the sidewalls from the concave to convex surface and eventually onto the convex surface. The effect of this flow (converging flow on the convex surface and diverging flow on the concave) is in the same direction as that caused by streamline curvature and therefore should be reduced to a small percentage of the curvature effect. In the present experiment, yaw measurements at the most downstream position and 10 cm on each side of the test section centerline showed that the secondary flow was mainly confined to the boundary layers and that the maximum convergence on the convex surface and divergence on the concave was about 2 deg. This secondary flow produces an error in Stanton number of about 2 percent which, at the same streamwise position, amounts to about 7 percent of the curvature effect.

The test section, shown in Fig. 1, was 51 cm wide and 12 cm high at its entrance. The curved test surfaces formed 90 deg portions of a cylindrical surface 51 cm wide with a radius of 61 cm attached to a 20.3 cm long flat leading edge section. Only one test surface at a time was installed in the test section. The opposite wall was made of a flexible steel sheet reinforced by bars across its width in order to maintain a rectangular cross-section passage at any streamwise position. The bars were spaced at 14 cm intervals and were connected to rigid tunnel structural members by means of adjustable turnbuckles. This wall was adjusted to provide a negligible streamwise pressure gradient along the test surface.

The curved test walls were constructed by casting a 2.5 cm thick layer of rigid urethane foam, having a thermal conductivity two to three times that of air, in aluminum molds which had been formed to produce the 61 cm radius. A flat 2.5 cm-thick foam section was then attached to form the leading edge. Heater elements consisting of a thin etched foil with a Kapton substrate (0.20 mm thick) were attached to the foam walls. Copper plates (0.81 mm thick and rolled to a 61 cm radius for the curved section) were attached to the heater elements to provide the surface in direct contact with the airstream. These plates measured 5.0 cm in the streamwise direction on the curved portion, 2.5 cm on the flat, and were about 17 cm long. Three rows of plates across the width of the foam wall covered the surface. The heat transfer measurements were taken along the center row; the other rows served as guard heaters. The plates were thermally isolated from each other using a low thermal conductivity filler and the heaters below each plate electrically isolated so that the electrical power input to each heater-plate unit could be related directly to the local convective heat transfer. Chromel-Alumel thermocouples were soldered into small holes drilled in the copper plates from the back to measure the surface temperature. During the test the power to each heater was adjusted to obtain a uniform value of this temperature over the entire

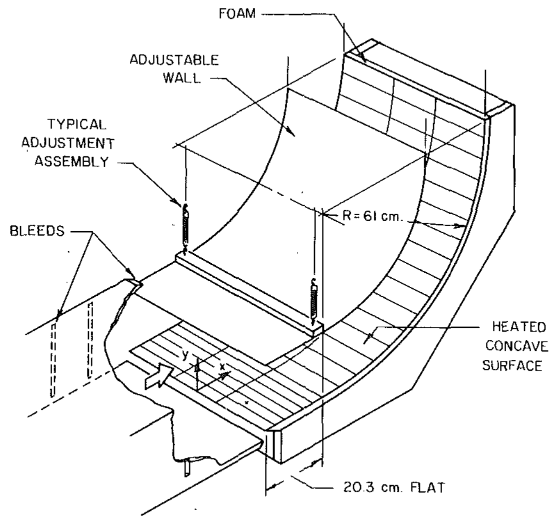

Fig. 1 Curved surface test section

surface. The entire test surface was painted with several thin coats of varnish to produce a hydraulically smooth surface and then sprayed with a carbon black paint to provide a surface with an emissivity of nearly unity. A 1.5 mm thick Micarta sheet was attached to the backside of the foam wall for strength and an additional 10 cm of fiberglass building insulation was fitted against the Micarta sheet.

The convective heat flux from the test surface was obtained from the measured power dissipated in the heaters after correcting for heat losses. The corrections included those for surface radiation which took into account the curved wall view factor and for back losses attributed to conduction through the foam, heater leads and thermocouple leads. These corrections amounted to about 10 and 3 percent of the measured power, respectively.

The velocity measurements were obtained with a DISA 55D01 constant-temperature anemometer using a 0.05 mm dia hot-film boundary layer probe. Temperature measurements were taken almost simultaneously at a given point by switching the unit from a constant-temperature operating mode to a resistance-thermometer mode. With this arrangement the velocities and temperatures could be determined to within 0.5 m/s and 0.5°C, respectively.

At the test section entrance, the free-stream velocity and temperature were found to be uniform and were nominally maintained at 21 m/s and 18°C, respectively. On the flat portion of the test surface, 7.5 cm downstream from the leading edge, a 0.64 mm dia trip wire was attached to provide a turbulent boundary layer over the curved surface. For an unheated surface (see [12])[2] the momentum thickness Reynolds number and shape factor at $x/R = 0.29$, which was slightly before the curved portion, were found to be 835 and 1.48, respectively, and a comparison with Coles' correlations [13] revealed it to be in equilibrium.

Results and Discussion

In contrast to the well-behaved, two-dimensional flow on the convex surface, the flow on the concave surface was found to be quite complex. Spanwise traverses of a 1.6 mm dia pitot tube near the surface

[2] The trip wire diameter was 0.64 mm for that experiment also, and not the value reported.

Nomenclature

c_p = specific heat at constant pressure
d = pitot tube diameter
G_t = turbulent Görtler number
ℓ = turbulent length scale
M = Mach number
p_s = static pressure
p_t = total pressure
Pr = Prandtl number
q = surface convective heat flux
R = radius of curvature of surface
Re_x = Reynolds number based on distance from leading edge
Ri = incompressible Richardson number
St = Stanton number
T = absolute temperature
u, v = velocity components in x and y direction, respectively
x, y, z = spatial coordinate system with x and y coordinates parallel and normal to the surface, respectively
γ = ratio of specific heats
δ = boundary layer thickness
δ^{**} = momentum thickness
λ = wavelength
ν = molecular kinematic viscosity
ρ = density

Subscripts

∞ = free-stream state
w = evaluated at wall

revealed lateral variations in the total pressure within the boundary layer which can be attributed to Taylor-Gortler type vortices. The results of the measurements for the tube touching the surface are shown in Fig. 2 where the ordinate, a dimensionless dynamic pressure, is proportional to the wall shear stress [14]. The momentum thickness at the start of curvature, δ_0^{**}, (measured without surface heating) and the radius, R, has been used to scale the lateral and streamwise distance, respectively, where $\delta_0^{**}/R = 1.05(10)^{-3}$. With the streamwise distance, x, measured from the leading edge of the test surface, curvature started at $x/R = 0.33$. Two items are immediately apparent from Fig. 2. The first is that the three-dimensional disturbances, which are initially barely detectable, grow in the streamwise direction and second they become more regular downstream. These two observations indicate that a regular system of vortices was evolving within the boundary layer. The same measurements taken without heating revealed an identical situation which in turn indicates that the inertia forces dominated the buoyancy forces in the process. As will be seen shortly, the growth of the vortices is particularly important since it affects the heat transfer dramatically.

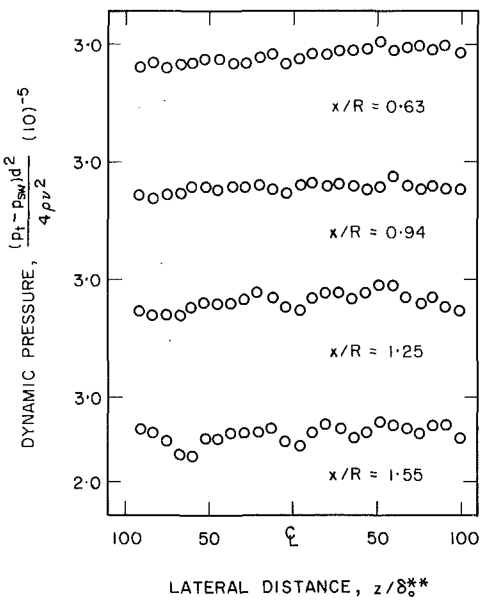

Fig. 2 Lateral dynamic pressure variation near concave surface

The likelihood of vortices being formed within a turbulent boundary layer on a concave surface can be determined from Tani's work. If the wavelength, λ, of the disturbance at $x/R = 1.55$ and the initial momentum thickness are used, a turbulent Gortler number, $G_t = 43\,(\delta_0^{**}/R)^{1/2}$, and a dimensionless wave number, $2\pi\,\delta_0^{**}/\lambda$, of 1.39 and 0.11 are found, respectively. This result, when plotted on Tani's graph for stability of a turbulent boundary layer on a concave surface lies well within the unstable region. Hence, a disturbance should amplify and a system of streamwise vortices should form as found.

Boundary layer velocity and temperature measurements were made at $x/R = 0.375$ which was slightly downstream of the point where curvature began. At this location the flow on the concave surface was still two-dimensional. The resulting profiles for both the convex and concave surfaces are shown in Figs. 3 and 4, and exhibit only slight differences. The momentum and enthalpy Reynolds numbers are 907 and 797, respectively, for the convex surface and 981 and 979 for the concave. The shape factors for both surfaces are identical and equal to 1.52. The enthalpy Reynolds numbers obtained by integrating the measured surface heat flux up to the profile measurement location are 917 for the convex surface and 937 for the concave. The discrepancies between these values and the profile values are attributed to the inaccuracies in the profile temperature measurements. For comparison, the momentum and enthalpy Reynolds numbers for a turbulent boundary layer on a flat surface developing over the same length are about 913 and 1022, respectively.

The curved surface heat transfer results are presented in Fig. 5. The Stanton number, St, defined as

$$\mathrm{St} = \frac{q}{\rho u_\infty c_p (T_w - T_\infty)}$$

with q the surface convective heat flux, is plotted against the Reynolds number based on the inlet mainstream conditions and distance from the leading edge. Although there is a fair amount of scatter in the data, particularly for the concave surface, the effect of surface curvature is unmistakable. Again, as found by Thomann, the heat transfer is much greater on the concave surface than on the convex; in this case, by almost a factor of two at the most downstream position. The correlation of Reynolds, et al. [15] for heat transfer in a turbulent boundary layer on a flat plate, i.e.

$$\mathrm{St}\left(\frac{T_w}{T_\infty}\right)^{0.4} = 0.0296\,\mathrm{Pr}^{-0.4}\,\mathrm{Re}_x{}^{-0.2}$$

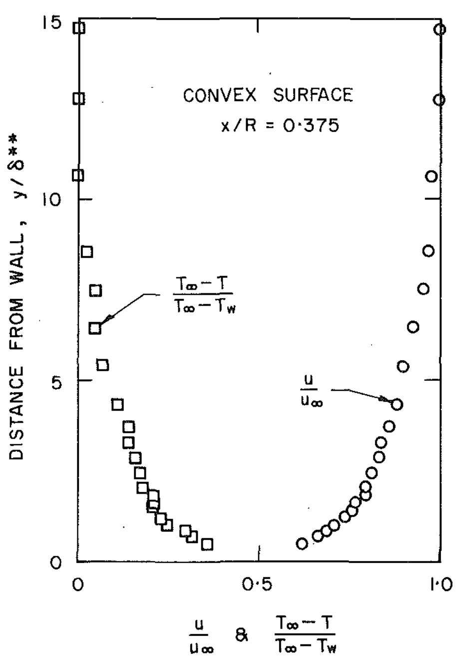

Fig. 3 Initial velocity and temperature profiles on convex surface

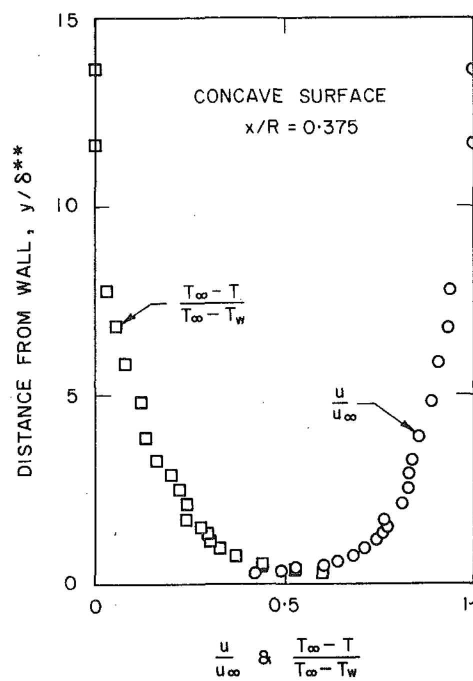

Fig. 4 Initial velocity and temperature profiles on concave surface

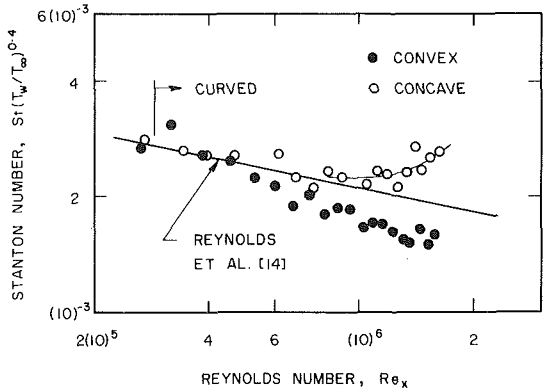

Fig. 5 Stanton number measurements on curved surfaces and comparison with flat plate correlation

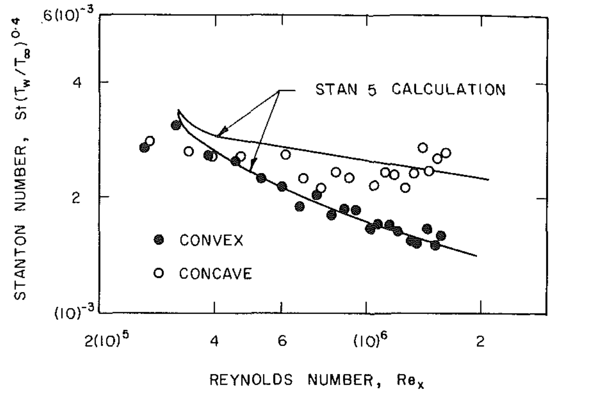

Fig. 6 Comparison of measured Stanton numbers with boundary layer calculations

is also shown in Fig. 5. A comparison of the data with this indicates an increasing departure as the viscous and thermal boundary layers develop along the curved surfaces. At the most downstream position, which is about 120 boundary layer thicknesses from the start of curvature, the heat transfer on the convex surface is about 20 percent less than the correlation while on the concave surface it is about 33 percent greater. The deviation of the concave surface data is particularly interesting since it occurs mostly on the downstream half of the surface and is still changing rapidly (compared to the data on the convex surface) at the most downstream position. This does not appear to be caused by curvature modifying the turbulent heat flux in a two-dimensional sense, as on the convex surface, and is most likely a result of the observed streamwise growth of vortices.

A comparison of the present results to Thomann's at the same number of boundary layer thicknesses downstream from the start of curvature reveals that curvature had much less of an effect in the present experiment even though the Richardson number, as defined earlier and averaged through the boundary layer, was virtually identical for both. Apparently, this is a consequence of the different Reynolds numbers and Mach numbers. The present experiment was carried out at Reynolds numbers based on the momentum thickness (1000 to 2000) about four times smaller than Thomann's and at a Mach number of about 0.06 compared to Thomann's value of 2.5. At the lower momentum thickness Reynolds numbers, viscous effects are more important and reduce curvature's influence. Therefore, curvature will have a smaller effect in the present situation. The effect of compressibility as shown in [10] is to increase the ratio of centrifugal to inertia forces by a factor of $[1 + (\gamma - 1)M^2/2]$. Accordingly, the appropriate Richardson number for Thomann's experiment is not as defined earlier but $[1 + (\gamma - 1)M^2/2]$ larger or slightly more than twice that for the present experiment. As a result, curvature will have a greater effect in the supersonic flow examined by Thomann.

Comparison with a Calculation Method

The calculation of two-dimensional turbulent boundary layer flow for a variety of mainstream and surface conditions has been made practical in recent years through the use of the digital computer. As a result, a number of boundary layer programs, both integral and finite difference types, have been developed. One of these programs called STAN 5, a finite difference type developed at Stanford University [16] using basically the Patankar-Spalding scheme [17], was modified to include streamline curvature. The modification was essentially that proposed by Bradshaw and is identical in form to that used by Eide and Johnston. That is, the mixing length without curvature was altered according to

$$\ell/\ell_0 = 1 - \beta\left(2\frac{u}{R}\right)\Big/\left(\frac{\partial u}{\partial y}\right)$$

for $(\partial u/\partial y) > 0.3\ (u_\infty/\delta)$ and

$$\ell/\ell_0 = 1 - \beta\left(2\frac{u}{R}\right)\Big/\left(0.3\frac{u_\infty}{\delta}\right)$$

for $(\partial u/\partial y) < 0.3\ u_\infty/\delta$. The restriction on $(\partial u/\partial y)$ is an artifice to prevent the mixing length from becoming infinitely large in the outer region of the boundary layer where $(\partial u/\partial y)$ becomes zero. The value of β used in the calculations was 6 for both the convex and concave surfaces. Also, the turbulent Prandtl number was assumed to be unaffected by curvature but the option was used in STAN 5 which allowed for its variation through the boundary layer. The wake value of the turbulent Prandtl number was taken as 0.86. Finally, in each case the velocity and temperature profiles shown in Figs. 3 and 4 were used to start the calculation.

The results of the calculations and the data are presented in Fig. 6 where again the distance from the leading edge of the test surface has been used in the Rynolds number. On the convex surface the calculation agrees reasonably well with the measurements which indicates that the heat load on a convex surface may be determined satisfactorily by using an appropriately modified two-dimensional boundary layer program. But on the concave surface the agreement is poor. In light of the transition from a two-dimensional to a three-dimensional flow which took place on the concave surface, this result is not unexpected and indicates rather clearly the area in which further work must be done before accurate heat transfer calculations for a turbulent boundary layer on a concave surface become feasible.

Concluding Remarks

As found by Thomann, the heat transferred to a turbulent boundary layer on a convex surface is less than that on a flat surface, while the heat transferred to a turbulent boundary layer on a concave surface is greater. However, the difference is much less for the present experiment than that reported by Thomann. This is attributed in part to the effects of compressibility in curved flows and indicates that curvatures effects become more important at higher Mach numbers.

The good agreement between the heat transfer results on the convex surface and the boundary layer calculation appears to indicate that only a modification to the turbulent shear stress (and turbulent heat flux via the turbulent Prandtl number) is required to successfully calculate the heat load on a convex surface. However, for flow on a concave surface, a two-dimensional calculation can only be expected to provide a reasonable average value of the heat load over the whole surface. The term "reasonable," of course, depends on whether or not the presently accepted value of β is appropriate for all flow situations over a concave surface. At that, the streamwise distribution of heat flux cannot be satisfactorily calculated and, eventually, the three-dimensional nature of the flow will have to be investigated and modeled.

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