Model Based Damage Identification Using Output Spectral Densities

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A damage identification method utilizing an existing computational model and output spectral densities is presented. The problem covered here is the detection, localization and quantification of damage in real vibrating elastomechanical structures. The damages are localized by means of changes in the dynamic characteristics between a reference model and the actual, measured system. The main contribution is that the exact measurement of the input signals is ignored. These signals are assumed to be an ergodic random process, whose statistical properties such as mean and covariances must be estimated. Power spectral densities allow random excitations to be dealt with. The lack of measurement information is treated by means of the dynamic condensation technique and the Kalman Bucy filter technique. In the first case the size of the model matrices are reduced to the number of measured degrees of freedom (dof). In the second procedure the measured responses are expanded to the size of the model matrices. With equally sized measurement and model matrices a linear equation system for the desired parameter changes is derived by using the sensitivity approach. The equation system for this inverse problem is usually ill-conditioned and must be regularized in some way. One possibility is to reduce the subset of parameters to be in error. The algorithm is applied to a beam structure and a measured laboratory structure, a multi story frame, in which artificial damage is introduced by weakening one column between two stories. So, it is shown that the location and the size of the corresponding stiffness decrease can be detected.
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1 Introduction

The ability to detect damage or faults is an important factor in the usage of machinery and plants. It is desired to get information about the structural integrity during normal operation. The deviation of the dynamic behavior from normal operating conditions allows a diagnosis of the damages. As symptoms for damage, changes of eigenfrequencies and mode shapes [1–3] and equally frequency response functions (FRFs) or time signals can be used e.g. [4–7]. To get the FRFs or modal data usually a forced vibration test with additional devices like shakers etc. is required. The determination of modal parameters from output-only data using the ambient excitation of the structure from traffic, wind etc. is described by [8]. Their approach works under the assumption that the unknown excitation is white noise. Another method which is based on the random decrement technique was proposed by [9]. In this article the physical parameters are determined directly from the power spectral densities of the output-only signals. Similar to Peeters and De Roeck the assumption of a broadband excitation under operating conditions is made.

Those methods based on parameter identification techniques particularly require an accurate model. This reference model represents the undamaged state of the system and is used to generate residuals between its own characteristic dynamic properties and the corresponding, measured data. In addition, the model allows the calculation of sensitivities of the dynamic characteristics with respect to any parameter describing the system. The sensitivities are important in solving the inverse problem. The number of parameters necessary to describe all possible locations and types of damage may be very large. One major problem is to achieve a small subset of parameters which characterize the real damage. Often the analyst may not be aware of the nature of the fault. The behavior of a damaged structure must be sensitive to variations of the parameter but that does not necessarily mean that a fault is associated with this parameter [4].

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2 The Mathematical Model

The dynamic behavior of the vibrating elastomechanical system with n degrees of freedom is described by the linear, viscously damped model

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{f}(\mathbf{t}) \tag{1}$$

$\mathbf{M},\mathbf{C},\mathbf{K}$	$\in\mathbb{R}^{n\times n}$	are the symmetric mass, damping, and stiffness matrices,
$\ddot{\mathbf{x}},\dot{\mathbf{x}},\mathbf{x}$	$\in\mathbb{R}^{n}$	are the accelerations, velocities, and displacements of the system,
$\mathbf{f(t)}$	$\in\mathbb{R}^{n}$	is the external excitation.

If the system is excited by random loads, as they occur during normal operating conditions, Eq. (1) is hard to solve. In place of Eq. (1), a formulation based on power spectral densities does not have the difficulties of the differential equation in the time domain. The Wiener-Khintchine transformation allows the transformation of a linear system with stochastic inputs in the frequency domain

$$\mathbf{K}_D\mathbf{S}_{XX}\mathbf{K}_D^*=\mathbf{S}_{FF}\,, \tag{2}$$

with $\mathbf{K}_D=\mathbf{K}+i\omega\mathbf{C}-\omega^2\mathbf{M}$;
$(\ldots)^*$: conjugate transposed of $(\ldots)$;

$$\mathbf{S}_{XX}(\omega)=E\left\{\frac{1}{T}\mathbf{X}(\omega)\mathbf{X}^*(\omega)\right\};\quad \mathbf{S}_{FF}(\omega)=E\left\{\frac{1}{T}\mathbf{F}(\omega)\mathbf{F}^*(\omega)\right\}.$$

$\mathbf{S}_{XX}$ and $\mathbf{S}_{FF}$ are the matrices of the power spectra of the displacements and forces, respectively. The diagonal elements are the real auto power spectra e.g., $E\{(1/T)\mathbf{X}_i\mathbf{X}_i^*\}$, while the off elements

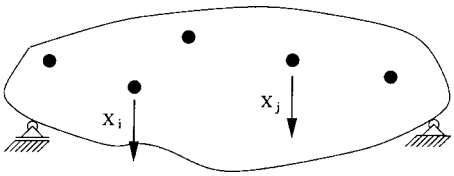

Fig. 1 Measured displacements

are the complex cross power spectra e.g., $E\{(1/T)\mathbf{X}_i\mathbf{X}_j^*\}$. The operator $E\{\dots\}$ symbolizes the expectation of a quantity. The factor T is the measurement duration Fig. 1.

2.1 Localization. If the excitation is an ergodic random signal, which provides statistical the same power spectral densities $\mathbf{S}_{FF}$ in the actual and in the reference state, then the relation between the actual measured system (index m) and the analytical reference model (index a) follows from Eq. (2)

$$\mathbf{K_D}\mathbf{S}_{XX_m}\mathbf{K}_\mathbf{D}^*=\mathbf{K}_{\mathbf{D}a}\mathbf{S}_{XX_a}\mathbf{K}_{\mathbf{D}a}^*. \tag{3}$$

$\mathbf{K_D}$ is the dynamical stiffness matrix of the damaged system. $\mathbf{K}_{\mathbf{D}a}$ represents the known, undamaged, reference model. All terms on the right side of Eq. (3) are known. On the left side only the measured power spectra $\mathbf{S}_{XX_m}$ are known. The differences in the intensity of the excitation are taken into account by calculating the average integral I_m of all auto power spectral densities of the measured signals and the corresponding model spectra I_a in the observed frequency range $\mathbf{S}_{XX_a}=(I_m/I_a)\mathbf{S}_{XX_a}^{(h)}$, with $\mathbf{S}_{XX_a}^{(h)}$ as the initial, not intensity adjusted reference power spectral densities. The differences between the matrix $\mathbf{K_D}$ and the matrix $\mathbf{K}_{\mathbf{D}a}$ are small expected as

$$\mathbf{K_D}=\mathbf{K}_{\mathbf{D}a}+\Delta\mathbf{K_D}. \tag{4}$$

For the unknown difference matrix $\mathbf{\Delta K_D}$, a sensitivity approach as in [5] is used. The differences are approximated by a linear Taylor series, whose arguments are the correction parameters Δa_j $j=1,2,\dots,n_p$.

$$\Delta\mathbf{K_D}=\sum_j \mathbf{K}_{\mathbf{D}a,_j}\Delta a_j, \tag{5}$$

with the partial derivative $\mathbf{K}_{\mathbf{D}a,_j}=\partial\mathbf{K}_{\mathbf{D}a}/\partial a_j$.

Indeed the linear relation between the correction parameters Δa_j and the matrix changes $\mathbf{\Delta K_D}$ is exact in many cases. Parameters with a similar influence on the model as Young's modulus and the Moment of Inertia should not be used concurrently because they amplify the ill-conditioned problem. In this case it is better to use the bending stiffness. For numerical reasons the correction parameter should be dimensionless. Therefore, a correction parameter is the change of the physical parameter $\Delta p_j=p_j-p_{jo}$ with respect to the original model parameter p_{jo} (e.g., mass, bending stiffness etc.) $\Delta a_j=p_j-p_{jo}/p_{jo}$ [10]. Inserting Eq. (4) and (5) into Eq. (3) and neglecting the higher order terms, yields the localization equation

$$\underbrace{\sum_j\{\mathbf{K}_{\mathbf{D}a}\mathbf{S}_{XX_m}\mathbf{K}_{\mathbf{D}a,_j}^*+\mathbf{K}_{\mathbf{D}a,_j}\mathbf{S}_{XX_m}\mathbf{K}_{\mathbf{D}a}^*\}\Delta a_j}_{\mathbf{A_j}}=\underbrace{\mathbf{K}_{\mathbf{D}a}(\mathbf{S}_{XX_a}-\mathbf{S}_{XX_m})K_{\mathbf{D}a}^*}_{\mathbf{B}}. \tag{6}$$

Arranging the correction parameters in a vector $\mathbf{\Delta a}\in\mathbb{R}^{n_p}$, building this equation system for every measured frequency ω_k, $k=1,2,\dots,n_f$ and splitting it into real and imaginary part provides the final overdetermined equation system

$$\mathbf{A}\Delta\mathbf{a}=\mathbf{b} \tag{7}$$

with $\mathbf{A}\in\mathbb{R}^{m\times n_p}$; $\mathbf{b}\in\mathbb{R}^m$; $m=2n_f(n^2/2)$
which has to be solved. The solution of Eq. (6) requires equal sized matrices. But in most cases there is a lack of measurement information $\mathbf{S}_{XX_m}\in\mathbb{C}^{n_m\times n_m}$ with $n_m\leqslant n$. The following two sections will deal with this problem.

2.2 Dynamic Condensation. One possibility to overcome the sizing problem is to reduce the n model dof to the $n_m\leqslant n$ measured dof. If the reduction is performed on the basis of physical laws, it is termed ''condensation'' Reference [10] gives a survey. One method is the dynamic condensation, using the minimum energy as a constraint. The condensation process has to be done for every observed frequency ω_k and provides

$$\mathcal{K}_\mathbf{D}=\mathbf{K}_{\mathbf{D}MM}-\mathbf{K}_{\mathbf{D}MN}\mathbf{K}_{\mathbf{D}_{NN}}^{-1}\mathbf{K}_{\mathbf{D}_{NM}} \tag{8}$$

with $\mathcal{K}_\mathbf{D}\in\mathbb{C}^{n_m\times n_m}$.

Index M stands for the measured dof, index N for the not measured and reduced dof. The partial derivative $\mathcal{K}_{\mathbf{D}a,_j}$ is built by the derivative of every term of Eq. (8) with respect to the correction parameter

$$\mathcal{K}_{\mathbf{D}a,_j}=\mathbf{K}_{\mathbf{D}MM,_j}-2\mathbf{K}_{\mathbf{D}MN}\mathbf{K}_{\mathbf{D}_{NN}}^{-1}\mathbf{K}_{\mathbf{D}NM,_j}+\mathbf{K}_{\mathbf{D}_{NN}}^{-1}\mathbf{K}_{\mathbf{D}NN,_j}\mathbf{K}_{\mathbf{D}_{NN}}^{-1}. \tag{9}$$

The corresponding power spectra to $\mathbf{S}_{XX_m}$ are obtained by extracting the appropriate components $\mathbf{S}_{XX_a}$ from the complete matrix $\mathbf{S}_{XX_a}$. Following this step the localization equation is

$$\sum_j\{\mathcal{K}_{\mathbf{D}a}\mathbf{S}_{\mathbf{XX}_m}\mathcal{K}_{\mathbf{D}a,j}^*+\mathcal{K}_{\mathbf{D}a,_j}\mathbf{S}_{\mathbf{XX}_m}\mathcal{K}_{\mathbf{D}a}^*\}\Delta a_j=\mathcal{K}_{\mathbf{D}a}(\mathbf{S}_{XX_a}-\mathbf{S}_{\mathbf{XX}_m})\mathcal{K}_{\mathbf{D}a}^*. \tag{10}$$

If a correct reference model does not exist, the difference in the dynamic characteristics between measurement and model are, in principal, caused by modelling errors. Adapting the model to the measurement seems to be the more sensible way. Still the question is open on which dof the output signals should be measured to get reliable results. The mode shapes of a previous finite element model could be helpful indicators. Those dof should be taken which have deflections, in as many as possible mode shapes.

2.3 Expansion in the Frequency Domain. An alternative possibility to solve the sizing problem is to expand the measured power spectra $\mathbf{S}_{XX_m}$ to the size of the model matrices. A suitable tool is the Kalman Bucy Filter (KBF) [11]. This filter is developed as the optimal state estimator for linear systems under stochastic noise. The starting point is the state equation

$$\dot{\mathbf{z}}=\mathbf{A_s}\mathbf{z}+\mathbf{B_s}\mathbf{u}+\mathbf{G_s}\mathbf{w} \tag{11}$$

with the accompanying measurement equation

$$\mathbf{y}=\mathbf{C_s}\mathbf{z}+\mathbf{D_s}\mathbf{u}+\mathbf{v}. \tag{12}$$

The state vector $\mathbf{z}$, the system matrix $\mathbf{A_s}$ and the input matrix $\mathbf{B_s}$ follow from the equation of motion (1)

$$\mathbf{z}=\begin{pmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{pmatrix}\in\mathbb{R}^{2n},\quad \mathbf{A_s}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}\in\mathbb{R}^{2n\times 2n},$$

$$\mathbf{B_s}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\end{bmatrix}\in\mathbb{R}^{2n\times n}.$$

$\mathbf{0},\mathbf{I}\in\mathbb{R}^{n\times n}$ are a zero and a identity matrix.

The vector $\mathbf{u}$ stands for measurable external input. $\mathbf{w}$ is the random excitation or system noise due to disturbances and modelling inaccuracies with known means and covariances. If $\mathbf{u}$ and $\mathbf{w}$ both are forces, then $\mathbf{G_s}=\mathbf{B_s}$

The matrices $\mathbf{C_s}$ and $\mathbf{D_s}$ link $\mathbf{z}$ and $\mathbf{u}$ with the measurements $\mathbf{y} \in \mathbb{R}^{n_m}$, which are corrupted by additive noise $\mathbf{v}$ with known means and covariances. The components of $\mathbf{C_s}$ and $\mathbf{D_s}$ are dependent upon the measured values. Let $\mathcal{H}$ be the observation matrix with entries one and zero for measured and not measured dof. In [6] it is shown, if the measured signals are displacements

$$\mathbf{C_s}=[\mathcal{H}\ \ \mathbf{0}] \in \mathbb{R}^{n_m \times 2n}, \quad \mathbf{D_s}=[\mathbf{0}] \in \mathbb{R}^{n_m \times n}, \tag{13}$$

if they are measured velocities

$$\mathbf{C_s}=[\mathbf{0}\ \ \mathcal{H}] \in \mathbb{R}^{n_m \times 2n}, \quad \mathbf{D_s}=[\mathbf{0}] \in \mathbb{R}^{n_m \times n} \tag{14}$$

and for measured accelerations results

$$\mathbf{C_s}=[\mathbf{0}\ \ \mathcal{H}]\ \ \mathbf{A_s} \in \mathbb{R}^{n_m \times 2n}, \quad \mathbf{D_s}=[\mathbf{0}\ \ \mathcal{H}]\ \ \mathbf{B_s} \in \mathbb{R}^{n_m \times n}. \tag{15}$$

Random excitation $\mathbf{w}$, measurement noise $\mathbf{v}$ and the initial state vector $\mathbf{z}_0$ should be mutual uncorrelated:

$$E\{\mathbf{w}\mathbf{v}^T\}=\mathbf{0};\ \ E\{\mathbf{z}_0\mathbf{v}^T\}=\mathbf{0};\ \ E\{\mathbf{z}_0\mathbf{w}^T\}=\mathbf{0}.$$

The mathematical background described above are the basis for KBF equation (16), which provides the optimal $2n$-dimensional estimation $\hat{\mathbf{z}}$ for the wanted state vector $\mathbf{z}$

$$\dot{\hat{\mathbf{z}}}=\mathbf{A_s}\hat{\mathbf{z}}+\mathbf{B_s}\mathbf{u}+\mathbf{K_G}(\mathbf{y}-\hat{\mathbf{y}})$$

with

$$\hat{\mathbf{y}}=\mathbf{C_s}\hat{\mathbf{z}}+\mathbf{D_s}\mathbf{u}. \tag{16}$$

$\mathbf{K_G}$ is the stationary Kalman gain matrix, which is determined that the symmetric covariance matrix $\mathbf{P}=E\{(\mathbf{z}-\hat{\mathbf{z}})(\mathbf{z}-\hat{\mathbf{z}})^T$ is minimized. In the case of big measurement noise $\mathbf{K_G}$ should effect a low amplification. $\hat{\mathbf{y}}$ should rather follow the signal $\mathbf{C_s}\mathbf{z}$ than the noise $\mathbf{v}$. Strong random excitation $\mathbf{w}$ excits the state $\mathbf{z}$ strong and $\mathbf{K_G}$ has to amplify the difference between measurement and estimation $(\mathbf{y}-\hat{\mathbf{y}})$ to show a good following characteristic. It can be shown that

$$\mathbf{K_G}=\mathbf{P}\mathbf{C_s}^T\mathbf{R}^{-1} \tag{17}$$

is the optimal filter [11–13]. The inverse of the positive definite covariance matrix of the measurement noise $\mathbf{R}=E\{\mathbf{v}\mathbf{v}^T\}$ effects a low amplification for big measurement noise and the linearity to $\mathbf{P}$ consider the case of strong random excitation. If the KBF equation (16) is transformed into the frequency domain, for $t \geqslant 0$ and initial condition $\mathbf{z}_0=\mathbf{0}$, one obtains for measured displacements and velocities

$$\mathbf{L}^{-1}\hat{\mathbf{Z}}=\mathbf{B_s}\mathbf{U}+\mathbf{K_G}\mathbf{Y},$$

with

$$\mathbf{L}^{-1}=i\omega\mathbf{I}-(\mathbf{A_s}-\mathbf{K_G}\mathbf{C_s}) \tag{18}$$

and $\hat{\mathbf{Z}}(\omega), \mathbf{U}(\omega), \mathbf{Y}(\omega)$. The corresponding equation for measured accelerations is Eq. (19).

$$\mathbf{L}^{-1}\hat{\mathbf{Z}}=(\mathbf{B_s}-\mathbf{K_G}\mathbf{D_s})\mathbf{U}+\mathbf{K_G}\mathbf{Y}. \tag{19}$$

Expressed by power spectra Eq. (19) gives

$$\begin{aligned}\mathbf{S}_{\hat{Z}\hat{Z}}=&\mathbf{L}\mathbf{K_G}\mathbf{S}_{YY}\mathbf{K_G}^T\mathbf{L}^*+\mathbf{L}\mathbf{K_G}\mathbf{S}_{YU}(\mathbf{B}_s-\mathbf{K_G}\mathbf{D_s})^T\mathbf{L}^*\\&+\mathbf{L}(\mathbf{B_s}-\mathbf{K_G}\mathbf{D_s})\mathbf{S}_{UY}\mathbf{K_G}^T\mathbf{L}^*\\&+\mathbf{L}(\mathbf{B_s}-\mathbf{K_G}\mathbf{D_s})\mathbf{S}_{UU}(\mathbf{B_s}-\mathbf{K_G}\mathbf{D_s})^T\mathbf{L}^*\end{aligned} \tag{20}$$

from which the n-dimensional matrix $\mathbf{S}_{\hat{X}\hat{X}_m}$ can be obtained directly.

$$\mathbf{S}_{\hat{Z}\hat{Z}}=\begin{bmatrix}\mathbf{S}_{\hat{X}\hat{X}_m} & \mathbf{S}_{\hat{X}\dot{\hat{X}}_m}\\ \mathbf{S}_{\dot{\hat{X}}\hat{X}_m} & \mathbf{S}_{\dot{\hat{X}}\dot{\hat{X}}_m}\end{bmatrix} \in \mathbb{C}^{2n \times 2n}.$$

In the more sophisticated case, if the measurable external input $\mathbf{u}$ is zero or unknown ($\mathbf{B_s}=\mathbf{D_s}=\mathbf{0}$) and the system is excited only by ergodic random signals $\mathbf{w}$, which are considered in $\mathbf{K_G}$, Eq. (20) and the corresponding equation for displacements and velocities is shortened to

$$\mathbf{S}_{\hat{Z}\hat{Z}}=\mathbf{L}\mathbf{K_G}\mathbf{S}_{YY}\mathbf{K_G}^T\mathbf{L}^*. \tag{21}$$

This Eq. (21) is the expansion equation for output-only measurements in the frequency domain. The detour of transforming expanded time signals in the frequency domain is not necessary [14].

2.4 Regularization. To get a stable solution of Eq. (7) with the least square error method is in general not successful. The reason is, the coefficient matrix $\mathbf{A}$ is an ill-conditioned matrix. Measurement noise and truncation errors amplify the instability. To obtain a stable solution the equation system must be regularised in some way. Here the regularisation is performed by reducing the subset of parameters to be in error. The proposal of [15] transform the equation system (7) gradually into the orthogonal equation system

$$\mathbf{W}\mathbf{g}=\mathbf{b}, \tag{22}$$

which is solved with the least square method $\mathbf{W}^T\mathbf{W}\mathbf{g}=\mathbf{W}^T\mathbf{b}$. $\mathbf{W}^T\mathbf{W}$ is an easy invertable diagonal matrix with the components

$$\mathbf{W}_i^T\mathbf{W}_j=\begin{cases}\|\mathbf{W}_j\|^2 & \text{for } i=j\\ 0 & \text{for } i\neq j.\end{cases}$$

In the first transformation step for every column $\mathbf{A}_i$ the equation error

$$\varepsilon_i=\mathbf{b}-\mathbf{A}_i\Delta a_i \tag{23}$$

is calculated. This column, which provides the minimum error, is written in the first column of a new orthogonal matrix $\mathbf{W}$ and the chosen column number is registered in a transformation vector. The maximum error in the least square sense is $\boldsymbol{\epsilon}^T\boldsymbol{\epsilon}=\mathbf{b}^T\mathbf{b}$. Every parameter-column combination $\mathbf{W}_j g_j$, which is taken into account reduces the error. The error reduction with respect to the maximum error is the error reduction ratio (*err*)

$$err_j=\frac{g_j\mathbf{W}_j^T\mathbf{W}_j g_j}{\mathbf{b}^T\mathbf{b}}. \tag{24}$$

The *err* is the selection and order criterion for every following column $\mathbf{W}_j$ and the crucial criterion to reduce the solution space. In every step that column $\mathbf{W}_j^{(h)}$ is selected, which maximizes the *err*.

$$\mathbf{W}_j^{(h)}=\mathbf{A}_i-\sum_{k=1}^{j-1}\mathbf{W}_k\frac{\mathbf{W}_k^T\mathbf{A}_i}{\mathbf{W}_k^T\mathbf{W}_k} \quad \text{and} \quad g_j^{(h)}=\frac{\mathbf{W}_j^{(h)^T}\mathbf{b}}{\mathbf{W}_j^{(h)^T}\mathbf{W}_j^{(h)}}. \tag{25}$$

$\mathbf{W}_k$, $k=1,\ldots,j-1$ are the previously determined columns of **W**. The transformation ends, when all column of **A** has been considered. The columns in **W** are ordered now by decreasing *err*. Looking at the *err* values shows that only the first few values are significantly greater than zero. This means that only the corresponding columns of **W** provide a significant contribution to solve the equation system. All the other columns are necessary to reproduce the noise and modelling error polluted right hand side **b** best but they contribute nothing to the desired solution. Having this background only those column of **W** are used, whose *err* values are significantly greater than a predefined bound. The correction parameters, which belong to the remaining columns, are set to zero. This strategy reduces the solution space and the solution of Eq. (22) becomes stable.

3 Example

The first example tests the damage identification by means of condensed model matrices and expanded displacements from asimulated beam structure. In the second example, the damage identification and condensation method is applied to a real multi story frame. Output signals in this case are measured accelerations.

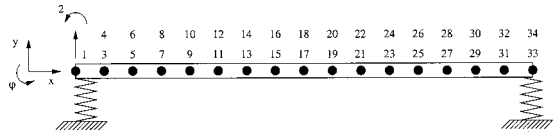

Fig. 2 Dof of the beam structure

Table 1 Design parameter

length	$l=0.08\ m$
cross section area	$A=1.26\cdot 10^{-4}\ m^2$
Moment of Inertia	$J=7.077\cdot 10^{-9}\ m^4$
Young's modulus	$E=1.95\cdot 10^{11}\frac{N}{m^2}$
density	$\rho=7850\frac{kg}{m^3}$
spring stiffness	$c_f=10^6\frac{N}{m}$
modal damping	$D_i=0.01$

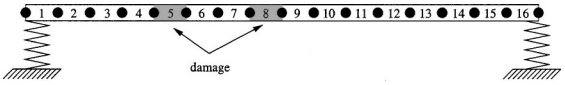

Fig. 3 Beam structure with simulated damages

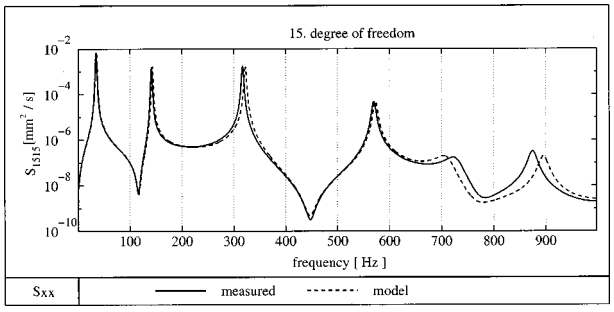

Fig. 4 Starting situation

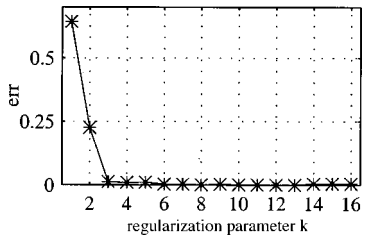

Fig. 5 Error reduction ratios of the first iteration

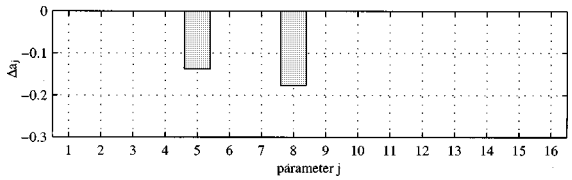

Fig. 6 Localization result of the first iteration

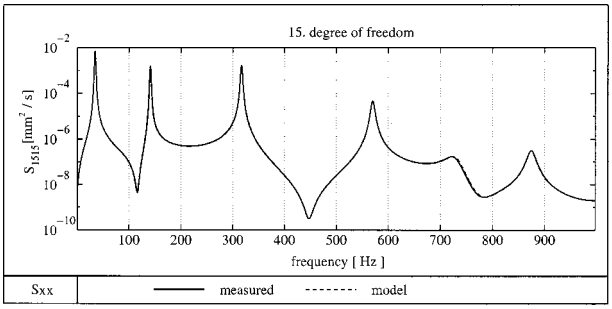

Fig. 7 Power spectral density of the first iteration

3.1 Beam Structure. The finite element model of the beam structure consists of 16 beam elements with 34 dof as shown in Fig. 2. Spring elements represent the elastic bearings at both ends. The dof in *x*-direction are neglected. Table 1 contents the design parameter for the beam elements of the reference model.

To generate a simulated damage the bending stiffness *EJ* of element five and element eight are reduced by 20%. This represents damages at two locations, as shown in Fig. 3.

The reference model and the ''damaged structure'' are excited by zero mean white noise on each dof, because only unknown forces **w** should act on the systems. Knowing the modeshapes of the structure the dof (5, 7, 9, 11, 13, 15, 25, 27, 29) are choosen as master dof, to test the condensation technique. Figure 4 shows the auto power spectral density of the 15. dof in the starting situation. It demonstrates, the higher the frequency, the bigger the difference between actual and reference state. The frequency ranges between the peaks, has not much information. Therefore one should take the highest eigenfrequencies one could get. To keep the equation system Eq. (7) small, only frequencies around the eigenfrequencies in the range between 140.5 Hz–896 Hz are used (Fig. 4).

The first iteration provides the results shown in Figs. 5–7. In Fig. 5 the error reduction ratios corresponding to the columns of matrix **W** are listed. As described above the *err* decreases fast. Only the first two values are significant greater than zero. Now the regularisation starts. The parameters $\mathbf{g}_3-\mathbf{g}_{16}$ are set to zero and Eq. (22) is solved with the first two columns of **W** only. The result for $\mathbf{g}_1$ and $\mathbf{g}_2$ back transformed to the corresponding parameters is shown in Fig. 6. As expected, the parameters five and eight, respectively, EJ_5 and EJ_8 are detected as wrong. EJ_5 has to be reduced by 0.14 respectively $\Delta EJ_5=-14\%$ and EJ_8 by ΔEJ_8

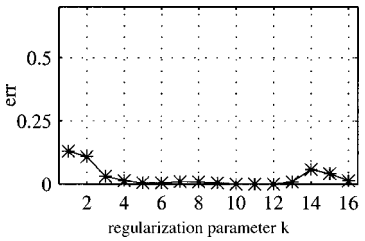

Fig. 8 Error reduction ratios of the first iteration

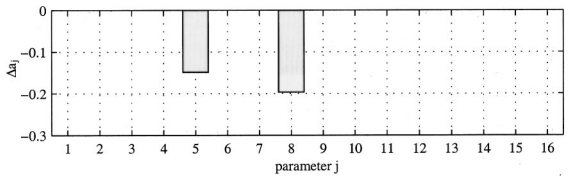

Fig. 9 Localization result of the first iteration

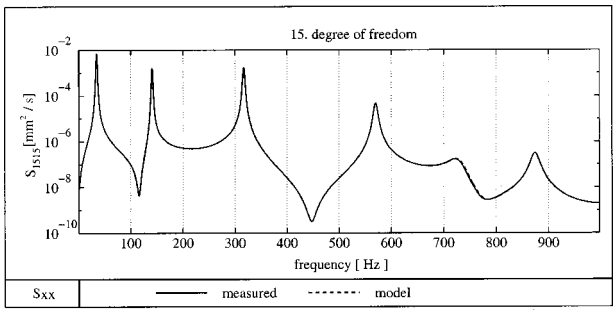

Fig. 10 Power spectral density of the first iteration

$=-17\%$. The damage location is detected but the severity is not correct. Figure 7 shows the auto power spectral density after the model is updated with the calculation result for EJ_5 and EJ_8.

The difference between the measurement and the model output is now considerable smaller. After two further iteration the results for EJ_5 and EJ_8 are nearly excact. But with the decreasing difference between real structure and the model the algorithm gets susceptible for noise or model inaccuracies.

The same beam is now used to prove the expansion method, all rotational dof of the damaged structure are unknown and have to be calculated. The result of the first iteration is shown in Figs. 8–10. Unfortunately, all *err*-values are small. The first two *err* values belong to the damaged elements, but with $err_1=0.131$ and $err_2=0.109$ the regularization criteria is not very sharp. Solving Eq. (22) with the first two columns of $\mathbf{W}$ and setting $\mathbf{g}_3-\mathbf{g}_{16}$ to zero provides the result for $\mathbf{g}_1$ and $\mathbf{g}_2$ resp. ΔEJ_5 and ΔEJ_8 shown in Fig. 9. EJ_5 have to be updated by $\Delta EJ_5=-15\%$ and EJ_8 by $\Delta EJ_8=-20\%$. In Fig. 10 this step is done.
Because of the small *err*-values the model cannot be improved by additional iterations. This example demonstrates, that under the same requirements the damage identification with condensed model matrices works better than the identification with expanded measurement data.

3.2 Multi-Story Frame. The next example is the multi story frame shown in Fig. 11. The whole frame has a height of approximately 500 mm. The six aluminum plates have a size of $310\,\text{mm}\times 280\,\text{mm}\times 15\,\text{mm}$ and a mass including screws of $m_P=3.6\,\text{kg}$. The steel strips between the plates have a cross sectional area of $A=8\cdot 10^{-5}\,\text{m}^2$, a Moment of Inertia of J

Fig. 11 Laboratory structure

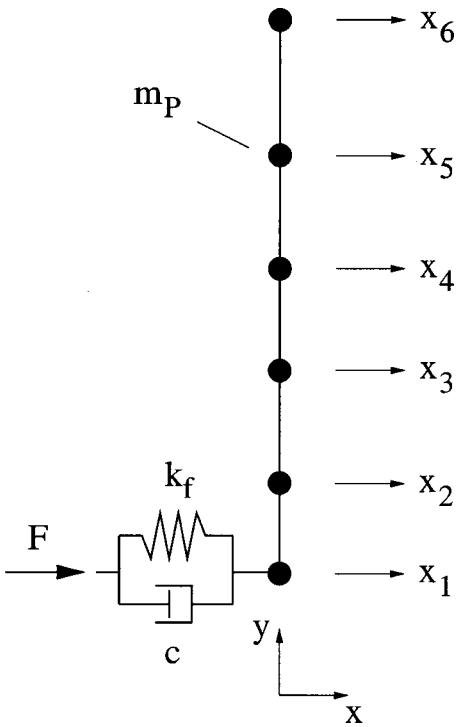

Fig. 12 Elastomechanical model

Table 2 Parameter of the reference model

story	1	2	3	4	5
$EJ[Nm^2]$	27.28	31.85	28.28	33.15	24.68
$k_f[N/m]$	$5.24\cdot 10^6$				

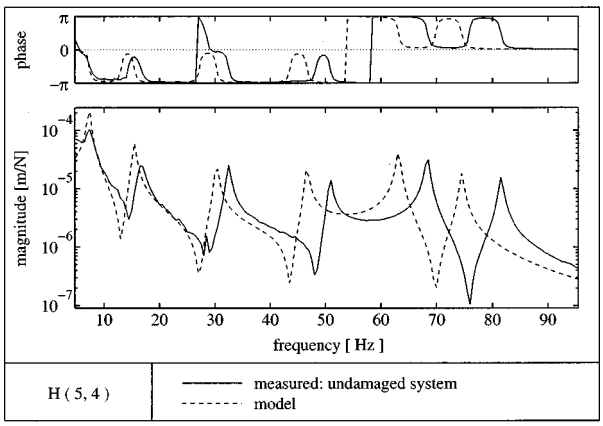

Fig. 13 FRFs of the measurement and the initial model

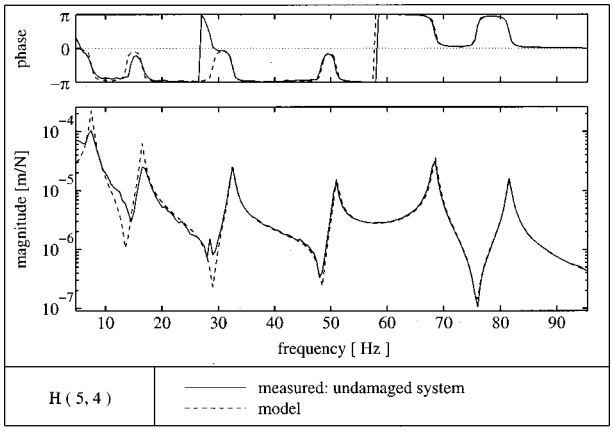

Fig. 14 FRFs of the measurement and the reference model

Fig. 15 Removed steel strip as artificial damage

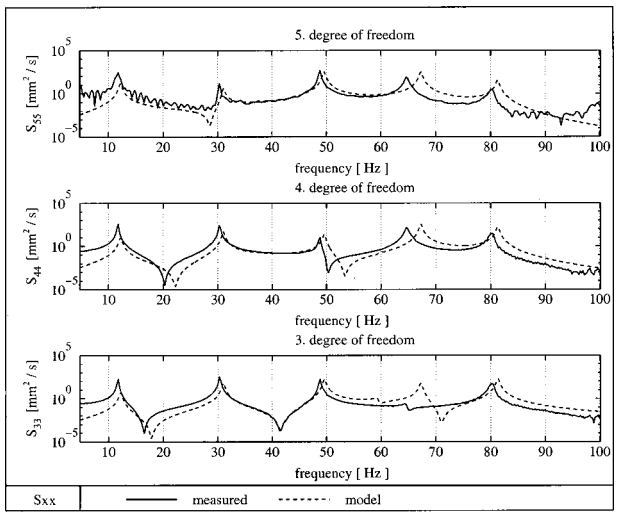

Fig. 16 Starting situation

$=2.66\cdot 10^{-11}\,\mathrm{m}^4$, a Young's modulus of $E=2.1\cdot 10^{11}\ \mathrm{N/m}^2$ and a density of $\rho=7850\ \mathrm{kg/m}^3$. Plates and steel strips are connected so that the frame can only vibrate in horizontal and vertical directions. But vertical vibrations are small. Rotational dof are constrained. The structure is excited with a random signal by a shaker at the bottom plate.

The corresponding elastomechanical model is shown in Fig. 12. The steel strips are modeled as Timoshenko beam elements. Plate masses are considered as lumped masses. The proportional damping approach $\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K},\ \alpha,\beta\in\mathbb{R}$ is used as damping model.

The shaker is a good excitation source for an ergodic random signal, but it is fixed on the structure and affects the dynamic properties of the structure. To get the influence of the shaker a detour in the updating process is done. The shaker is turned off and the whole system, story frame and shaker are excited successively by a Dirac impulse at all floors and the output accelerations at the third, fourth and fifth dof are measured. The linearity of the whole system is checked by the reciprocity of the measured FRFs. Comparing the measured and the modeled FRFs in Fig. 13 shows, that the model represents the shape of the measured FRFs good. But the position of the eigenfrequencies indicates that the model is too stiff. With formula (26) presented by [16], the reference model is adapted in two steps.

$$\sum_{j=1}\frac{1}{\lambda_{m_i}}\boldsymbol{\varphi}_{ai}^T,(\mathbf{K}_{a,j}-\lambda_{ai},\mathbf{M}_{a,j})\boldsymbol{\varphi}_{ai},\Delta a_j=\frac{\lambda_{m_i}-\lambda_{a_i}}{\lambda_{m_i}},\qquad(26)$$

$\lambda_{m_i},\lambda_{a_i}$, measured and analytical eigenvalues,

$\boldsymbol{\varphi}_{a_i}$, analytical eigenvector.

In the first step the correction parameters Δa_j are the shaker stiffness k_f and a common bending stiffness EJ_B for the beam elements. Based on the real structure, the bending stiffness of the first, third, and fifth floor are four times EJ_B and in the second and fourth floor five times EJ_B. With a shaker stiffness $k_f=5.24\cdot 10^6\ \mathrm{N/m}$ and a bending stiffness of $EJ_B=6.30\ \mathrm{N\,m}^2$, the model represents the first two peaks, which depends most of the shaker and his mounting on the frame. In the next step the correction parameters are the bending stiffnesses of the five floors. The result is shown in Table 2. Figure 14 compares the developed reference model with the measurement.

A test for damage identification with the condensed model follows. Therefore, artificial damage is introduced by removing a steel strip in the fourth floor, as shown in Fig. 15. The bending stiffness of the fourth story EJ_4 of the real system is now reduced by 20%. The shaker excites the frame with random noise. The

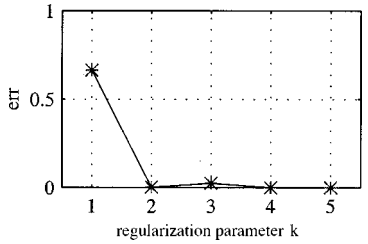

Fig. 17 Error reduction ratios of the first iteration

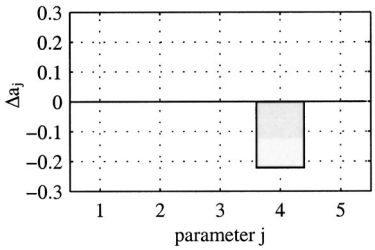

Fig. 18 Localization result of the first iteration

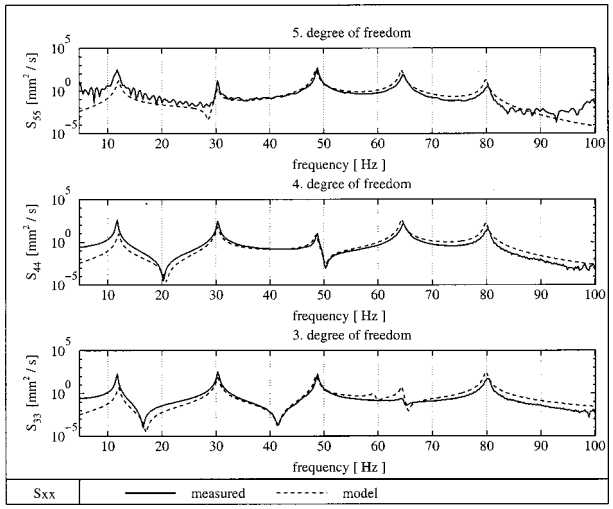

Fig. 19 Comparison of the power spectral densities after updating

model is excited by white noise. Measured are the acceleration signals in *x* direction at the third, fourth, and fifth dof, respectively at the third, fourth, and fifth aluminum plate counting bottom up. To compare even scaled power spectral densities of the measurement and the condensed model, the measured spectral densities have to be scaled by ω^{-4}. The scale factor depends on the Fourier transformation of accelerations, which provides $\mathcal{F}\{\ddot{\mathbf{x}}(\mathbf{t})\} = -\omega^2\mathbf{X}(\omega)$ for $t \geqslant 0$ and $\mathbf{x}_0 = \mathbf{0}$; $\dot{\mathbf{x}}_0 = \mathbf{0}$.

Figure 16 shows the resulting deviations in the power spectral densities. The task of the damage identification algorithm is now to locate the position and the severity of damage.

As in the previous example the equation system (7) is built only for some frequencies around the second to the fifth eigenfrequency in the range of 29.5–85 Hz. The solution of the damage identification algorithm is shown in Figs. 17–19.

Figure 17 shows a sharp decreasing of the *err* and that only one parameter provides an appreciable contribution to the solution. Solving the Eq. (22) with column $\mathbf{W}_1$, setting $\mathbf{g}_2 - \mathbf{g}_5$ to zero, checking the selection sequence and back transformation of $\mathbf{g}$ to $\mathbf{\Delta a}$ provides the result shown in Fig. 18. As expected it is parameter four, which should be decreased by $\Delta EJ_4 = -22\%$. If the model is updated with this result and the damage identification algorithm is started a second time, the value is adjusted and as a final result a correction of $\Delta EJ_4 = -19\%$ is made. The corresponding power spectral densities are shown in Fig. 19

In both examples the excitation signals are unknown. But the damages are localized and the serverity of the damages are well identified.

4 Conclusion

A method for the identification of damages in structural models has been presented, which is particularly used for the localization and quantification of structural faults. The main contribution is that the identification is based on power spectral densities. These offer the possibility of working with an ambient excitation and using output-only signals. The only assumption is that the input spectral density can be approximated by ergodic white noise. The advantage is that the damage identification can be done during normal operating conditions. Time consumptions to perform dynamical tests with artifical test signals are not necessary. Without test signals, the required excitation equipment like shakers etc. could also be saved. The deficiency of measurement information is treated by condensing the model matrices and by expanding the measurement data. A further contribution is that the expansion technique is able to deal with measured signals in the frequency domain. The example shows that the condensation technique works better and should be preferred in solving the sizing problem. The ill-conditioned linear equation system is regularized and solved by an orthogonal parameter estimation algorithm, which provides the error reduction ratios for the single columns of the coefficient matrix. With this important information it is possible to reduce the solution space and stabilize the solution of the equation system. Comparing the algorithm with the model based methods using FRFs like in [5], in this algorithm the equation system (7) is $n_m/2$ times bigger. This results because the algorithm needs the upper or lower triangle matrix of the hermitian power spectral density matrices $\mathbf{S}_{XX} \in \mathbb{C}^{n_m \times n_m}$ instead of one column $\mathbf{H}_i \in \mathbb{C}^{n_m}$ of a FRF matrix $\mathbf{H}$. Therefore, the time consumption for the algorithm is higher as for the methods using FRFs. But a lot more time could be saved because the method dispense, with breaks for artifical test signals. Another aspect is that if the excitation cannot be measured, the FRF based methods cannot be applied.

The algorithm has been applied to two examples. Both show that the location and the size of the deviation between the measured structure and the finite element model could be obtained. But the application to laboratory structures can only be a preliminary goal and the extension to industrial parts should be the next step.

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