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Instrumentation Selection and Uncertainty Analysis for Performance Test of Small Centrifugal Compressors

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ABSTRACT

In the design phase of centrifugal compressors, it is essential to have some experimental results on performance. The extent of usefulness of the experiments depends on quality and accuracy of the results. Part of proper experimental procedure is the correct selection of instrumentation leading to lower uncertainty in the final results. ASME PTC 10 (Performance Test Code on Compressors and Exhausters) requires fluctuation limits on the measured performance parameters. This does not guarantee limits for accuracy of performance parameters. Also, different experimental setup will affect uncertainty of the results, even with similar instrumentation accuracy. The present research deals with uncertainty analysis for performance evaluation of small-scale centrifugal compressor. The instrumentation errors are accommodated in the relation to ASME PTC 19.1 (test uncertainty). The analysis takes into consideration the correlated bias limits. Selection of proper type of instruments for measuring associated parameters is based on literature review. A case study is included as an example to illustrate the selection on instrumentation accuracy and preferred bias correlations. The analysis is a useful tool in designing experiments for testing compressor and optimizing accuracy of results.

KEYWORDS: instrumentation selection, uncertainty, performance test, bias correlation and small centrifugal compressors.

INTRODUCTION

The aim of designing high performance compressors is to achieve a high-pressure ratio with high efficiency. Compressor testing is essential in the early stage to improve the design parameters. Moreover, the performance map of a compressor is the plot of the pressure ratio and efficiency versus mass flow rate for different rotational speeds. The first three parameters cannot be measured directly and need combination of instrumentations while the rotational speed is a measurable parameter.

Each one of these four parameters is analyzed keeping into consideration American Society of Mechanical Engineering, Perfomance Test Code 10 (ASME PTC 10) [1] regulations. Instrumentation selection is an essential part of the experimental setup. Proper selection of instrument and experimental setup leads to more reliable results. This can be achieved through the combination of studying alternatives of instrumentation with proper uncertainty analysis. The present research discusses the selection of instrumentation to be used in testing centrifugal compressor performance under atmospheric condition (driven by electric motor), as well as performing extensive uncertainty analysis.

NOMENCLATURE

Symbols

A	Area
C_d	Discharge coefficient
C_p	Constant pressure specific heat at average temperature
ΔP	Differential static pressure of the flow meter
$\dot{m}$	Mass flow rate
P	Pressure
Pr	Total pressure ratio
Q	Flow rate
R	Gas constant
R	Result (used for the uncertainty analysis)
T	Temperature

U_B Bias uncertainty
U_p Precision uncertainty
U_R Resultant uncertainty
Y Expansion factor
$\dot{W}$ Power

Greek letters

δ_{ik} Kronecker delta
γ Specific heat ratio
η Isentropic efficiency
Ω Angular velocity ($2\pi/60$ * revolution per minutes)
ζ Correlation coefficient of the bias limit
τ Input torque

Subscripts

01 Inlet compressor stagnation condition
04 Exit compressor stagnation condition
f Flow meter
1f Flow meter inlet
2f Flow meter throat
I Piping inlet

Definitions

$$Z = \frac{\Delta P \, P_{2f}}{R\left(1 - \frac{A_{2f}^2}{A_{1f}^2}\right) T_{2f}}$$

INSTRUMENTATION SELECTION

Pressure Ratio

Two methods can be used to find the total pressure, either direct measurement with total pressure probes or combination of static pressure measurements and flow rate. Three types of total pressure probes are widely used in literature: the pitot, Kiel and multi-hole probes. Figliola and Beasley [2] explained briefly the characteristics of the first two probes. A combination of stagnation pressure measuring technique may be used for a setup. For example Shirley [3] used the pitot probe at the inlet of the compressor where the flow was almost undisturbed and the Kiel probe at exit where the flow was disturbed.

The multi-hole probe: three, four, five or seven holes proved to have reliable measurement results. Reunanen [4] used three-hole Cobra-probe to measure the static pressure and total pressure, as well as flow direction on a plane at the diffuser exit of a centrifugal compressor.

Another method to find the pressure ratio is the indirect technique in which the total pressure is calculated using other measured values (static pressure, temperature and flow rate), Whitefield et al. [5]. Actually, within each measurement there is an upper limit of accuracy. That is, accuracy of a calculated result decreases with increased number of measured variables. So, this method is not recommended for the proposed compressor test facility.

The direct measurement of the total pressure is widely done by the pitot, Kiel or multi-hole probe. The proposed test facility will be used for different compressors with different geometries and high rotational speeds. Also, these compressors will be tested for perfomance evaluation. This assumes to operate the compressors around the surge limit, where separation and back flow may take place. With these operating conditions the pitot probe is not recommended, especially at the compressor exit since it is relatively less insensitive to the flow angle comparing to the Kiel and multi-hole probes. The main drawback of the Kiel probe comes from the blockage that may build up inside its shroud. However, clean air is the working fluid and the operation time is relatively short. In addition, pretest run will be done to insure that all the instrumentations function properly, as required by ASME PTC 10 paragraph 3.10 [1]. Also, the pressure transducers will be calibrated before and after each measurement, following ASME PTC 10 paragraph 4.6.5 [1]. So, the problem of uncertainty in Kiel probe due to blockage is eliminated. The multi-hole probe does not have the problem of the blockage. Adding to that it can measure the total and static pressure. Also, some recent types of multi-hole probe, such as cobra probes, can measure flow turbulence Chen et al. [6]. To sum up, it is recommended to choose the Kiel probe over the multi-hole probe, as long as there is no need to measure the turbulent flow fields due to economical advantage. ASME PTC 19.2 [7] and Chue [8] are good references for general information on pressure measurement instruments.

Isentropic Efficiency

The isentropic efficiency of the compressor can be defined as the ratio of the ideal (isentropic) power to actual power of the compressor. If a restriction flow passage meter is used to measure mass flow rate, then the ideal power can be defined as:

$$\dot{W}_{ideal} = \frac{P_f}{\mathrm{R}T_f} Q_f C_p T_{01} \left(\left(\frac{P_{04}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \tag{1}$$

Measuring technique of total pressures was explained above. Static pressures can be measured through tabs.

Regarding the rotational speed of the compressor shaft, there are three general methods: devices that record the number of revolutions within a known time interval; devices which record time averaged rotational speed; and devices which continuously record instantaneous angular velocity, ASME PTC 19.7 [9]. Krain and Hoffmann [10] used an inductive system that functions by providing electrical signals of 60 equally spaced slots on the rotor shaft. ASME PTC 10 paragraph 4.10 [1] requires that the rotational speed instrument must have the ability to provide a continuous speed indication without restriction to specific type of instrument. Several types of rotational speed sensors are available in the market and the selection depends on the accuracy requirement for compressor testing, range of reading and its compatibility with controlling system of the compressor drive unit.

The choice of a flow meter device depends mainly on the size, accuracy, cost, pressure losses and compatibility with the fluid, Figliola and Beasley [2]. There are two suitable kinds of flow measurement methods for compressors, either the restriction flow passage meter (pressure differential meter) or insertion volume flow meter method. The most practical and widely used in compressors tests are the restriction flow passage meters. There are mainly three types of them: orifice plate, flow nozzle and venturi meters. Roduner et al. [11] compared the results of mass flow measurements from three types of probes (insertion volume flow meter) with a standard orifice as the reference for comparison and they stated, "Certainly, measuring the mass flow with a standard orifice is a more suitable method". The main drawback of these probes comes from highly mass flow measurement sensitivity to the flow angle. Regarding the ASME PTC 10, paragraph 4.8 [1], all the three mentioned types of the restriction flow passage meter are acceptable and can be installed either at the inlet or the exit piping of the compressor. ASME PTC 19.5 [12] represents a good reference for general instruction and detailed description of various primary flow measurement and their applications.

If the restriction flow passage meters are not practical to utilize, insertion volume flow meter can be used. For example, when nozzles or orifice plates cannot be installed owing to the configurations of the piping, velocity traverse technique can be used. Care must be taken where the measured pressure must represent the average value; otherwise an estimated correction factor or calibration is needed. Also, the prediction of the right flow angle is important to reduce the uncertainty in the reading.

To sum up, since the accuracy of the flow value is important, the restriction flow passage meter is more suitable than insertion volume flow meter method. To test the performance of a compressor, especially near the surge limit, disturbance sources near the compressor openings must be minimized. In the case of using the restriction flow passage meter, there has to be enough pipe length to overcome their disturbance on the flow. Also, the flow meter with larger measurement range and higher accuracy is preferred. So, the orifice plate is less appropriate since it has lower range, Boyce [13], compared to venturi and nozzle flow meters. Also, venturi meter when calibrated, is more accurate compared to flow nozzle and orifice plate, Doebelin [14]. The proposed test facility is an open loop with an electric motor drive unit and in this case the compressor piping system is relatively short to avoid occupying unnecessary large space. This means there is a need for a device, which has low-pressure loses and minimum disturbance. So, the best available choice is the venturi meter.

Two types of temperature probes are widely used in the literature, the thermocouples and resistance temperature detectors (RTD). Colantuoni and Colella [15] and Shirley [3] used thermocouples whereas Krain and Hoffmann [10] and Whitefield et al. [5] used the resistance temperature devices, Platinum (Pt) type. Nicholas and White [16] discussed the characteristics of both types of the probes. Pt-RTD is a non-expensive type that can operate over wide range of temperature, -60 ^{o}C to 960 ^{o}C, with accuracy that can reach to 0.001 Kelvin with good stability compared to thermocouples. The thermocouples are mainly used for less accurate, faster response measurement and higher range of temperature applications (over 1400^{o}C). On the other hand, the Pt-RTD is delicate while the thermocouple is robust, Walsh and Fletcher [17].

In summary, since the Pt-RTD has high accuracy and adequate range of temperature measurement, it is recommended to use it as a total temperature instrument as long as its time response is acceptable. If a thermocouple is accurate enough for the measurement and has better time response compared to a Pt-RTD, it is preferred. It is essential to notice that the indicated reading from the temperature probe is normally between the static and total temperature and in this case a recovery factor needs to be used to get the right total temperature, ASME PTC 10 [1] paragraph 5.4.4.1. The static temperature can be found from the total temperature equation. For general guidance on instruments for temperature measurement, see ASME PTC 19.3 [18].

Calculating the actual power from the total temperature measurement, where adiabatic condition is assumed, may not be practical in the proposed compressor test facility. The reason behind that is the possibility of having heat loses through the compressor structure. The power of the compressor can be calculated from either measurement of electrical input to the driving motor or the combination of torque and rotational speed. Both methods are used in literature and comply with ASME PTC 10 [1].

If the mechanical losses of the transmission system are low and can be accurately estimated, then the actual power can be calculated from the motor power. This method provides simplicity in constructing compressor mount and coupling, and is effective in many cases as long as it can satisfy the required uncertainty limits.

Using the other method, the impeller's power can be directly calculated from the product of measured torque and rotational speed. In the present setup it is recommended to use this technique. Torque can be measured through several techniques. A brief review of some torquemeters, with concentration on laser torquemeter, was done by Tullis [19]. A survey for some torque transduction methodologies for industrial applications (excluding laser technique) was discussed by Beihoff [20].

Several types of torque measurement devices are used in practical applications but each has advantages and disadvantages. For example, torque can be measured through a free structure around the compressor assembly that is attached to torque measurement device. In this case, proper alignment with the transmission axis is required. This method minimizes uncertainty from the measurement techniques but requires complex structure. Slip rings and telemetry torquemeter are other two types of torquemeters. An integrated-optical, non-contact torque measurement micro-system can be used to find the torque, Ebi et al. [21] but still under investigation. This type of torque meter is based on calculating torsional angle on the drive shaft. In general, the characteristics of a torquemeter type are varied from manufacturer to other. So, torquemeter selection depends on how it can comply with the testing requirements.

Table 1 summarizes the recommended instrumentation discussed in the research for testing centrifugal compressor performance together with recommended permissible fluctuation by ASME PTC 10 [1]. Figure 1 illustrates the recommended instrumentation setup for the experiment.

Table 1: Recommended instrumentation list for compressor performance test

Measured Parameter	**Recommended instrument type**	**Permissible Fluctuation, ASME PTC 10**
Rotational speed	Inductive	0.5 %
flow meter is at the inlet:		
T_1	Pt-RTD or thermocouple	0.5%
P_1&P_f	Static pressure tab	2%
flow meter is at the exit:		
P_{1f} and P_{2f} (ΔP)	Static pressure tab	2%
T_f	Pt-RTD or thermocouple	0.5%
P_{01} and P_{04}	Kiel probe	2%
T_{01}	Pt-RTD or thermocouple	0.5%
Torque	---------	1%

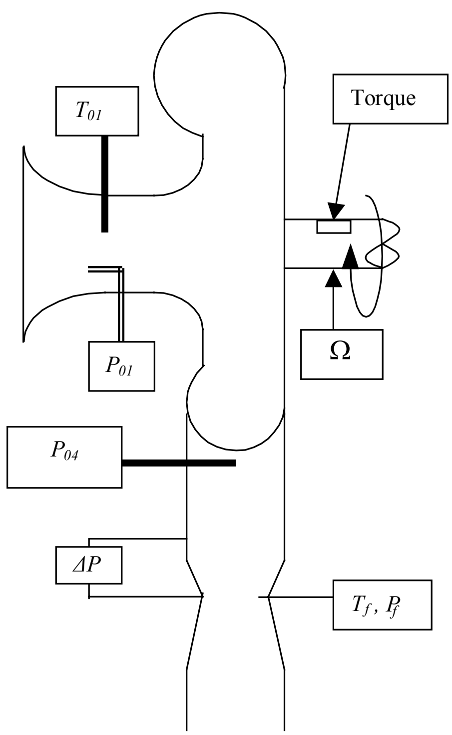

Fig. 1: Instrumentation setup on the centrifugal Compressor

INSTRUMENTATIONS UNCERTAINTY ANALYSIS

In the planning phase of experiments it is essential to perform uncertainty analysis. This will aid in selecting proper specifications for instruments, to improve the accuracy of the results. Many literatures are available that involves uncertainty analysis for centrifugal compressor testing. For example, Kurz and Brun [22] performed uncertainty analysis for field performance of gas turbine driven compressor. As a result a set of errors limits for pressure, temperature and flow rate were set to provide acceptable uncertainty limits for efficiency, power, fuel consumption, and head. It was also reported that to achieve similar accuracy as the factory test method, the field-testing should include metering coupling (torque measurement) for power measurement.

Poti and Rabe [23] used uncertainty analysis to verify compressor data accuracy. Their analysis was based on a single sample and did not include any error correlation. For micro scale radial compressor, Kang et al. [24] performed uncertainty analysis for the experimental results of efficiency and total pressure ratio. The compressor was driven by a cold flow turbine on a common shaft without torque measurement device. They reported uncertainty for the pressure ratio of $\pm$ 0.04 (for pressure ratio 1.22 – 1.31), and for the efficiency the uncertainty was ± 4 % (for efficiency 60% - 75%). The main sources of errors were the total pressure and temperature measurements. However, no correlation of error was considered. Coleman and Steele [25] emphasized on the importance of incuding the correlated error terms in the uncertainty analysis.

Bettocchi et al. [26] performed extensive uncertainty analysis for a multistage compressor test facility. Most of bias uncertainties included estimated installation and location errors, as well as instrumentation errors. This analysis did not include correlated bias errors effect. Furthermore, the experimental setup and data reduction equations for efficiency and pressure ratio are different from the present research.

The present research handles the uncertainty analysis for compressor performance testing with emphasis on potential correlated bias errors.

Analysis

General uncertainty analysis equations, precision, bias, and resultant uncertainty, can be found in Coleman and Steele [25] and ASME PTC 19.1 [27]. The Same procedure is applied in the present analysis, handling precision and bias error independently, keeping in mind the same confidence level. The analysis is for a single sample which is applied for the four parameters that determines the compressor performance map.

Precision Uncertainty

The general dimensionless precision uncertainty equation is:

$$\left(\frac{U_p}{R}\right)^2 = \sum_{i=1}^{J}\left(\frac{1}{R}\frac{\partial R}{\partial x_i}U_{p,x_i}\right)^2 \qquad (2)$$

Were, U_p is the precision uncertainty, R is the experimental result, U_{p,x_i} are the precision uncertainties in the measured variables x_i.

Bias Uncertainty

The general dimensionless bias uncertainty equations is:

$$\left(\frac{U_B}{R}\right)^2 = \sum_{i=1}^{J}\left(\frac{1}{R}\frac{\partial R}{\partial x_i}U_{B,x_i}\right)^2 + \sum_{i=1}^{J}\sum_{k=1}^{J}\frac{1}{R}\frac{\partial R}{\partial x_i}\frac{1}{R}\frac{\partial R}{\partial x_k}\zeta_{ik}U_{B,x_i}U_{B,x_k}(1-\delta_{ik}) \quad (3)$$

Here the subscript B refers to the bias error. The second term on the right hand side of Eq. 3 is the bias correlation term. The correlation coefficient of the bias limit, ζ_{ik}, has range from zero to one, depending on the degree of correlation.

Resultant Uncertainty

The resultant uncertainty is the square root summation of Eqs. 2 and 3. That is:

$$\left(\frac{U_R}{R}\right) = \sqrt{\left(\frac{U_p}{R}\right)^2 + \left(\frac{U_B}{R}\right)^2} \quad (4)$$

Uncertainty associated with rotational speed, mass flow rate, pressure ratio and isentropic efficiency is given below.

Uncertainty in the Rotational Speed

The rotational speed is directly measured through the instrument. The resultant uncertainty is the combination of bias and precision uncertainty in the measurement such that:

$$\left(\frac{U_\Omega}{\Omega}\right) = \sqrt{\left(\frac{U_{p,\Omega}}{\Omega}\right)^2 + \left(\frac{U_{B,\Omega}}{\Omega}\right)^2} \quad (5)$$

Both bias and precision errors are obtained through specifications of the device.

Uncertainty in the Mass Flow Rate

The mass flow rate is a function of the static density and flow rate. In the case of using differential pressure flow meter, mass flow rate can be defined as:

$$\dot{m} = A_{2f}C_dY\sqrt{\frac{P_{2f}}{\mathrm{R}T_{2f}}\frac{2\Delta P}{(1-(\frac{A_{2f}}{A_{1f}})^2)}} \quad (6)$$

The expansion factor, Y, can be calculated, assuming isentropic flow and using the same equation used by Benedict [28]:

$$Y = \left(\left(\frac{P_{2f}}{P_{2f}+\Delta P}\right)^{\frac{2}{\gamma}}\frac{\gamma}{\gamma-1}\frac{1-\left(\frac{P_{2f}}{P_{2f}+\Delta P}\right)^{\frac{\gamma-1}{\gamma}}}{1-\frac{P_{2f}}{P_{2f}+\Delta P}} \times \frac{1-\left(\frac{A_{2f}}{A_{1f}}\right)^2}{1-\left(\frac{A_{2f}}{A_{1f}}\right)^2\left(\frac{P_{2f}}{P_{2f}+\Delta P}\right)^{\frac{2}{\gamma}}}\right)^{0.5} \quad (7)$$

Taking into consideration only the measured values, which have significant uncertainty, the mass flow rate is a function of:

$$\dot{m} = f\,(C_d, P_{2f}, T_{2f}, \Delta P)$$

Flow area is not included in the measured variables since it has negligible error in the diameter (with the use of high precision micrometers, errors are in the order of 0.01 mm). Now substitute Eq. 6 into Eq. 2 to get single sample uncertainty:

$$\left(\frac{U_{p,\dot{m}}}{\dot{m}}\right)^2 = \left(\frac{U_{p,C_d}}{\dot{m}}\frac{\partial\dot{m}}{\partial C_d}\right)^2 + \left(\frac{U_{p,P_{2f}}}{\dot{m}}\frac{\partial\dot{m}}{\partial P_{2f}}\right)^2 + \left(\frac{U_{p,T_{2f}}}{\dot{m}}\frac{\partial\dot{m}}{\partial T_{2f}}\right)^2 + \left(\frac{U_{p,\Delta P}}{\dot{m}}\frac{\partial\dot{m}}{\partial\Delta P}\right)^2 \quad (8)$$

Partial derivatives for all variables, performed using Mathematica software, are presented in Appendix A. Similarly, as done in Eq. 8 the bias limit is:

$$\left(\frac{U_{B,\dot{m}}}{\dot{m}}\right)^2 = \left(\frac{U_{B,C_d}}{\dot{m}}\frac{\partial\dot{m}}{\partial C_d}\right)^2 + \left(\frac{U_{B,P_{2f}}}{\dot{m}}\frac{\partial\dot{m}}{\partial P_{2f}}\right)^2 + \left(\frac{U_{B,T_{2f}}}{\dot{m}}\frac{\partial\dot{m}}{\partial T_{2f}}\right)^2 + \left(\frac{U_{B,\Delta P}}{\dot{m}}\frac{\partial\dot{m}}{\partial\Delta P}\right)^2 + 2\zeta_{P_{2f},\Delta P}\frac{U_{B,P_{2f}}}{\dot{m}}\frac{\partial\dot{m}}{\partial P_{2f}}\frac{U_{B,\Delta P}}{\dot{m}}\frac{\partial\dot{m}}{\partial\Delta P} \quad (9)$$

The resultant uncertainty in mass flow rate is:

$$\left(\frac{U_{\dot{m}}}{\dot{m}}\right) = \sqrt{\left(\frac{U_{p,\dot{m}}}{\dot{m}}\right)^2 + \left(\frac{U_{B,\dot{m}}}{\dot{m}}\right)^2} \quad (10)$$

Uncertainty in the Pressure Ratio

Performing the uncertainty analysis for the pressure ratio and substituting for the partial derivatives leads to:

$$\left(\frac{U_{p,Pr}}{Pr}\right)^2 = \left(\frac{U_{p,P_{01}}}{Pr}\frac{\partial Pr}{\partial P_{01}}\right)^2 + \left(\frac{U_{p,P_{04}}}{Pr}\frac{\partial Pr}{\partial P_{04}}\right)^2 \quad (11)$$

and

$$\left(\frac{U_{B,Pr}}{Pr}\right)^2 = \left(\frac{U_{B,P_{01}}}{Pr}\frac{\partial Pr}{\partial P_{01}}\right)^2 + \left(\frac{U_{B,P_{04}}}{Pr}\frac{\partial Pr}{\partial P_{04}}\right)^2 + 2\zeta_{P_{01},P_{04}}\left(\frac{U_{B,P_{01}}}{Pr}\frac{\partial Pr}{\partial P_{01}}\frac{U_{B,P_{04}}}{Pr}\frac{\partial Pr}{\partial P_{04}}\right) \quad (12)$$

The resultant uncertainty in the pressure ratio is:

$$\left(\frac{U_{Pr}}{Pr}\right) = \sqrt{\left(\frac{U_{p,Pr}}{Pr}\right)^2 + \left(\frac{U_{B,Pr}}{Pr}\right)^2} \quad (13)$$

Uncertainty in the Isentropic Efficiency

The equation of the isentropic efficiency is:

$$\eta = \frac{\left(A_{2f} C_d Y \sqrt{\frac{P_{2f}}{\mathrm{R}T_{2f}} \frac{2\Delta P}{(1-(\frac{A_{2f}}{A_{1f}})^2)}}\, C_p T_{01}\left((\frac{P_{04}}{P_{01}})^{\frac{(\gamma-1)}{\gamma}} - 1\right)\right)}{\tau\Omega} \quad (14)$$

Taking into consideration there is no uncertainty in the flow meter area; the efficiency is a function of

$$\eta = \eta(C_d, P_{2f}, T_{2f}, \Delta P, T_{01}, P_{01}, P_{04}, \tau, \Omega)$$

Now the precision error of the efficiency is:

$$\left(\frac{U_{p,\eta}}{\eta}\right)^2 = \left(\frac{U_{p,C_d}}{\eta}\frac{\partial\eta}{\partial C_d}\right)^2 + \left(\frac{U_{p,P_{2f}}}{\eta}\frac{\partial\eta}{\partial P_{2f}}\right)^2 + \left(\frac{U_{p,T_{2f}}}{\eta}\frac{\partial\eta}{\partial T_{2f}}\right)^2 + \left(\frac{U_{p,\Delta P}}{\eta}\frac{\partial\eta}{\partial \Delta P}\right)^2 + \left(\frac{U_{p,T_{01}}}{\eta}\frac{\partial\eta}{\partial T_{01}}\right)^2 + \left(\frac{U_{p,P_{01}}}{\eta}\frac{\partial\eta}{\partial P_{01}}\right)^2 + \left(\frac{U_{p,P_{04}}}{\eta}\frac{\partial\eta}{\partial P_{04}}\right)^2 + \left(\frac{U_{p,\tau}}{\eta}\frac{\partial\eta}{\partial \tau}\right)^2 + \left(\frac{U_{p,\Omega}}{\eta}\frac{\partial\eta}{\partial \Omega}\right)^2 \quad (15)$$

and the bias error is:

$$\begin{aligned}\left(\frac{U_{B,\eta}}{\eta}\right)^2 &= \left(\frac{U_{B,C_d}}{\eta}\frac{\partial\eta}{\partial C_d}\right)^2 + \left(\frac{U_{B,P_{2f}}}{\eta}\frac{\partial\eta}{\partial P_{2f}}\right)^2 + \left(\frac{U_{B,T_{2f}}}{\eta}\frac{\partial\eta}{\partial T_{2f}}\right)^2 \\ &+ \left(\frac{U_{B,\Delta P}}{\eta}\frac{\partial\eta}{\partial \Delta P}\right)^2 + \left(\frac{U_{B,T_{01}}}{\eta}\frac{\partial\eta}{\partial T_{01}}\right)^2 + \left(\frac{U_{B,P_{01}}}{\eta}\frac{\partial\eta}{\partial P_{01}}\right)^2 \\ &+ \left(\frac{U_{B,P_{04}}}{\eta}\frac{\partial\eta}{\partial P_{04}}\right)^2 + \left(\frac{U_{B,\tau}}{\eta}\frac{\partial\eta}{\partial \tau}\right)^2 + \left(\frac{U_{B,\Omega}}{\eta}\frac{\partial\eta}{\partial \Omega}\right)^2 \\ &+ 2\zeta_{P_{01},P_{04}}\left[\frac{U_{B,P_{01}}}{\eta}\frac{\partial\eta}{\partial P_{01}}\frac{U_{B,P_{04}}}{\eta}\frac{\partial\eta}{\partial P_{04}}\right] \\ &+ 2\zeta_{P_{01},P_{2f}}\left[\frac{U_{B,P_{01}}}{\eta}\frac{\partial\eta}{\partial P_{01}}\frac{U_{B,P_{2f}}}{\eta}\frac{\partial\eta}{\partial P_{2f}}\right] \\ &+ 2\zeta_{P_{01},\Delta P}\left[\frac{U_{B,P_{01}}}{\eta}\frac{\partial\eta}{\partial P_{01}}\frac{U_{B,\Delta P}}{\eta}\frac{\partial\eta}{\partial \Delta P}\right] \\ &+ 2\zeta_{P_{04},P_{2f}}\left[\frac{U_{B,P_{04}}}{\eta}\frac{\partial\eta}{\partial P_{04}}\frac{U_{B,P_{2f}}}{\eta}\frac{\partial\eta}{\partial P_{2f}}\right] \\ &+ 2\zeta_{P_{04},\Delta P}\left[\frac{U_{B,P_{04}}}{\eta}\frac{\partial\eta}{\partial P_{04}}\frac{U_{B,\Delta P}}{\eta}\frac{\partial\eta}{\partial \Delta P}\right] \\ &+ 2\zeta_{P_{2f},\Delta P}\left[\frac{U_{B,P_{2f}}}{\eta}\frac{\partial\eta}{\partial P_{2f}}\frac{U_{B,\Delta P}}{\eta}\frac{\partial\eta}{\partial \Delta P}\right] \\ &+ 2\zeta_{T_{01},T_{2f}}\left[\frac{U_{B,T_{01}}}{\eta}\frac{\partial\eta}{\partial T_{01}}\frac{U_{B,T_{2f}}}{\eta}\frac{\partial\eta}{\partial T_{2f}}\right]\end{aligned} \quad (16)$$

The resultant uncertainty in the isentropic efficiency is:

$$\left(\frac{U_{\eta}}{\eta}\right) = \sqrt{\left(\frac{U_{p,\eta}}{\eta}\right)^2 + \left(\frac{U_{B,\eta}}{\eta}\right)^2} \quad (17)$$

Based on the results of partial derivatives (Appendix A) and Eq. 16, it is recommended to have correlated bias for the parameters:

- T_{01} with T_{2f} and
- P_{01} with P_{04} or
- P_{01} with P_{2f} or
- P_{01} with ΔP

The bias correlations have to be investigated to select one or more for optimizing the resultant uncertainty for a specific set of conditions. An example illustrating the uncertainty analysis procedure discussed in the paper follows.

Example

A typical example can be implemented to illustrate the effects of major sources of uncertainty on the reported performance of centrifugal compressor. Also effect of bias correlations on the uncertainty of mass flow rate, total pressure ratio, and isentropic efficiency will be investigated. The results are based on an example setup shown in Fig. 1, for a centrifugal compressor. Table 2 is a typical list of performance test results for measured variables of the compressor.

Table 2: Typical performance test results of centrifugal compressor

Measured Parameter	Value
P_{01}	100 kPa
P_{04}	550 kPa
T_{01}	20 °C (293.15 K)
T_{2f}	180 °C (453.15 K)
P_{2f}	490 kPa
ΔP	20 Kpa
C_d	0.99
Ω	5,236 rad/s (50,000 rpm)
τ	76.46 N.M

Additional information is:

- $A_{2f} = 0.004479$ m^2
- $(A_{1f} / A_{2f}) = 4$
- R = 287 J/kg.K
- $\gamma = 1.4$
- $C_p = 1.011$ kJ/kg.K

The performance results, based on the data reduction equations above, are:

- Mass flow rate = 1.74 kg/s
- Total Pressure ratio = 5.5
- Isentropic efficiency = 81 %

To perform uncertainty analysis for the results typical errors of instrumentation are used as shown in Table 3. Uncertainties assigned per ASME PTC 10 [1] requirements are based on half of the tolerable fluctuations in measurements.

Table 3: Typical errors in measurements and ASME PTC 10 fluctuation limit requirements

Parameter	$U_{B,x}$	$U_{P,x}$	$U_{R,x}$	ASME PCT 10, $U_{R,x}$
P_{01}	0.15 kPa	0.2 kPa	0.25 kPa	1 kPa
P_{04}, P_{2f}	0.75 kPa	1.0 kPa	1.25 kPa	5 kPa
T_{01}, T_{2f}	0.1 K	0.4 K	0.41 K	0.73 K, 1.1 K
ΔP	0.03 kPa	0.04 kPa	0.05 kPa	-
C_d *	-	-	0.005	-
Ω	-	-	10 rad/s	13.1 rad/s
τ	-	-	0.2 N.m	0.36 N.m

* Calibrated

Uncertainties for the results are calculated based on single sample. No correlation terms were considered on the results of Table 4. Following ASME PTC 10 [1] as minimum requirement would provide 0.74% of uncertainty in mass flow rate (single sample), which is a slightly higher when typical instrumentation specifications are used, 0.54%. However, in many applications 0.742% error in mass flow rate is considered acceptable. On the other hand, uncertainty in (P_{04}/P_{01}) and efficiency are considered relatively high, for instruments complying with minimum requirements of ASME PTC 10 [1]. Using instrumentation with typical specification, Table 3, would provide much lower uncertainty for (P_{04}/P_{01}) and efficiency, which is acceptable for most applications.

Table 4: Resultant single sample uncertainty (no error correlation)

Result	Typical (U_R / R) ×100 (abs)	ASME PTC 10 (U_R / R) ×100 (abs.)
Mass flow rate	0.54 % (0.0095 kg/s)	0.74 % (0.0131 kg/s)
Total Pressure ratio	0.35 % (0.0177)	1.4 % (0.0707)
Isentropic efficiency	0.7 % (0.57%)	1.46 % (1.2 %)

Further improvement to uncertainty in the results can be achieved through repetition of performance tests. This will reduce the precision components of the uncertainty, ASME PTC 19.1 [27]. It should be clear that uncertainties listed in table 4 include only instrumentation errors effect. Installation and location errors may add to the uncertainties in the results, which is not within the scope of present research.

In mass flow rate measurements, the major source of error is due to uncertainty in discharge coefficient (C_d), as shown in Fig. 2 in which uncertainties of other measuring parameters are kept typical as shown in Table 3. The uncertainty of mass flow rate is less sensitive to errors in the pressure measurements. Uncertainty in the efficiency is also greatly affected by uncertainty in the discharge coefficient; see Fig. 3. So more attention should be given to the selection and installation of the venturi-meter as compared to pressure and temperature measuring devices.

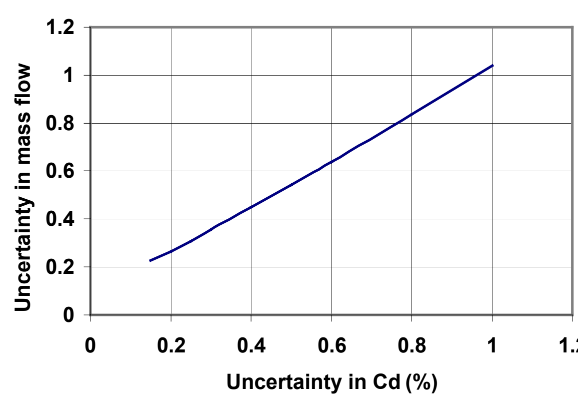

Fig. 2: Discharge coefficient error effect on mass flow rate uncertainty

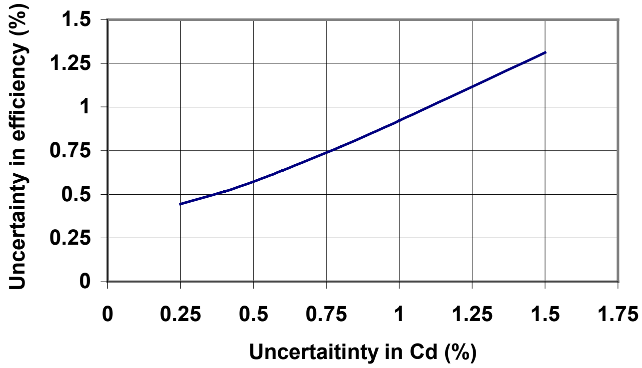

Fig. 3: Discharge coefficient error effect on efficiency uncertainty

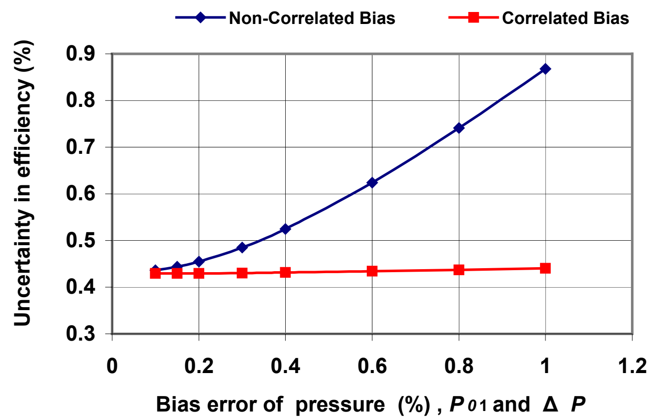

Fig. 4: Effect of bias correlation of ΔP and P_{01} on the efficiency uncertainty

It is therefore recommended to use calibrated venturi meters. In this case, we have two options to handle the calibrated data. The first option, as used in the present analysis, is to assume true value of Y, Eq. 7**,** and have the discharge coefficient calibrated. The other option is to have the product ($Y\ C_d$) calibrated and treated as a single independent variable, as illustrated by Benedict [28]. In both options ($\Delta P/P_{2f}$) has to be considered as independent variable in the calibration procedure.

Further improvements can be made on efficiency measurements by correlating bias limit of P_{01} with other pressure measurements bias limits. Correlation of bias errors may occur when the two variables are measured using the same transducer or when two transducers are calibrated against the same standard, Coleman and Steele [25]. Typical instance where the variables are measured with the same transducer is when a scani-valve is used to measured pressure at different locations using one pressure transducer. The improvement in the resultant uncertainty is illustrated in Figs. 4 and 5. Precision errors are kept at typical values with variation in the correlated bias only. It is clear that correlation of pressure bias with P_{01} lowers the uncertainty in the efficiency, as well as making it insensitive to magnitude of the correlated bias limits. The implementation of both bias correlations (P_{01} with P_{04} and P_{01} with ΔP) would result in higher uncertainty due to the automatic bias correlation between P_{04} and ΔP. In general, the benefit of bias correlation of pressure measurement is obvious at relatively high bias limits (inexpensive pressure instrument). Temperature bias correlation, T_{01} with T_{2f}, has negligible effect in the uncertainty of the efficiency. This is due to the fact that the relative bias limits for temperatures are practically low. For the present example, the uncertainty in the measured efficiency of compressor is greatly affected by the uncertainty associated with torque measurement and the uncertainty in the discharge coefficient of the flow meter.

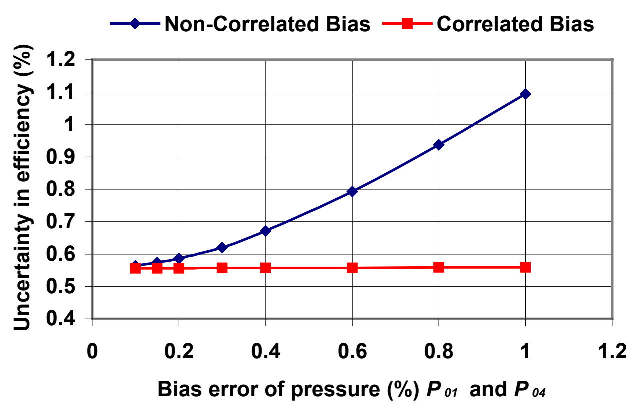

Fig. 5: Effect of bias correlation of P_{01} and P_{04} on the efficiency uncertainty

CONCLUSION

In the present study, the selection of appropriate instrumentation for testing of small-size high-speed centrifugal compressors was considered. The selection was based on extensive literature survey, including ASME PTC 10 and 19.1. Venturi meter was selected for flow rate, Kiel probes for total pressure and Pt-RTD or thermocouples for temperature measurements. The detailed example uncertainty analysis was performed for mass flow rate, total pressure ratio and isentropic efficiency. Using instrumentation with typical errors proves to provide reasonable uncertainties in the performance test results. Selecting instrumentation based on satisfying ASME PTC 10 criteria only, may lead to unacceptable errors in the perfomance results. The major sources of instrumentation uncertainty for the present test facility are the errors associated with the discharge coefficient of the flow meter, the torque measurement and, to less extent, the total pressure measurements. Correlation in bias error of inlet total pressure with one of the other pressure bias errors is proved to lower the uncertainty in the efficiency results, specifically at larger bias errors. However, this would be useful especially when inexpensive pressure transducers (low bias error) are used. Combing more than one correlation of pressure bias error may increase the resulting uncertainty. Correlation of temperature bias errors proves to have negligible effect on uncertainty in efficiency results. It is strongly recommenced to study correlated uncertainty in the design phase of compressor performance test.

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APPENDIX A

Partial Derivative of the variables used to in the uncertainty analysis
(Performed using Mathematica software)

Parameter	Derivative result
$\frac{\partial \dot{m}}{\partial c_d}$	$\sqrt{2} A_{2f} Y \sqrt{Z}$
$\frac{\partial \dot{m}}{\partial P_{2f}}$ $^{\spadesuit}$	$\frac{\Delta P A_{2f} C_d Y}{\sqrt{2} R \left(1-\frac{A_{2f}^2}{A_{1f}^2}\right)\sqrt{Z}\, T_{2f}}$
$\frac{\partial \dot{m}}{\partial T_{2f}}$	$-\frac{\Delta P A_{2f} C_d Y P_{2f}}{\sqrt{2} R \left(1-\frac{A_{2f}^2}{A_{1f}^2}\right)\sqrt{Z}\, T_{2f}^2}$
$\frac{\partial \dot{m}}{\partial \Delta P}$ $^{\spadesuit}$	$\frac{A_{2f} C_d Y P_{2f}}{\sqrt{2} R \left(1-\frac{A_{2f}^2}{A_{1f}^2}\right)\sqrt{Z}\, T_{2f}}$
$\frac{\partial Pr}{\partial P_{01}}$	$-\frac{P_{04}}{P_{01}^2}$
$\frac{\partial Pr}{\partial P_{04}}$	$\frac{1}{P_{01}}$
$\frac{\partial \eta}{\partial c_d}$	$\frac{\sqrt{2} A_{2f} Y C_p \left(-1+\left(\frac{P_{04}}{P_{01}}\right)^{\frac{-1+\gamma}{\gamma}}\right) T_{01}\sqrt{Z}}{\tau\Omega}$
$\frac{\partial \eta}{\partial P_{2f}}$ $^{\spadesuit}$	$\frac{\Delta P A_{2f} C_d Y C_p (-1+(\frac{P_{04}}{P_{01}})^{\frac{-1+\gamma}{\gamma}}) T_{01}}{\sqrt{2} R \tau \Omega (1-\frac{A_{2f}^2}{A_{1f}^2})\sqrt{Z}\, T_{2f}}$
$\frac{\partial \eta}{\partial T_{2f}}$	$-\frac{\Delta P A_{2f} C_d Y C_p (-1+(\frac{P_{04}}{P_{01}})^{\frac{-1+\gamma}{\gamma}}) P_{2f} T_{01}}{\sqrt{2} R \tau \Omega (1-\frac{A_{2f}^2}{A_{1f}^2})\sqrt{Z}\, T_{2f}^2}$

$\frac{\partial \eta}{\partial \Delta P}$ ♠	$\frac{A_{2f} C_d Y C_p (-1+(\frac{P_{04}}{P_{01}})^{\frac{-1+\gamma}{\gamma}}) P_{2f} T_{01}}{\sqrt{2} R \tau \Omega (1-\frac{A_{2f}^2}{A_{1f}^2}) \sqrt{Z}\, T_{2f}}$
$\frac{\partial \eta}{\partial T_{01}}$	$\frac{\sqrt{2} A_{2f} C_d Y C_p (-1+(\frac{P_{04}}{P_{01}})^{\frac{-1+\gamma}{\gamma}}) \sqrt{Z}}{\tau \Omega}$
$\frac{\partial \eta}{\partial P_{01}}$	$-\frac{\sqrt{2}(-1+\gamma) A_{2f} C_d Y C_p P_{04} (\frac{P_{04}}{P_{01}})^{-1+\frac{-1+\gamma}{\gamma}} T_{01} \sqrt{Z}}{\gamma \tau \Omega P_{01}^2}$
$\frac{\partial \eta}{\partial P_{04}}$	$\frac{\sqrt{2}(-1+\gamma) A_{2f} C_d Y C_p (\frac{P_{04}}{P_{01}})^{-1+\frac{-1+\gamma}{\gamma}} T_{01} \sqrt{Z}}{\gamma \tau \Omega P_{01}}$
$\frac{\partial \eta}{\partial \tau}$	$-\frac{\sqrt{2} A_{2f} C_d Y C_p (-1+(\frac{P_{04}}{P_{01}})^{\frac{-1+\gamma}{\gamma}}) T_{01} \sqrt{Z}}{\tau^2 \Omega}$
$\frac{\partial \eta}{\partial \Omega}$	$-\frac{\sqrt{2} A_{2f} C_d Y C_p (-1+(\frac{P_{04}}{P_{01}})^{\frac{-1+\gamma}{\gamma}}) T_{01} \sqrt{Z}}{\tau \Omega^2}$

♠ Approximation to actual derivative values and may be used as actual. Error in this can be proven mathematically as negligible.