Proceedings of IMECE2008
2008 ASME International Mechanical Engineering Congress and Exposition
October 31-November 6, 2008, Boston, Massachusetts, USA

IMECE2008-68095

ON THE MODELING OF A RECTANGULAR SYNTHETIC JET

Gopi Krishnan
University of Colorado at Boulder
Boulder, CO, USA

Kamran Mohseni
University of Colorado at Boulder
Boulder, CO, USA

ABSTRACT

The flow field of a rectangular synthetic jet is studied in this paper. It is known that synthetic jets exhibit similarities to continuous turbulent jets in that the far field velocity profile of synthetic jets displays self-similar behavior as well. In this paper we systematically model the a rectangular synthetic jet by applying the argument that synthetic jets can be described by the same equations used to describe continuous laminar jets, with the replacement of the kinematic viscosity of laminar flow, with the virtual kinematic viscosity obtained from the experiments on a synthetic jet. The virtual kinematic viscosity is obtained through experimental measurements of the time average velocity profiles using hot wire anemometry. The virtual kinematic viscosity of the synthetic jet under study was found to exceed that of a turbulent jet of equivalent momentum. The inherent periodic excitation of the synthetic is attributed to the increased virtual kinematic viscosity, which results in the faster spreading and increased entrainment observed in experiments. It is observed that the variation in the centerline velocity and jet width with axial distance, for various actuator stroke lengths collapse onto single curves when scaled appropriately.

INTRODUCTION

A synthetic jet is a type of pulsatile jet that results from the formation and interaction of vortex rings or pairs. The term 'synthetic' refers to the concept that the jet is synthesized from the surrounding medium unlike continuous jets. One method of generating a synthetic jet employs a cavity-diaphragm setup and is the method employed in this experiment. The setup consists of a sealed cavity with a flexible vibrating diaphragm on one end, with an orifice on the other. Methods for actuating the diaphragm include electrostatic, electromagnetic, mechanical, and piezoelectric mechanisms. In this investigation a piezoelectric disc serves as the vibrating membrane in the actuator. As in Fig. 1, the synthesis is comprised of a cycle with two strokes; suction and an expulsion. During the suction stroke the diaphragm moves away from the orifice, increasing the volume of the cavity and subsequently decreasing the pressure within, which results in the entrainment of low momentum fluid into the cavity.

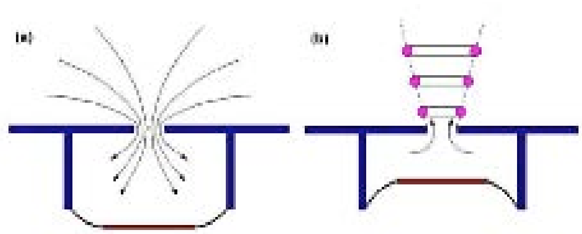

Figure 1. A schematic of synthetic jet operation

During the expulsion stroke, the diaphragm moves towards the orifice, resulting in the ejection of fluid though the orifice. At the edge of the orifice the shear layer formed, rolls up to form a vortex ring that travels downstream due to its self induced velocity. In this fashion a series of the suction-ejection strokes results in the formation of a train of vortex rings moving away from the orifice. The rings coalesce and interact to form a jet that may be laminar or turbulent in nature. A salient point is that over a cycle of operation while there is no net mass flux across the orifice, there is a net momentum flux, and to this end a synthetic jet is also called a Zero Net Mass Flux (ZNMF) jet [1]. The ramification of this fact is that no fluid reservoir or complex piping is necessary in a synthetic jet, which may keep size and weight of the flow actuator devices down. Additionally, the jets scale with the dimension of the orifice, thereby allowing for the production of a wide range of jet scales. These benefits make the actuators attractive in a number of applications that include active flow control [2,3,4,5], electronic cooling [6], fluid mixing [7] and, micro propulsion [8]. A recent review of synthetic jet evolution may be found in Glezer and Amitay [1] while Zhong et al. [9,10] provide a report on experiments and modeling of synthetic jet actuators applied towards full-scale flight conditions.

Due to the promise of synthetic jets, research interest in these actuators has grown considerably in the last few years. In order to design actuators or apply them to active flow control applications, a model of the evolution of the external jet at least in the average sense is required. There are several theoretical and numerical [11,12] modeling approaches in the literature. In this paper however we focus on a semi-analytical modeling approach, which circumvents the computational cost of a detailed numerical analysis, while at the same time captures the essential physics required for further optimization studies. This study is part of a research project focused on synthetic jets at the department of Aerospace Science at the University of Colorado, Boulder. This includes design, fabrication, simulation, and experimentation on synthetic jets in air at the mesoscale [13,14] and water at the macro scale [15,16]. Applications include active aerodynamic flow control, biologically inspired thrusters in underwater vehicles, as well as future work in propulsion for micro vehicles within the human body.

Smith and Glezer [17] were first to examine synthetic jets of the diaphragm–cavity type where hot-wire anemometry was used to study the flow field of a 2D planar synthetic jet. They showed that synthetic jets bore similarities to conventional steady 2D turbulent jets, in that both the time average velocity and fluctuating components collapsed into self-similar profiles when normalized. While synthetic jets in contrast, became fully developed quicker, the centerline velocity decayed more rapidly and that they spread faster than an equivalent plane turbulent jet. Mallinson et al [18] investigated several axisymmetric synthetic jets configurations in air, both numerically and experimentally. They found that the axisymmetric synthetic jet behaved similar to a steady round turbulent jet as well. The mean velocity in the far field beyond ten orifice diameters was show to be self similar in nature, while the mass flow rate was found to be larger than a steady turbulent jet, due to the greater entrainment of ambient fluid in the vortices seen in synthetic jets. Cater and Soria [19] studied round synthetic jets in water using particle image velocimetry and reaffirmed the larger spreading rate and more rapid decay when compared to an equivalent axisymmetric continuous turbulent jet and this was attributed to the near-field structural differences of the flow. Smith and Swift [20] experimentally compared 2D continuous and synthetic jets in air, of equivalent Reynolds numbers. While there were differences in the near field, the far field flows resembled each in their self-similar nature, while the synthetic jet was wider and slower. More recently Shuster and Smith [21], as part of their study investigated the mean flow fields of round synthetic jets in water while varying the stroke length and Reynolds number. They showed that the far field did exhibit self-similar behavior, however the decay and spreading rate were dependent of the two aforementioned scaling parameters. Amitay and Canelle [22] studied the evolution of a finite span plane synthetic jet and the dependence of Reynolds number, stroke length and Strouhal number. On account to three dimensional effects due to edges it was shown that a 3D model was desired for accurate modeling.

In summary, it has been shown that synthetic jets exhibit similarities to turbulent jets in that the far fields of synthetic jets display self-similar behavior as well. However synthetic jets are observed to exhibit a greater spreading, due to increased entrainment in the near field. In modeling the self-similar behavior past workers have ‘fit’ the analytical velocity profile of a turbulent jet to the experimental data to ascertain an empirical constant, without ascribing a physical meaning to this constant. In this paper we attempt to systematically model the spherical jet and obtain the empirical constant from the spreading rate of the jet and ascribe a physical meaning to it.

This paper is outlined as follows; an overview of the modeling approach is laid out, and then the planar continuous jet model is outlined. The experimental setup is then described after which the results for time average velocity profiles, and velocity decay and spreading rate are presented. The variation in the centerline velocity and jet width with axial distance, for various actuator stroke lengths is also shown. The results are then discussed along side the synthetic jet analytical models.

NOMENCLATURE

h=diameter of orifice
u=axial velocity component
v=transverse velocity component
U=centerline axial velocity
K=kinematic momentum flux
x=axial coordinate
y=transverse coordinate
z=longitudinal coordinate
η=self similar variable
ρ=density of fluid
ε_v=virtual kinematic viscosity
$b_{1/2}$=half width
τ_τ= turbulent shear stress

SYNTHETIC JET MODEL

We now briefly explain our modeling approach. Bickley [23] and Schlicting [24] found similarity solutions of the boundary layer equations of a laminar plane jet emanating from a narrow slit into a surrounding fluid. Reichardt [25] extended this analysis to turbulent plane jets based on Prandtl’s second hypothesis, which indicates that the turbulent shear stress can be modeled by an eddy viscosity as

$$\tau_t = \rho \varepsilon_v \frac{\partial u}{\partial y}. \quad (1)$$

As a result the turbulent jet could be described using the same differential equations as those used to describe a laminar jet with the replacement on the kinematic viscosity of laminar flow, ν with a virtual kinematic viscosity of the turbulent flow ε_v, which was obtained from the spreading rate of the turbulent jet.

In replacing the kinematic viscosity of the laminar solution with the eddy viscosity of turbulent jet, the velocity distribution was in good agreement with experiments. In this study we extend the above idea to synthetic jets. Just as the equations describing turbulent jets are identical to those describing laminar jets with the replacement of an empirical constant εo, we suggest that a synthetic jet can be described by equations for a laminar jets, with the replacement of the kinematic viscosity of laminar flow, with the virtual kinematic viscosity obtained from the experiments for a synthetic jet.

The classic self-similar solution for a plane jet yields the following expressions for the axial and transverse velocity components

$$u = \frac{\sqrt{3}}{2}\sqrt{\frac{K\sigma}{x}}\left(1-\tanh^2\eta\right), \tag{2}$$

$$v = \frac{\sqrt{3}}{4}\sqrt{\frac{K}{x\sigma}}\left\{2\eta\left(1-\tanh^2\eta\right)-\tanh\eta\right\}, \tag{3}$$

where the similarity variable η is

$$\eta = \sigma\frac{y}{x}, \tag{4}$$

where σ is an empirically determined constant and is obtained from the spreading rate of the jet as follows. At the centerline the axial velocity is

$$U = \frac{\sqrt{3}}{2}\sqrt{\frac{K\sigma}{x}}, \tag{5}$$

where it is seen that the axial centerline velocity decays as $x^{-1/2}$. The width of the jet at a particular axial station may be characterized by a half width ($b_{1/2}$), defined as the transverse distance from the center at which the axial velocity drops to half the centerline velocity. At the half width, the axial velocity may be expressed as

$$u_{1/2} = \frac{\sqrt{3}}{2}\sqrt{\frac{K\sigma}{x}}\left(1-\tanh^2\eta_{1/2}\right). \tag{6}$$

Equating eq. 6 to half of eq. 5, we obtain

$$1-\tanh^2\eta_{1/2} = \frac{1}{2}, \tag{7}$$

from which

$$\eta_{1/2} = \tanh^{-1}\sqrt{\frac{1}{2}} \approx 0.88 \tag{8}$$

From eq. 4

$$\sigma = \frac{0.88}{b_{1/2}/x}. \tag{9}$$

In this fashion the empirical constant σ is obtained from the spreading rate. The virtual viscosity is assumed to vary as

$$\varepsilon_v = \frac{Ux}{4\sigma^2}, \tag{10}$$

and using eq. 5 we see that

$$\varepsilon_v \propto \frac{1}{\sigma^{3/2}} \propto \left(b_{1/2}/x\right)^{3/2}, \tag{11}$$

indicating that an increase in spreading rate implies an increase in the virtual viscosity.

EXPERIMENTAL SETUP

The experimental setup in Fig. 2 consists of a synthetic jet actuator, computer controlled stages, and a hot wire probe, all of which are placed in a clear Plexiglas enclosure.

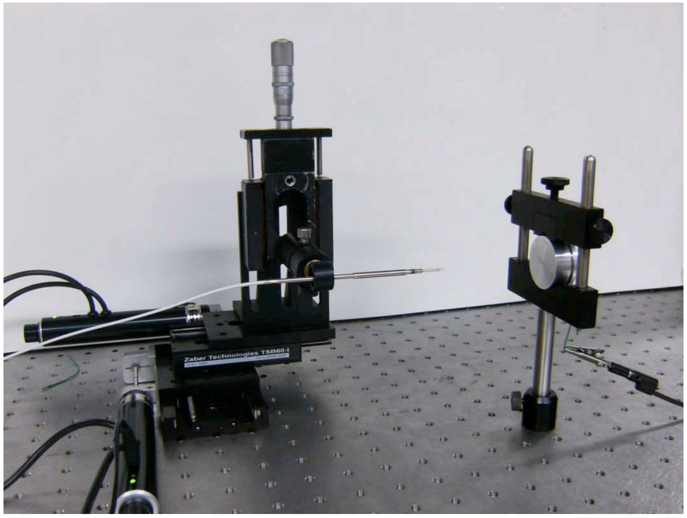

Figure 2. Experimental Setup

The actuator shown in Fig. 3 was a piezoelectric driven apparatus that consists of a circular piezo membrane, and an aluminum housing. The piezo element is sandwiched between the two circular aluminum elements, which when screwed together form a cavity with a rectangular orifice on one end and a flexible membrane on the other. The width of the rectangular slot orifice, h is 1 mm, and width, w is 38 mm. The piezo element is connected to a power amplifier, which receives a signal from the function generator. A sinusoidal signal was used in these experiments where the input voltage amplitude and frequency could be controlled. The piezo was operated at a frequency of 540 Hz, with the voltage varied to change the stroke length. The frequency was selected to maximize the velocity at the exit.

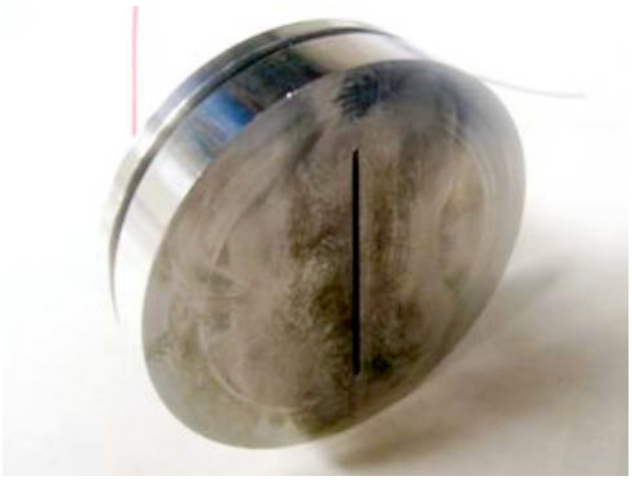

Figure 3. Rectangular Slot Actuator

Velocity measurements are made using a single normal constant temperature anemometer (CTA) hotwire probe. The probe wire was 1.25 mm long and 5 µm in diameter. The probe axis was positioned parallel to the jet axis with the wire axis being vertical. The hotwire is affixed to two computer control stages that are capable of traversing in a horizontal plane. A stepper motor with a spatial resolution of 1 micron drives the stages. The transverse and longitudinal velocity profiles were measured at 5 axial locations downstream of the actuator. For the transverse and longitudinal profiles, at each axial location the velocity was measured for 10 and 30 slot widths respectively, on either side of the jet axis in intervals of 1 h. With the hot wire sampling the data at 50 KHz, the velocity at a particular location was sampled for 5 seconds to obtain a time average value. The uncertainty in the mean measurements is estimated to be 2%. The hot wire calibration process used is much like the iterative procedure illustrated in Johnstone et al. [26].

RESULTS AND DISCUSSION

The time averaged velocity profiles at different axial location were acquired and analyzed. We show that the time average transverse axial velocity profiles of the synthetic jet are similar to continuous turbulent jets in that the velocity profiles collapse into a single profile. We next show that the analytical description models the synthetic jet well. Figure 4 – 7 are results for actuator operating parameters of f = 540 Hz, and V = 25 V, corresponding to a Reynolds number and Stroke ratio of 334 and 9.3 respectively, as calculated from a slug model.

In Fig. 4, the transverse axial velocity profiles at five different axial locations are shown. It has been shown that within a few diameters of the orifice the vortex pairs are not longer coherent and have coalesced to form a jet. As is seen the centerline velocity decreases as the jet starts to spread, as it moves progressively away from the orifice. As with a turbulent jet, a synthetic jet is not immediately self-similar and develops within a few orifice diameters downstream.

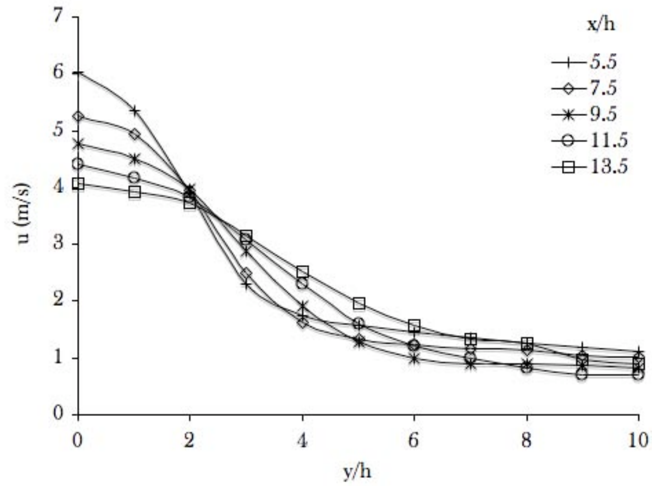

Figure 4. Transverse Axial Velocity Profiles

Figure 5 shows the longitudinal velocity profiles at different axial locations. A peak is observed whose magnitude becomes smaller, and position draws closer to the jet axis as the jet progresses axially. This feature is a manifestation of the edge effects due to the finite width of the slot.

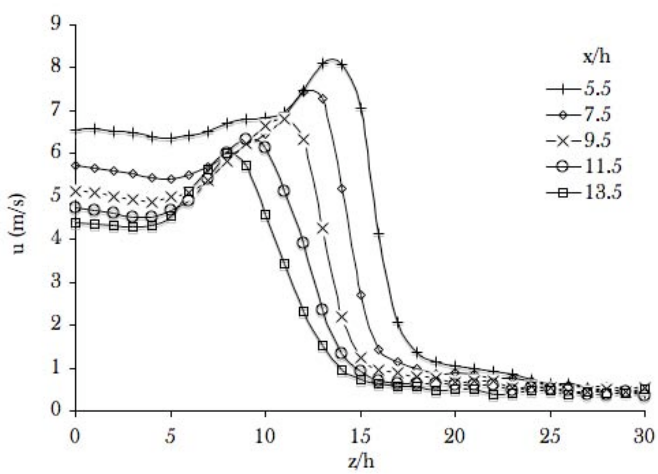

Figure 5. Longitudinal Axial Velocity Profile

The centerline axial velocity of the synthetic jet decreases as $x^{-1/2}$ and Fig. 6 shows this dependence. This is similar to the decay in a turbulent jet, and is consistent with others observations [18, 19]. As with a turbulent jet the self-similar region appears to grow from a virtual origin and may be considered the imaginary point source of the jet. The location of the virtual origin appears to be dependent on the experimental setup and may exist either upstream or downstream of the orifice. The location of the virtual origin is obtained from the extrapolation of the centerline velocity graph. In this case it is found to found to be 1 mm upstream of the orifice.

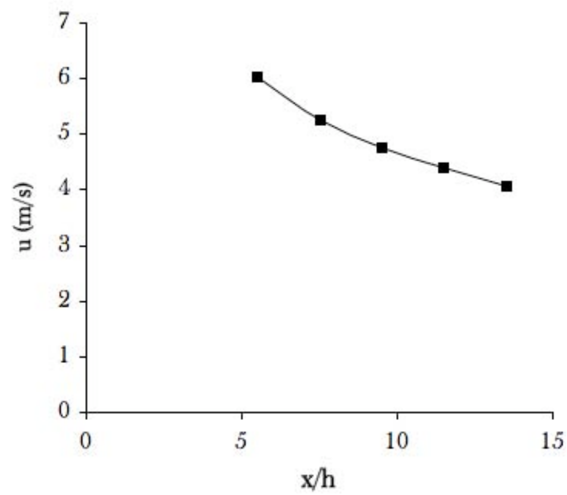

Figure 6. Centerline velocity variation

The growth of the synthetic jet is characterized by the spreading rate and is constant with distance from the origin once the jet is self-similar as in Fig. 7

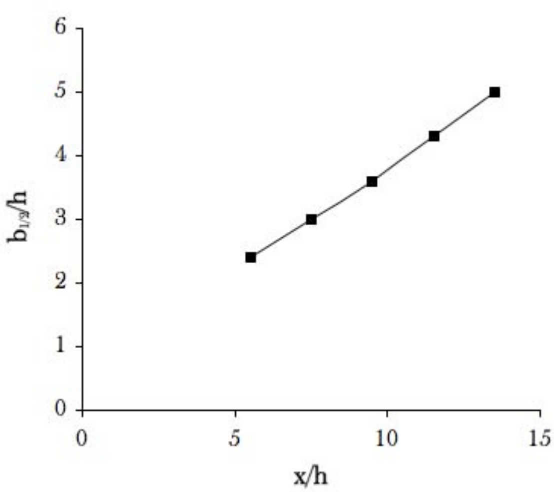

Figure 7. Jet Width

It was observed in the experiment that before 5 h, the spreading rate deviates from this trend, suggesting that perhaps the jet has not fully reached its self-similar state at these locations and in non-dimensionalizing the velocity profiles at the stations before 5 h, the profiles did not collapse appropriately. The spreading rate is calculated to be 0.31, which is in contrast to that of a turbulent jet (0.12). Thus from equation 11, the virtual kinematic viscosity of this particular synthetic jet is shown to be approximately 4 greater than an equivalent turbulent jet. As viscosity is an important mechanism to transmit momentum to the surrounding fluid, this higher virtual kinematic viscosity will give the synthetic jet the capability to influence the surrounding fluid in a more effective way.

Based on the normalizing variables suggested earlier and empirical data obtained, the velocity profiles collapse into a single profile as revealed in Fig. 7. The synthetic jet is shown to be wider than an equivalent turbulent jet, and thus is capable of influencing the surrounding environment more. The analytical model agrees well with the data in the central portion of the jet, however towards the edges there is a noticeable deviation. This may be attributed to the limitation of both the model and the hot wire measurements towards the edge of the jet where the mean velocity is low and turbulent intensity is high.

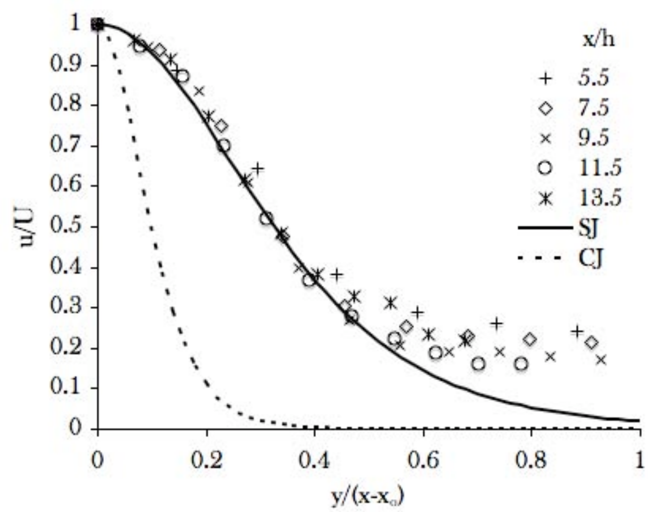

Figure 8. Analytical Model

We attribute the difference between the time averaged behavior of a synthetic jet behavior and a turbulent one, to the inherent enhanced periodic excitation introduced into the flow. The added disturbances in the synthetic jet increase the virtual kinematic viscosity, thereby enhancing momentum transfer This higher virtual viscosity of the synthetic jet makes it appropriate for applications where changes in the surrounding fluid are desired, as in fluid mixing or flow control.

It is knows that the stroke length is one parameter that controls the behavior of the jet. With this in mind the actuator was driven at a fixed frequency varying the voltage, and thus varying the stroke length. Figure 8, shows the centerline velocity decay for three stroke ratios. When normalized by the average exit velocity of the synthetic jet, the curve collapses onto a single one, thus suggesting that the average velocity from the plug model is a suitable scaling variable.

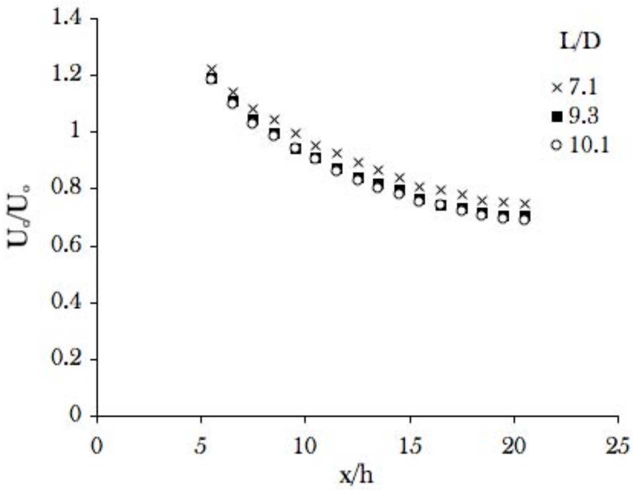

Figure 8. Normalized centerline velocity decay

As the stroke ratio was increased, the spreading rate increased as well, within the limited range of the experiment. When both the half width and axial distance were scaled by the stroke length, the jet width's collapse onto a single line.

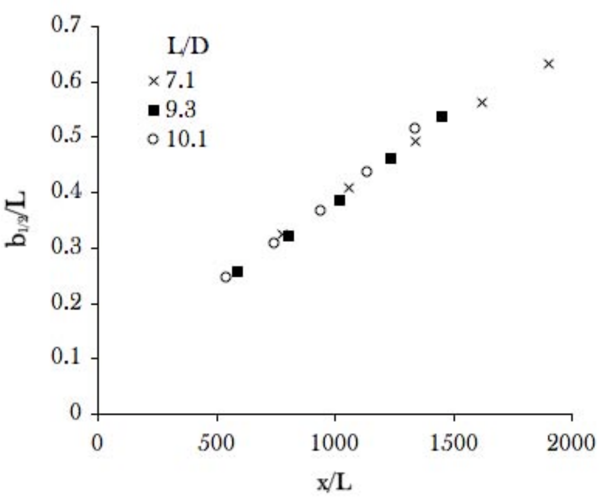

Figure 9. Normalized Jet Width

Further experiments examining the effect of operating frequency, and geometry need to be conducted to look for a universal scaling that perhaps may exist, thereby allowing for a simple but effective method for designing a the synthetic jet actuators.

REFERENCES

1. Glezer, A., Amitay, M., 2002, "Synthetic jets", Annual Review of Fluid Mechanics, 34

2. Amitay, M., Honoban, A., Trautman, M. & Glezer, A., 1997 "Modification of the aerodynamic characteristics of bluff bodies using fluidic actuators", AIAA Paper 97-2004.

3. Amitay, M., Smith, B. L. & Glezer A., 1998 "Aerodynamic flow control using synthetic jet technology", AIAA Paper 98-0208.

4. Seifert, A. and Pack, L., "Oscillatory Control of Separation at High Reynolds Numbers," 1999, AIAA J., Vol. 37, No. 9, 1999

5. Seifert, A., Eliahu, S., Greenblatt, D., and Wygnanski, I., "Use of Piezoelectric Actuators for Airfoil Separation Control (TN)," , 1998, AIAA J., Vol. 36, No. 8, 1998

6. Mahalingam, R., and Glezer, A., " Design and Thermal Characteristics of a Synthetic Jet Ejector Heat Sink",
Journal of Electronic Packaging, 2005 v127, n1

7. Davis SA, Glezer A.,1999. "Mixing control of fuel jets using synthetic jet technology", 1999, AIAA 37th Aerosp. Sci. Meet. 99-0447, Reno

8. Parviz, B.A.; Najafi, K., Muller, M.O.; Bernal, L.P., Washabaugh, P.D., "Electrostatically driven synthetic microjet arrays as a propulsion method for micro flight.", 2005, Microsystem Technologies 11 (12)

9. Tang, H., Zhong, S., Jabbal, M., Guo, F., Garcillan, L.: "Towards the design of synthetic jet actuators for full-scale flight conditions. Part 2: Low-dimensional performance prediction models and actuator design method.", 2007, Flow Turbulence % Combustion.

10. Zhong, S., Jabbal, M., Tang, H., Garcillan, L., Guo, F., Wood, N., Warsop, C.,"Towards the design of synthetic jet actuators for full-scale flight conditions. Part 1: The fluid mechanics of synthetic jet actuators.", 2007, Flow Turbulence & Combustion., 79, N. 3,4

11. Utturkar, Y., Holman, R., Mittal, R., Carroll B., Sheplak, M. and Cattafesta, L., ``A Jet Formation Criteria for Synthetic Jet Actuators", 2003, AIAA Paper, 2003—0636

12. Kotapati, R.B., Mittal, R., and Cattafesta, L., "Numerical Study of a Transitional Synthetic Jet in Quiescent External Flow", 2007, J. Fluid Mech, Vol. 581

13. Finley, T., and Mohseni, K., "Micro Pulsatile Jets for Thrust Optimization, ", 2004, ASME International Mechanical Engineering Congress and R&D Expo, IMECE 2004-62042

14. Vargas, Y., Finley, T., Mohseni, K., Hertzberg, J. "Flow Characteristics of a Synthetic Jet", 2006, 44th AIAA Aerospace Sciences Meeting and Exhibit, AIAA 2006-1422

15. K. Mohseni, "Pulsatile vortex generators for low-speed maneuvering of small underwater vehicles, Ocean Engineerings", 2006, 33(16)

16. Krieg, M., Mohseni, K., "Thrust Characterization of a Bio-inspired Vortex Ring Generator for Locomotion of Underwater Robots", in press 2008, IEEE J. of Oceanic Eng.

17. Smith, B. L., Glezer, A.,"The formation and evolution of synthetic jets,", 1998, Phys. Fluids, 10
18. Mallinson, S. G., Hong, G. & Reizes, J. A., "Some characteristics of synthetic jets", 1999, AIAA Paper 99-3651.
19. Cater, J., Soria, J., "The evolution of round zero-net-mass-flux jets,", 2002, J. Fluid Mech., 472
20. Smith, B. L., Swift, G.W., “A comparison between synthetic jets and continuous jets”, 2003, Experiments in Fluids, 34
21. Shuster, J.M., Smith, D.R., “Experimental study of the formation and scaling of a round synthetic jet”, 2007, Physics of Fluids, 19
22. Amitay, M., Cannelle, F., “Evolution of Finite Span synthetic Jets”, 2006, Physics of Fluids, 18
23. Bickley, W., “The plane jet”,1939, Phil. Mag. Ser 7, 23
24. Schlichting, H.,1933, “Laminare Strahlenausbreitung”, ZAMM, 13
25. Recihardt, H., 1942, “Gesetzmassigkeiten der freien Turbulenz”, VDI 414
26. Johnstone, A., Uddin, M., Pollard, A., “Calibration of hot wire probes using non unifrom mean velocity profiles”, 2005, Experiments in Fluids, 39