Analysis of Dynamic Behavior of Switched Reluctance Motor-Design Parameters Effects

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Abstract— **In the design of switched reluctance motor (SRM) and its drive, analysis and simulation of the dynamic behavior of the motor is essential. In the simulation process, model of the SRM and its controller is taken into account. In this paper motor characteristics are examined and effects of the motor design parameters, such as inductance, resistance and supply voltage, on the system response is studied. Variations of phase current and developed torque of the motor are predicted and discussed.**

Keywords- ***Switched reluctance motor, dynamic behavior, design parameters.***

I. Introduction

Advance of power electronics components and circuits leads to a wide application of SRMs in industry. High efficiency over a wide operating range, low number of power electronics devices, absence of rotor windings are some advantages of SRMs over other types of electrical machines [1, 2]. Structure of SRM seems to be the simplest type of electric machines and it is designed and built for a high efficiency energy conversion. Each phase of SRM is physically independent of the other phases. The rotor includes no conductors or permanent magnets and only stator has windings [3, 4]. SRM is not expensive and can reach the speed higher than that of other conventional motors. Disadvantages of these motors include difficult speed control, high noise, high ripples in the developed torque and drive circuit requirements. In [5], an indirect method to control the rotor position based on measuring the corresponding induced voltage in the passive phase is presented. General methods to control SRMs and different ways to control the current in these motors are discussed in [6] and a new method is then introduced. In order to reduce the electromechanical torque ripples and effective current in SRMs, its current must be optimized. In [7], a new method has been introduced to produce an SRM drive control signal for optimizing stator current waveform. A number of nonlinear SRM models, using magnetic circuit theory have been developed. In [8], a magnetic circuit concept is used to compute SRM's mean torque and optimize motor performance according to the pole tooth width change.

In this paper, electrical and mechanical equations of the motor based on co-energy concept are developed. By reviewing the behavior characteristics of SRMs and solving the system of equations, the dynamic behavior of a three-phase 12/8 SRM is simulated. Variation of current and developed torque is examined by changing parameters of the SRM.

II. Power Circuit of SRM

SRM operates through power electronics drive. A higher efficiency, maximum developed torque and lower torque ripples may be achieved by an appropriate operating of the drive circuit. The magnetic field within the motor is produced by exciting a phase of stator winding. This leads to the alignment of the stator and rotor teeth. By changing the excitation to the next phase, the rotor will move and a new alignment of the teeth occurs and reluctance torque is developed [9]. To develop the torque a strong and nonlinear magnetic characteristic are required; this is the reason for complexity in the controlling and examining the SRMs. The SRM converter must be capable of connecting each motor phase independent of other phases. There are different drive circuits for supplying SRMs as reported in [10]. These converters have been classified based on their operation, design and switching modes (soft and hard). Three important angles regarding the rotor position are: the turn-on angle (θ_o), the turn-off angle (θ_C) which is slightly behind the rotor alignment and the directive angle (θ_D) which is equal to the difference of angles θ_o and θ_C. To achieve a maximum developed torque, angle θ_C must be properly chosen. Likewise, by changing angle θ_C and θ_o, the developed torque ripples can be lowered and the efficiency can be improved. To control the developed torque, selection and calculation of angles θ_o and θ_C and current level is important. By switching-on and off of electronics devices such as MOSFETs, current passing the phase windings of the motor and voltage and current of power electronics devices can be regulated. In the drive of the SRMs, two switches per phase are used as shown in Fig. 1. In this case, the phases are independent and the normal voltage of the switch and diodes are equal to the dc supply voltage. In this case, it is possible to return energy to the supply but higher number of power electronics per phase

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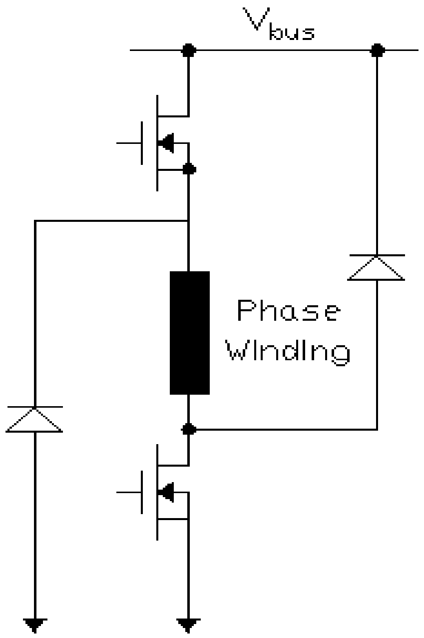


Fig. 1. Per-phase power drive of SRM

is the disadvantage of this drive. If both MOSFETs are switched-on, a dc voltage is applied across the phase where the diodes are also switched-on. There will be a negative dc voltage across them. In the case that one of the MOSFETs be switched-on with a diode, the voltage across the motor phase would be zero.

III. SYSTEM EQUATIONS

To analyze, design and simulate the control system of the non-linear SRM system, a mathematical model of the system is required.

A. Electromagnetic equations

The voltage equation of the k^{th} stator phase winding of the SRM is:

$$v_k = R_k i_k + \frac{d\lambda_k}{dt} \qquad k = 1,.......,q \tag{1}$$

where v_k, i_k, R_k and λ_k are the stator voltage, current, winding resistance and flux-linkage of phase k of the motor respectively. Flux-linkage of SRM depends on the current and rotor position:

$$\lambda_k = \lambda_k(i_1, i_2, ..., i_q, \theta) \tag{2}$$

Thus, (1) can be stated as follows:

$$v_k = R_k i_k + \sum_{m=1}^{q} \frac{\partial \lambda_k}{\partial i_m} \frac{di_m}{dt} + \frac{\partial \lambda_k}{\partial \theta} \frac{d\theta}{dt} \tag{3}$$

Coefficient $\partial\lambda_k / \partial i_m$ is the corresponding inductance between phase k and m, $d\theta/dt$ is the angular speed (ω) and $\partial\lambda/\partial\theta$ is the instant back-electromotive force.

If the saturation effect is ignored, the flux-linkage is stated as a linear function of the current as follows:

$$\lambda_k(i_1, i_2, .., i_q, \theta) = L_{k1}(\theta) i_1 + L_{k2}(\theta) i_2 + ... + L_{kj}(\theta) i_j + ... + L_{kq}(\theta) i_q \tag{4}$$

where $L_{kj}(\theta)$ is the corresponding inductance between phase winding i and k. Substituting (4) in (3), the phase voltage will be as follows:

$$v_k = R i_k + \sum_{m=1}^{q} \left[i_m \frac{\partial L_{km}}{\partial \theta} \frac{d\theta}{dt} + i_m \frac{\partial L_{km}}{\partial i_m} \frac{di_m}{dt} + L_{km}(i_m, \theta) \frac{di_m}{dt} \right] \tag{5}$$

It is assumed that the SRM phases are independent of each other and there is no mutual flux between the phases, and therefore their corresponding inductances are neglected. The flux-linkage characteristics, $\lambda_{ph} = f(i_{ph}, \theta)$, and the developed torque, $T_{ph} = g(i_{ph}, \theta)$,can be determined using finite element method, static measurement or analytic method. Therefore, electrical equation of SRM is simplified as follows:

$$v_k = R_k i_k + i_k \frac{\partial L_k}{\partial \theta} \omega + L_k \frac{di_k}{dt} \qquad k = 1,.......,q \tag{6}$$

The phase current of SRM can be estimated as follows:

$$\frac{di_k}{d\theta} = \frac{1}{\omega L_k} \left[v_k - R_k i_k - i_k \frac{\partial L_k}{\partial \theta} \omega \right] \qquad k = 1,.......,q \tag{7}$$

B. Mechanical equations

The developed torque of the SRM depends on the current level, current conducting period and rotor angular position. The mean torque and efficiency may be increased by controlling the stator current level and its on and off period. The developed torque by k^{th} phase at rotor position θ is as follows [11]:

$$T_k(\theta, i_k) = \left. \frac{\partial W'_{fk}(\theta, i_k)}{\partial \theta} \right|_{i = con.} \tag{8}$$

where W'_{fk} is the co-energy of the k^{th} phase. The mechanical power based on the torque and speed is:

$$\frac{dW_{mk}}{dt} = T_k \omega = T_k \frac{d\theta}{dt} \tag{9}$$

where W_{mk} is the mechanical energy of the k^{th} phase. The total torque of SRM is equal to the sum of developed torques in different stator phase which may be controlled independently [12]. Thus, the SRM mechanical equation is as follows:

$$J\frac{d\omega}{dt} + B\omega + T_L = T_e(\theta) = \sum_{k=1}^{q} T_k(\theta, i_k) \tag{10}$$

where T_L, B, J and T_k are the load torque, damping factor, shaft inertia and torque per phase respectively. By ignoring the mutual flux-linkage between the phases, the phase torque depends on the phase current and rotor angular position:

$$T_e(\theta) = \frac{1}{2} \sum_{k=1}^{q} i_k^2 \frac{dL_{kk}}{d\theta} \tag{11}$$

where L_{kk} is the k^{th} self-inductance of the stator phase winding and is equal to:

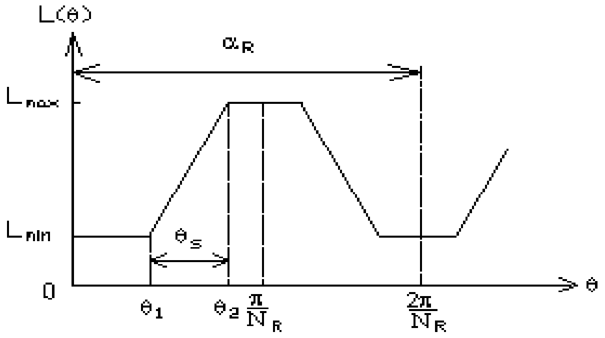


Fig. 2: Characteristic of phase inductance of SRM

$$L_{kk}(\theta)=\frac{N_{ph}^2}{\Re(\theta)} \qquad (12)$$

where N_{ph} is the turns number of phase winding. In general, the phase inductance is nonlinear and periodic in θ [13]. Fig. 2 shows the variation of SRM phase inductance when the saturation is ignored, L_{max} and L_{min} are the aligned and unaligned inductances, respectively. Higher maximum inductance L_{max} improves the efficiency and power density of the motor. If the inductance of the first phase is shown by $L_1(\theta)$, the k[th] phase inductance will be:

$$L_k(\theta)=L_1(\theta-\frac{2\pi(k-1)}{qN_R}) \qquad (13)$$

The phase inductances vary with period equal to α_R, the angle between the rotor teeth (in mechanical degree):

$$\alpha_R=\frac{2\pi}{N_R} \qquad (14)$$

where N_R is the number of rotor teeth. The angle between the stator teeth α_s is equal to:

$$\alpha_S=\frac{2\pi}{N_S} \qquad (15)$$

where N_s is the number of stator teeth. The phase developed torque is proportional to $dL_{kk}/d\theta$. To increase the developed torque, the difference between the aligned (L_{max}) and the unaligned (L_{min}) inductances must be maximized. To enhance L_{max}, the air gap length must be decreased; however mechanical tolerance limits the decrease of the air gap length. An alternative method is to increase the number of turns, but this also increases copper losses if the motor volume is kept constant. The ratio between the maximum and minimum inductance is usually equal to or larger than 2 [9].

IV. CHARACTERISTIC BEHAVIOR OF SRMS

Fig. 3 shows the cross section of a four-phase 8/6 SRM in which the number of stator teeth is 8 and rotor is 6. There are different combinations of the stator and rotor teeth. However, configurations 2/2 and 4/2 unable to develop the torque in SRMs. Higher phase number can decrease the torque ripples. Also increasing the stator and rotor teeth numbers can decrease the developed torque of the motor [14]. At least two

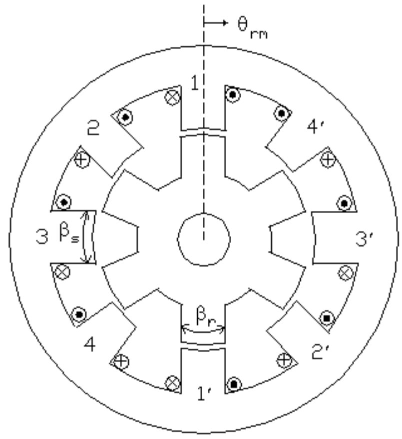


Fig. 3: Cross section of a 8/6 SRM

phases are needed for rotation of the motor; control of the rotating direction requires at least 3 phases.

When the excited phase changes to another one, rotor rotates by angle α_p:

$$\alpha_P=|\alpha_R-\alpha_S|=2\pi\frac{|N_S-N_R|}{N_S N_R} \qquad (16)$$

N_P, the number of steps over a complete cycle of the rotor is:

$$N_p=\frac{2\pi}{\alpha_p}=\frac{N_S N_R}{|N_S-N_R|} \qquad (17)$$

N_C, the number of rotating cycle is:

$$N_C=\frac{2\pi}{q\alpha_p}=\frac{N_S N_R}{q|N_S-N_R|} \qquad (18)$$

where q is the number of stator phase. For known value of N_P and N_S, two different values are obtained for the number of rotor teeth. To decrease the rotor current frequency, the value of N_s is chosen larger than N_R. The characteristics of different configurations of teeth in SRMs, taking into account the above-mentioned relationships, are summarized in Table 1.

V, SIMULATION RESULTS

To control the developed torque, angles θ_o and θ_C and current level must be selected and calculated. Knowing the exact rotor angular position is also important for controlling the developed torque and current level. For exact control of the developed torque the inductance variations and its minimum and maximum values must be known. Current in the

TABLE 1. CHARACTERISTICS OF DIFFERENT COMBINATIONS OF TEETH NUMBERS IN SRM

N_S/N_R	α_S	α_R	α_P	q	N_P	N_C	θ_D
4/2	90	180	90	2	4	2	45
6/4	60	90	30	3	12	4	30
8/6	45	60	15	4	24	6	22.5
10/4	36	90	54	5	6.7	1.3	18
10/8	36	45	9	5	40	8	18
12/6	30	6	30	2	12	6	15
12/8	30	45	15	3	24	8	15
12/10	30	36	6	6	60	10	15
16/12	22.5	30	7.5	4	48	12	11.25
24/16	150	22.5	7.5	3	48	16	7.5

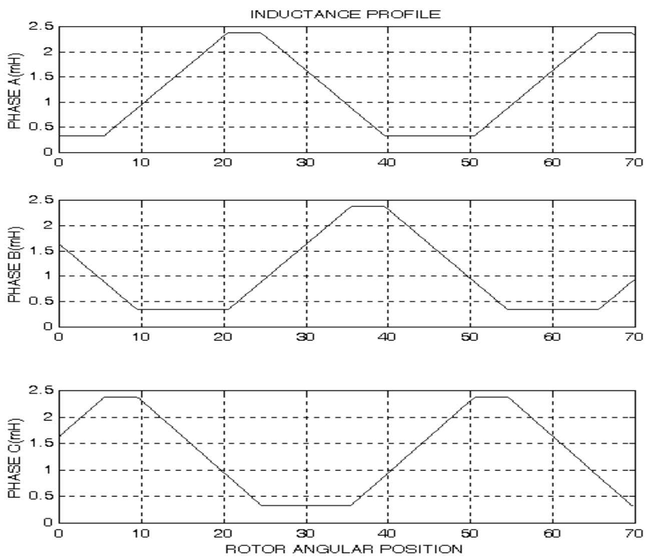


Fig. 4. Inductance variations of phases in three-phase 12/8 SRM versus rotor angular position

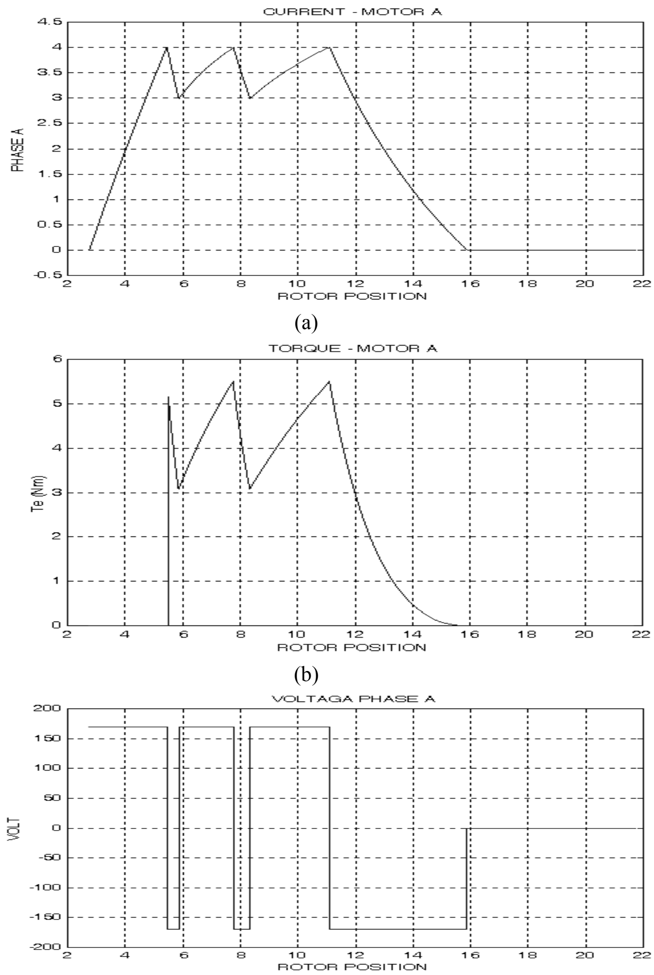
stator windings of SRM is controlled by turn-on angle θ_o and turn-off angle θ_C in the drive of the motor. θ_C must be chosen before the rotor enters to the negative torque region and excited phase winding is turn-off. When the excited phase changes to the next phase of the stator phase, three-phase 12/8 SRM rotates about 15 degrees; therefore, θ_o-θ_C will be 10 degrees at most. Fig. 4 shows the inductance variations in the three-phase 12/8 SRM. The inductances of phases b and c have phase difference of 15 and 30 mechanical degrees in respect to the phase a. The nonlinear differential equations describing the SRMs with the parameters given in Table 2 are solved in order to simulate the motor performance. In addition, the phase developed torque and phase current curves are determined where the parameters of the motor and its load change. Figs. 5 and 6 show the simulation results including phase current, developed torque and voltage for motors A and B.

The phase inductance is directly proportional to the square of the number of winding turns and is inversely proportional to the equivalent magnetic resistances in the flux paths. The minimum and maximum inductances can be varied by changing the number of winding turns.

Figs. 7 and 8 represent the simulation results for motors A and B with decreasing the number of the winding turns. By changing the minimum and maximum inductances and keeping their ratio identical, the maximum developed torque will not be changed that much.

TABLE 2. PARAMETERS OF SRMS

Parameter	A	B
Phase number	3	3
Stator teeth number	12	12
Rotor teeth number	8	8
Phase resistance (Ω)	8.1	2.5
Unaligned inductance (mH)	60	9.5
Aligned inductance (mH)	240	52
DC voltage (V)	170	162
Inductance ratio=L_{max}/L_{min}	4	5.5



(a)

(b)

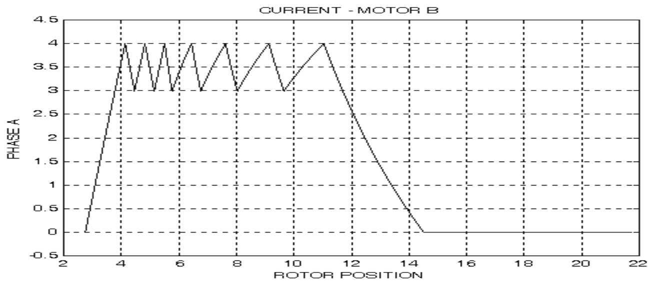
(c)

Fig. 5. Simulation results for motor A – (a) phase current, (b) phase developed torque, (c) phase voltage

By decreasing the number of winding turns, the current after turn-off angle θ_O tends to zero quicker. Fig. 9 shows the phase voltage variations in motor B with increasing the dc voltage. However, there is no change in the maximum torque and the current conducting angle. Fig. 10 shows the simulation results for motor A with decreasing the resistance of each stator winding phase. By reducing the resistance, the maximum developed torque is increased and the current conducting angle is decreased. Likewise, by reducing the stator winding resistance after turn-off angle θ_O the current quickly tends to zero.

IV. CONCLUSION

It is essential to analyze the dynamic operation of SRMs considering its power electronics drive. In this paper, the effects of changes in the number of phase winding turns, dc source voltage and stator phase windings resistance on the dynamic behavior of the motor were shown. Reducing the number of phase winding turns causes the phase current tends to zero faster following the turning-off and it prevents the motor to enter the negative torque region (generating mode).



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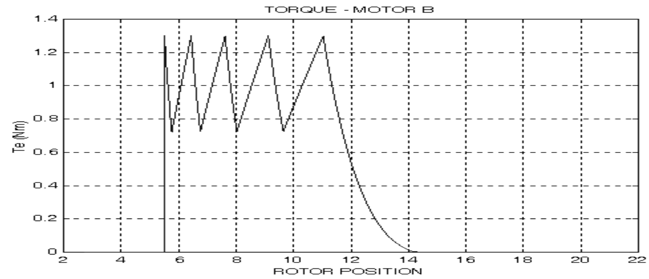

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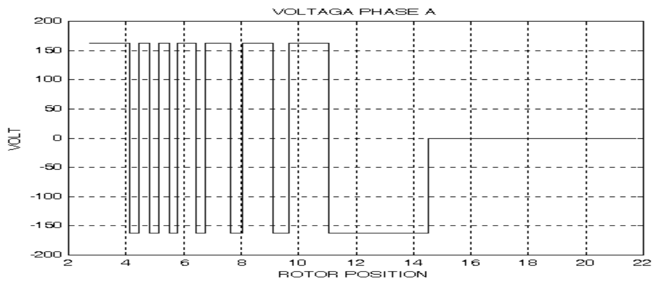

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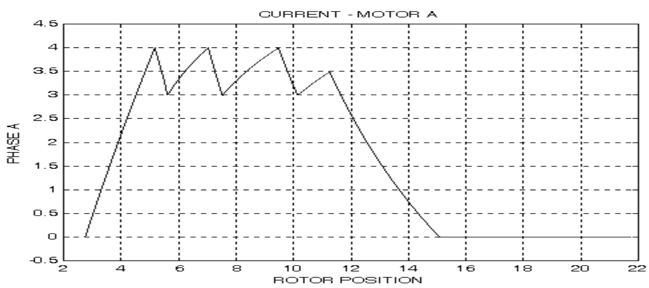
Fig. 6: Simulation results for motor B - (a) phase current (b) phase developed torque, (c) phase voltage



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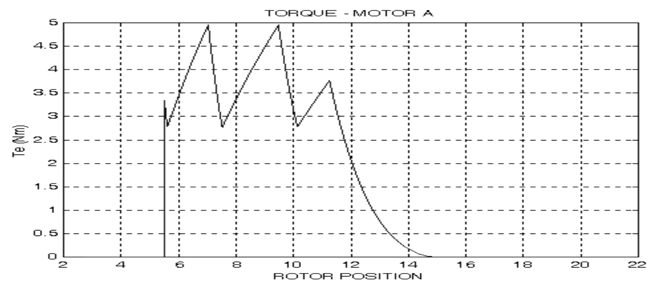

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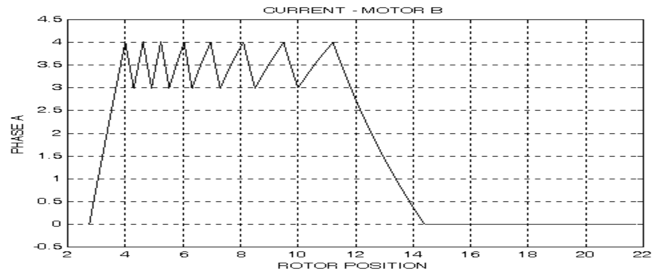
Fig. 7. Simulation results of motor A with decreasing number of winding turns in phases – (a) phase current, (b) phase developed torque



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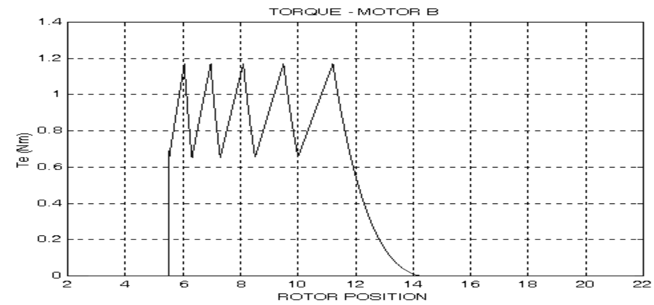

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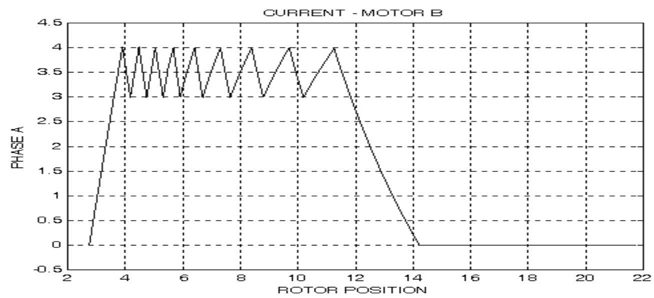
Fig. 8. Simulation results of motor B with decreasing number of winding turns in phases – (a) phase current variations, (b) phase torque variations



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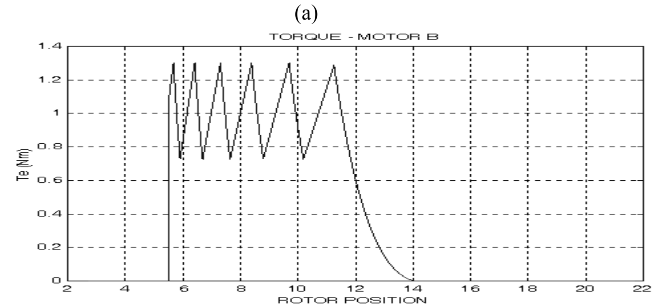

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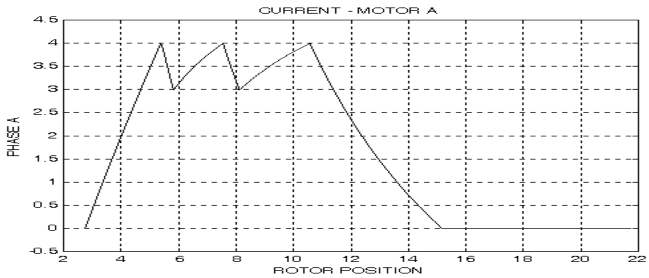
Fig. 9: Simulation results of motor B with increasing dc voltage – (a) phase current, (b) phase developed torque



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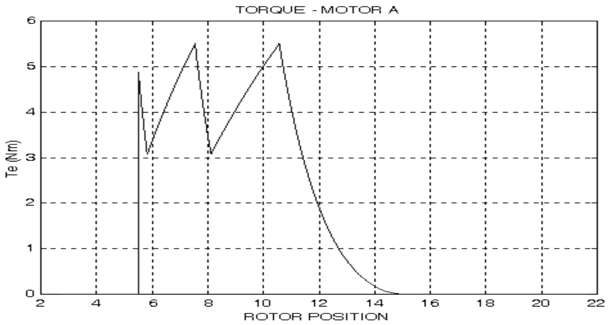

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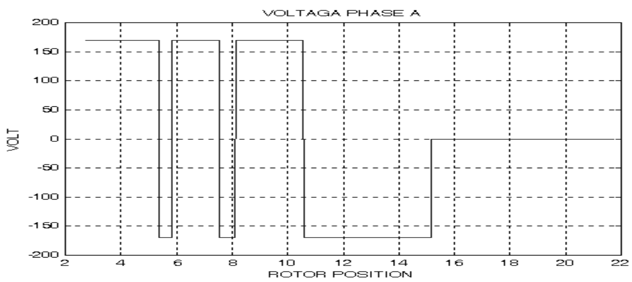

(c)

Fig. 10. Simulation results of motor A with decreasing stator phase winding resistance – (a) phase current, (b) phase developed torque), (c) phase voltage

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