FORMALIZED MATHEMATICS
Volume 12, Number **3**, 2004
University of Białystok

Logical Correctness of Vector Calculation Programs

Takaya Nishiyama
Shinshu University
Nagano

Hirofumi Fukura
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. In C-program, vectors of n-dimension are sometimes represented by arrays, where the dimension n is saved in the 0-th element of each array. If we write the program in non-overwriting type, we can give Logical-Model to each program. Here, we give a program calculating inner product of 2 vectors, as an example of such a type, and its Logical-Model. If the Logical-Model is well defined, and theorems tying the model with previous definitions are given, we can say that the program is logically correct. In case the program is given as implicit function form (i.e., the result of calculation is given by a variable of one of arguments of a function), its Logical-Model is given by a definition of a new predicate form. Logical correctness of such a program is shown by theorems following the definition. As examples of such programs, we presented vector calculation of add, sub, minus and scalar product.

MML Identifier: PRGCOR_2.

The articles [16], [18], [14], [20], [8], [4], [5], [11], [3], [10], [2], [6], [19], [17], [12], [9], [13], [1], [15], and [7] provide the terminology and notation for this paper.

In this paper m, n, i are natural numbers and D is a set.

The following proposition is true

(1) For all n, m holds $n \in m$ iff $n < m$.

Let D be a non empty set. One can check that there exists a finite 0-sequence of D which is non empty.

The following proposition is true

(2) For every non empty set D and for every non empty finite 0-sequence f of D holds $\operatorname{len} f > 0$.

Let D be a set and let q be a finite sequence of elements of D. The functor FS2XFS(q) yields a finite 0-sequence of D and is defined by:

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ISSN 1426–2630

(Def. 1) $\operatorname{len} \mathrm{FS2XFS}(q) = \operatorname{len} q$ and for every i such that $i < \operatorname{len} q$ holds $q(i+1) = (\mathrm{FS2XFS}(q))(i)$.

Let D be a set and let q be a finite 0-sequence of D. The functor $\mathrm{XFS2FS}(q)$ yielding a finite sequence of elements of D is defined as follows:

(Def. 2) $\operatorname{len} \mathrm{XFS2FS}(q) = \operatorname{len} q$ and for every i such that $1 \leqslant i$ and $i \leqslant \operatorname{len} q$ holds $q(i -' 1) = (\mathrm{XFS2FS}(q))(i)$.

One can prove the following two propositions:

(3) For every natural number k and for every set a holds $k \longmapsto a$ is a finite 0-sequence.

(4) Let D be a set, n be a natural number, and r be a set. Suppose $r \in D$. Then $n \longmapsto r$ is a finite 0-sequence of D and for every finite 0-sequence q_2 such that $q_2 = n \longmapsto r$ holds $\operatorname{len} q_2 = n$.

Let D be a non empty set, let q be a finite sequence of elements of D, and let n be a natural number. Let us assume that $n > \operatorname{len} q$ and $\mathbb{N} \subseteq D$. The functor $\mathrm{FS2XFS}^\star(q, n)$ yields a non empty finite 0-sequence of D and is defined by the conditions (Def. 3).

(Def. 3)(i) $\operatorname{len} q = (\mathrm{FS2XFS}^\star(q, n))(0)$,

(ii) $\operatorname{len} \mathrm{FS2XFS}^\star(q, n) = n$,

(iii) for every i such that $1 \leqslant i$ and $i \leqslant \operatorname{len} q$ holds $(\mathrm{FS2XFS}^\star(q, n))(i) = q(i)$, and

(iv) for every natural number j such that $\operatorname{len} q < j$ and $j < n$ holds $(\mathrm{FS2XFS}^\star(q, n))(j) = 0$.

Let D be a non empty set and let p be a non empty finite 0-sequence of D. Let us assume that $\mathbb{N} \subseteq D$ and $p(0)$ is a natural number and $p(0) \in \operatorname{len} p$. The functor $\mathrm{XFS2FS}^\star(p)$ yielding a finite sequence of elements of D is defined by:

(Def. 4) For every m such that $m = p(0)$ holds $\operatorname{len} \mathrm{XFS2FS}^\star(p) = m$ and for every i such that $1 \leqslant i$ and $i \leqslant m$ holds $(\mathrm{XFS2FS}^\star(p))(i) = p(i)$.

The following proposition is true

(5) For every non empty set D and for every non empty finite 0-sequence p of D such that $\mathbb{N} \subseteq D$ and $p(0) = 0$ and $0 < \operatorname{len} p$ holds $\mathrm{XFS2FS}^\star(p) = \emptyset$.

Let D be a non empty set, let p be a finite 0-sequence of D, and let q be a finite sequence of elements of D. We say that p is an xrep of q if and only if:

(Def. 5) $\mathbb{N} \subseteq D$ and $p(0) = \operatorname{len} q$ and $\operatorname{len} q < \operatorname{len} p$ and for every i such that $1 \leqslant i$ and $i \leqslant \operatorname{len} q$ holds $p(i) = q(i)$.

The following proposition is true

(6) Let D be a non empty set and p be a non empty finite 0-sequence of D. Suppose $\mathbb{N} \subseteq D$ and $p(0)$ is a natural number and $p(0) \in \operatorname{len} p$. Then p is an xrep of $\mathrm{XFS2FS}^\star(p)$.

Let x, y, a, b, c be sets. The functor $\mathrm{IFLGT}(x, y, a, b, c)$ yielding a set is defined by:

(Def. 6) $\mathrm{IFLGT}(x, y, a, b, c) = \begin{cases} a, & \text{if } x \in y, \\ b, & \text{if } x = y, \\ c, & \text{otherwise.} \end{cases}$

Next we state the proposition

(7) Let D be a non empty set, q be a finite sequence of elements of D, and n be a natural number. Suppose $\mathbb{N} \subseteq D$ and $n > \operatorname{len} q$. Then there exists a finite 0-sequence p of D such that $\operatorname{len} p = n$ and p is an xrep of q.

Let b be a finite 0-sequence of $\mathbb{R}$ and let n be a natural number. Then $b(n)$ is a real number.

Let a, b be finite 0-sequences of $\mathbb{R}$. Let us assume that $b(0)$ is a natural number and $0 \leqslant b(0)$ and $b(0) < \operatorname{len} a$. The functor $\mathrm{InnerPrdPrg}(a, b)$ yielding a real number is defined by the condition (Def. 7).

(Def. 7) There exists a finite 0-sequence s of $\mathbb{R}$ and there exists an integer n such that

(i) $\operatorname{len} s = \operatorname{len} a$,

(ii) $s(0) = 0$,

(iii) $n = b(0)$,

(iv) if $n \neq 0$, then for every natural number i such that $i < n$ holds $s(i+1) = s(i) + a(i+1) \cdot b(i+1)$, and

(v) $\mathrm{InnerPrdPrg}(a, b) = s(n)$.

The following propositions are true:

(8) Let a be a finite sequence of elements of $\mathbb{R}$ and s be a finite 0-sequence of $\mathbb{R}$. Suppose $\operatorname{len} s > \operatorname{len} a$ and $s(0) = 0$ and for every i such that $i < \operatorname{len} a$ holds $s(i+1) = s(i) + a(i+1)$. Then $\sum a = s(\operatorname{len} a)$.

(9) Let a be a finite sequence of elements of $\mathbb{R}$. Then there exists a finite 0-sequence s of $\mathbb{R}$ such that $\operatorname{len} s = \operatorname{len} a + 1$ and $s(0) = 0$ and for every i such that $i < \operatorname{len} a$ holds $s(i+1) = s(i) + a(i+1)$ and $\sum a = s(\operatorname{len} a)$.

(10) Let a, b be finite sequences of elements of $\mathbb{R}$ and n be a natural number. If $\operatorname{len} a = \operatorname{len} b$ and $n > \operatorname{len} a$, then $|(a, b)| = \mathrm{InnerPrdPrg}(\mathrm{FS2XFS}^{\star}(a, n), \mathrm{FS2XFS}^{\star}(b, n))$.

Let b, c be finite 0-sequences of $\mathbb{R}$, let a be a real number, and let m be an integer. We say that m scalar prd prg of c, a, b if and only if the conditions (Def. 8) are satisfied.

(Def. 8)(i) $\operatorname{len} c = m$,

(ii) $\operatorname{len} b = m$, and

(iii) there exists an integer n such that $c(0) = b(0)$ and $n = b(0)$ and if $n \neq 0$, then for every natural number i such that $1 \leqslant i$ and $i \leqslant n$ holds $c(i) = a \cdot b(i)$.

We now state the proposition

(11) Let b be a non empty finite 0-sequence of $\mathbb{R}$, a be a real number, and m be a natural number. Suppose $b(0)$ is a natural number and $\operatorname{len} b = m$ and $0 \leqslant b(0)$ and $b(0) < m$. Then

(i) there exists a finite 0-sequence c of $\mathbb{R}$ such that m scalar prd prg of c, a, b, and

(ii) for every non empty finite 0-sequence c of $\mathbb{R}$ such that m scalar prd prg of c, a, b holds $\mathrm{XFS2FS}^{\star}(c) = a \cdot \mathrm{XFS2FS}^{\star}(b)$.

Let b, c be finite 0-sequences of $\mathbb{R}$ and let m be an integer. We say that m vector minus prg of c, b if and only if the conditions (Def. 9) are satisfied.

(Def. 9)(i) $\operatorname{len} c = m$,

(ii) $\operatorname{len} b = m$, and

(iii) there exists an integer n such that $c(0) = b(0)$ and $n = b(0)$ and if $n \neq 0$, then for every natural number i such that $1 \leqslant i$ and $i \leqslant n$ holds $c(i) = -b(i)$.

The following proposition is true

(12) Let b be a non empty finite 0-sequence of $\mathbb{R}$ and m be a natural number. Suppose $b(0)$ is a natural number and $\operatorname{len} b = m$ and $0 \leqslant b(0)$ and $b(0) < m$. Then

(i) there exists a finite 0-sequence c of $\mathbb{R}$ such that m vector minus prg of c, b, and

(ii) for every non empty finite 0-sequence c of $\mathbb{R}$ such that m vector minus prg of c, b holds $\mathrm{XFS2FS}^{\star}(c) = -\mathrm{XFS2FS}^{\star}(b)$.

Let a, b, c be finite 0-sequences of $\mathbb{R}$ and let m be an integer. We say that m vector add prg of c, a, b if and only if the conditions (Def. 10) are satisfied.

(Def. 10)(i) $\operatorname{len} c = m$,

(ii) $\operatorname{len} a = m$,

(iii) $\operatorname{len} b = m$, and

(iv) there exists an integer n such that $c(0) = b(0)$ and $n = b(0)$ and if $n \neq 0$, then for every natural number i such that $1 \leqslant i$ and $i \leqslant n$ holds $c(i) = a(i) + b(i)$.

Next we state the proposition

(13) Let a, b be non empty finite 0-sequences of $\mathbb{R}$ and m be a natural number. Suppose $b(0)$ is a natural number and $\operatorname{len} a = m$ and $\operatorname{len} b = m$ and $a(0) = b(0)$ and $0 \leqslant b(0)$ and $b(0) < m$. Then

(i) there exists a finite 0-sequence c of $\mathbb{R}$ such that m vector add prg of c, a, b, and

(ii) for every non empty finite 0-sequence c of $\mathbb{R}$ such that m vector add prg of c, a, b holds $\mathrm{XFS2FS}^{\star}(c) = \mathrm{XFS2FS}^{\star}(a) + \mathrm{XFS2FS}^{\star}(b)$.

Let a, b, c be finite 0-sequences of $\mathbb{R}$ and let m be an integer. We say that m vector sub prg of c, a, b if and only if the conditions (Def. 11) are satisfied.

(Def. 11)(i) $\operatorname{len} c = m$,

(ii) $\operatorname{len} a = m$,

(iii) $\operatorname{len} b = m$, and

(iv) there exists an integer n such that $c(0) = b(0)$ and $n = b(0)$ and if $n \neq 0$, then for every natural number i such that $1 \leqslant i$ and $i \leqslant n$ holds $c(i) = a(i) - b(i)$.

One can prove the following proposition

(14) Let a, b be non empty finite 0-sequences of $\mathbb{R}$ and m be a natural number. Suppose $b(0)$ is a natural number and $\operatorname{len} a = m$ and $\operatorname{len} b = m$ and $a(0) = b(0)$ and $0 \leqslant b(0)$ and $b(0) < m$. Then

(i) there exists a finite 0-sequence c of $\mathbb{R}$ such that m vector sub prg of c, a, b, and

(ii) for every non empty finite 0-sequence c of $\mathbb{R}$ such that m vector sub prg of c, a, b holds $\mathrm{XFS2FS}^\star(c) = \mathrm{XFS2FS}^\star(a) - \mathrm{XFS2FS}^\star(b)$.

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Received July 13, 2004