Growth models with oblique asymptote

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Summary

Growth functions with horizontal asymptotes as Gompertz, Logistic, Richards (generalized Logistic), Bridge or Michaelis-Menten are commonly and widely used in various fields, e.g., agriculture [8, 9, 10, 11, 14], biology [7, 15, 16, 26, 27], economy [22], engineering [21], fishery [24], forestry [18, 29, 30], hydrology [1, 2, 4, 5, 6, 12, 17, 28], medicine [13], and other areas of applied research [19, 20, 23]. These growth functions are characterized by an horizontal asymptote and a single inflection point [3]. In some applications it could be usefull to have growth functions with one inflection point and an oblique asymptote. In this paper we extend our previous study [3] and modify the already mentioned models in such a way to obtain an oblique asymptote in keeping the single inflection point property. The structure of the present paper is as follows. In Section 2 we start with a general study of our modified model and introduce some elementary properties which are used in Section 3. In this section we modify some growth functions having a horizontal asymptote to obtain a model having an increasing linear asymptote. Finally in Section 4 we present some numerical examples on synthetic data.

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