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Discussion

A General Formulation of the Theory of Wire Ropes[1]

Steven A. Velinsky.[2] The author claims to have developed a simple and well-organized approach towards the formulation of wire strand and rope problems. He considers both the geometric nonlinear theory as well as the linearized theory, and while stating the ease in programming his theory, no numerical results are presented. The continued interest in wire strand and rope behavior exemplifies the importance of these elements. However, the author has missed a whole body of literature over the last ten years which has previously provided generalized theories for wire strands and ropes, and has done so in a much more usable form. In general, the theory of wire rope has been well developed by Costello and his associates, whose references are too numerous to mention, and they have examined a wide variety of problems from linear and nonlinear response of wire strands and ropes under static and dynamics loads to the response of viscoelastic ropes to the response of strands and ropes comprised of wires with various types of cross-sectional geometry. Typical of these works are the development of the basic theory and their use in the examination of specific wire rope problems. A few of the papers will be discussed to follow which have already accomplished far more than the paper under discussion.

First, as noted above, Jiang (1995) presents both the geometrically nonlinear and linear theories. Velinsky (1985) has already presented a general nonlinear theory for wire ropes and additionally, and of principal importance, through examination of a wide variety of complex configurations, he has shown that the geometrically nonlinear theory provides no value over the linear theory for the normal load range of wire ropes. This paper (Velinsky, 1985), thus further verified the linear theory. It might be added that the deviation from the linear theory is only of significance for strains that would be well beyond the linear elastic region for the rope material, and thus the material model would fail at loads far lower than that in which the linear geometric theory is no longer valid. As such, the first part of the Jiang paper provides no valuable contribution.

Costello and his associates use the wire rope axial and rotational strain to describe the deformation behavior of the total rope. Jiang in Eq. (18) of his paper also uses these parameters. One aspect should be noted, however, and that is that Jiang's rotational strain, ϕ, is not dimensionless.

Jiang states as one of his primary contributions is his showing that the strand structure can be characterized by seven stiffness and deformation constants. This is not a new idea. Velinsky (1988) stated, "We note that the global behavior of a strand can be completely described by the following strand quantities: the stiffness constants, S_1, S_2, S_3, and S_4, the effective strand radius, R^*, the strand effective Poisson's ratios, $\nu_{e\epsilon}$ and $\nu_{e\beta}$, and the strand bending stiffness, A^*." Velinsky has eight constants, and the reason is that the strand size is necessary which Jiang has omitted. It should also be noted that Jiang uses a different notation which is much less intuitive, but represents essentially the same parameters. It should also be stressed that these strand describing coefficients are all constants only for the linear theory (e.g., the stiffness varies with load for the nonlinear theory).

The Velinsky (1988) paper not only generalizes the analysis of strands of both Seale (close packed) and resting lay types, but also: examines the detailed geometry of these configurations, develops a design methodology for the configurations including methods for selecting the appropriate wire sizes, and examines the sensitivities of various strand properties to the design parameters. Velinsky's generalization recognizes that three independent variables, the wire axial strain, the change in helix radius and the change in helix angle, exist for each wire lay and requires the solution of three simultaneous linear equations for each lay. His formulation is performed in a dimensionless manner in order to quantify a class of strand configurations rather than a specific size and geometry. Furthermore, he examines parameters that describe global strand behavior which support the fact that optimal designs must exist.

Velinsky (1989) later extended the generalized approach of his 1988 paper to examine complex wire rope design. The Velinsky 1989 paper develops the general analysis for wire strand core, independent wire rope core, and fiber core types of wire ropes. The total rope analysis requires only the eight parameters for each strand, and a similar set of three linear equations are necessary for the deformations of each strand lay. Furthermore, as in the earlier paper, the theory is exercised in examining the sensitivities of various total rope properties to numerous strand and rope design parameters. In addition to total rope properties, Velinsky also examines the sensitivities of rope design parameters on individual wire stresses. The Velinsky formulation is easily programmed in a general manner (and has been), and is easily exercised as exhibited by the large amount of results that have been presented.

References

Velinsky, S. A., 1985, "General Nonlinear Theory for Complex Wire Rope," *International Journal of Mechanical Sciences*, Vol. 27, No. 7/8, pp. 497–507.

Velinsky, S. A., 1988, "Design and Mechanics of Multi-Lay Wire Strands," ASME *Journal of Mechanisms, Transmissions and Automation in Design*, Vol. 110, No. 2, pp. 152–160.

Velinsky, S. A., 1989, "On the Design of Wire Rope," ASME *Journal of Mechanisms, Transmissions and Automation in Design*, Vol. 111, No. 3, pp. 382–388.

Author's Closure[3]

Dr. Velinsky first criticizes that the paper presented "no numerical results" and "missed a whole body of literature over

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the last ten years.'' The original version of the paper did present considerable numerical examples and analyses, which, however, were dropped during the revision following reviewers' instructions that the paper was overlong. As a matter of fact, the published paper was still three pages over the limit, after considerable simplifications including the Introduction. As Dr. Velinsky points out, the relevant literatures are too numerous to mention. The author cannot see any reason to provide a complete publication list, in addition to those needed to be cited, especially in such an overlength situation. Dr. Velinsky then mentions a 1985 paper of his, and concludes that the ''nonlinear theory provides no value over the linear theory.'' The author disagrees with such a point of view. It is well known that almost all larger deformation problems encounter the possibility of plastic deformation. The elastic nonlinear theory, however, remains an important branch of mechanics. In general, Dr. Velinsky denies any contribution of the paper and only advocates his own accomplishment in this field. The author has no intention of making any comments on his claims, but believes that readers are professionals and have the best judgement.

Bifurcation of Orthotropic Solids[4]

A. Chattopadhyay[5,7] and H. Gu[6,7]. DeBotton and Schugasser (1996) recently presented an exact solution for bifurcation of orthotropic solids. In the paper, the following equilibrium equations were used.

$$\nabla \cdot [(\boldsymbol{\sigma} + \Sigma)\cdot \nabla(\mathbf{x} + \mathbf{u}) = \mathbf{0} \tag{1}$$

An assumption was made for plane-strain conditions that $\Sigma_{11} = -p$ is the only nonvanishing component of the initial stress in Eq. (1). Therefore, the remaining equilibrium equations can be derived by ignoring the product terms $\boldsymbol{\sigma}\cdot\nabla\mathbf{u}$ and their derivatives.

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} - p\frac{\partial^2 u_1}{\partial x_1^2} = 0$$

$$\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} - p\frac{\partial^2 u_2}{\partial x_1^2} = 0 \tag{2}$$

However, it is our purpose to point out that their approach contains a fundamental error resulting from ignoring the primary prebuckling displacements, which normally provide the same order of contributions as prebuckling stresses at the buckling state. Of course, these prebuckling displacement contributions are not included in simplified plate-type theories for simplicity. For an exact elasticity solution, which is the motivation of DeBotton and Schugasser's work, these terms should be included to ensure a rigorous analysis procedure.

If the superscript $(\ )^0$ is used to denote the prebuckling terms, the primary prebuckling state of for an orthotropic half-space whose material axes are parallel to the geometric axes can be truly simulated by

$$\sigma_{11}^0 = -p, \quad \sigma_{22}^0 = \sigma_{12}^0 = 0. \tag{3}$$

Using the constitutive equation for this orthotropic half-space, the derivatives of prebuckling displacements are derived as follows:

$$\frac{\partial u_1^0}{\partial x_1} = -\frac{C_{22}}{C_{11}C_{22} - C_{12}^2}p, \quad \frac{\partial u_2^0}{\partial x_2} = \frac{C_{12}}{C_{11}C_{22} - C_{12}^2}p$$

$$\frac{\partial u_1^0}{\partial x_2} = \frac{\partial u_2^0}{\partial x_1} = 0. \tag{4}$$

Including these displacement contributions, the buckling equations can be finally stated as

$$\frac{\partial}{\partial x_1}\left[\sigma_{11}\left(1 + \frac{\partial u_1^0}{\partial x_1}\right)\right] + \frac{\partial}{\partial x_2}\left[\sigma_{12}\left(1 + \frac{\partial u_1^0}{\partial x_1}\right)\right] - p\frac{\partial^2 u_1}{\partial x_1^2} = 0$$

$$\frac{\partial}{\partial x_1}\left[\sigma_{12}\left(1 + \frac{\partial u_2^0}{\partial x_2}\right)\right] + \frac{\partial}{\partial x_2}\left[\sigma_{22}\left(1 + \frac{\partial u_2^0}{\partial x_2}\right)\right] - p\frac{\partial^2 u_2}{\partial x_1^2} = 0 \tag{5}$$

and the buckling equation can also be expressed in terms of displacements as follows:

$$\left(1 - \frac{C_{22}}{C_{11}C_{22} - C_{12}^2}p\right)\left[C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + (C_{12} + C_{66})\frac{\partial^2 u_2}{\partial x_1 \partial x_2}\right] - p\frac{\partial^2 u_1}{\partial x_1^2} = 0$$

$$\left(1 + \frac{C_{12}}{C_{11}C_{22} - C_{12}^2}p\right)\left[(C_{12} + C_{66})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2}\right] - p\frac{\partial^2 u_2}{\partial x_1^2} = 0. \tag{6}$$

The prebuckling displacement contributions in these two equations can be written as

$$\Delta_1 = -\frac{C_{22}}{C_{11}C_{22} - C_{12}^2}p\left[C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + (C_{12} + C_{66})\frac{\partial^2 u_2}{\partial x_1 \partial x_2}\right]$$

$$\Delta_2 = \frac{C_{12}}{C_{11}C_{22} - C_{12}^2}p\left[(C_{12} + C_{66})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2}\right]. \tag{7}$$

Therefore, their effects in the buckling analysis can now be discussed.

Orthotropic Material With $C_{11} \gg C_{22}$

In this extreme case, the values of the material properties are assumed to be

$$C_{11} \gg C_{22}, \quad C_{12} \cong \frac{C_{22}}{3}, \quad C_{66} \cong \frac{C_{22}}{2}.$$

The prebuckling displacement contributions in the buckling equations are thus simplified as follows:

$$\Delta_1 \cong -p\frac{\partial^2 u_1}{\partial x_1^2} - \frac{C_{22}}{2C_{11}}p\frac{\partial^2 u_1}{\partial x_2^2} - \frac{C_{22}}{C_{11}}p\frac{\partial^2 u_2}{\partial x_1 \partial x_2} \cong -p\frac{\partial^2 u_1}{\partial x_1^2}$$

$$\Delta_2 \cong \frac{C_{22}}{3C_{11}}p\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{C_{22}}{6C_{11}}p\frac{\partial^2 u_2}{\partial x_1^2} + \frac{C_{22}}{3C_{11}}p\frac{\partial^2 u_2}{\partial x_2^2} \cong 0. \tag{8}$$

[4] deBotton, G., and Schulgasser, K., 1996, ''Bifurcation of Orthotropic Solids,'' ASME Journal of Applied Mechanics, Vol. 63, pp. 317–320.

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