Mathematical Modelling and Simulation of Magnetostrictive Materials by Comsol Multiphysics

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Abstract:
This paper presents the mathematical modeling for the analysis of the magnetostrictive materials, which describes the behaviour of the magneto-mechanical coupling in a rod of magnetostrictive material and capturing it on Comsol© to have a predictive and detailed analysis. The simulation about the behaviour under harmonic excitation is done in this work.

Keywords: magnetostrictive,   modeling, harmonic analysis, energy conversion.

1. Introduction

The magnetostrictive materials will change shape when it is subjected to a magnetic field. This occurs because magnetic domains in the material align with the magnetic field. Similarly, when the material is strained (stretched or compressed), its magnetic energy changes. This phenomenon is reversible and for this matter it can be use to build actuators, sensors or to make energy conversion.
It is possible to simplify the processes into two reversible energy conversion steps:
-    from electric to magnetic;
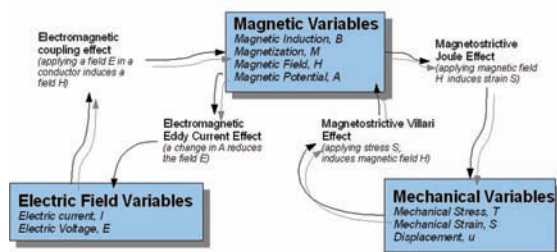-   from magnetic to mechanical.

Figure 1.2 Coupling diagram [4]

Dimensional change is generated from broadly two different processes: the first is migration of domains walls inside the material and second is the rotation of the domain. When the magnetic field is not applied (H=0), the domains returns at the random alignment (as shown in the Figure 1.1).
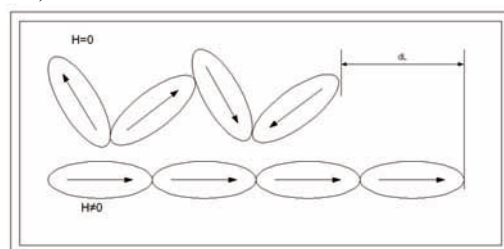

Figure 1.1 Domains alignment

The mechanical strain is very little and the order of magnitude is µm (micro meter), but the force that it produces is very strong. The magnetostrictive materials have very good specifications and show some interesting physical properties such as: a high density of available energy and a life time almost indefinite, that make these materials promising in several technological applications. In the magnetostrictive materials, the magnetic field causes elastic deformation or variation of the Young's modulus, and the mechanical stress modifies the magnetic properties: the magnetisation curve or the hysteresis loop [1].

1.1 Functional principle

The functional principle of the magnetostrictive materials is shown in Figure 1.2. The magnetostrictive rod, with length l and cross section A, when the magnetic field H is applied, is able to show the extension in shape such as $\Delta l + l$.
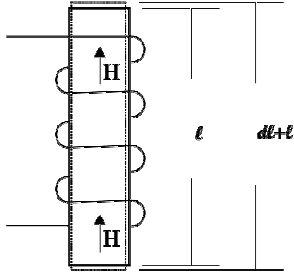
Figure 1.2 S(strain) caused by Joule magnetostriction when applied a magnetic field H. [2]

This phenomenon can be seen when some conditions are satisfied:
- All magnetic materials exhibit magnetostriction but if the constitutive materials, are the giant magnetostrictive materials the manifestation is more visible. [2]
- The magnetic field has to be along the axis of the mechanical extension.
- In order to observe the stress or the strain it is necessary to fix the mechanical boundary conditions such an allowing free movement or by lock the part and inhibiting the movement.

2. Mathematical modelling

The main purpose of the mathematic model is to predict the behaviour of the parts involved in the electromagnetic circuit, in the magnetostrictive materials and in the mechanical structure. Magneto-mechanical coupling in the magnetostrictive materials is very strong. The mechanical strain will occur when the magnetic field is applied in addition to the strain that comes from the normal applied stress. (See Eq. 2.1). The magnetization of that material changes due to the changes of the mechanical stress. (See Eq. 2.2).

The constitutive magnetostriction equations are:

$$S = \eta_{(H)} \cdot T + d \cdot H \qquad (2.1)$$

$$B = d \cdot T + \mu_{(T)} \cdot H \qquad (2.2)$$

Definitions:

- strain $S = \dfrac{\Delta l}{l}$ is the relative deformation of the shape,

- $\eta_{(H)}$ is the compliance at constant magnetic field (the reciprocal of Young's modulus),

- stress T is the forces per unit of area,
- t is the time,
- d is the piezomagnetic strain constant,
- H is the magnetic filed (intensity),
- B is the magnetic field (magnetic flux density, induction),
- $\mu_{(T)}$ is the permeability at constant stress.

Some important mathematical steps to achieve the modelling are the follows:
- For the first step the following parameters are assumed to be constants:

$$\eta_{(H)} \rightarrow \eta \text{ and } \mu_{(T)} \rightarrow \mu$$

Considering the mechanical variation only in x direction, the strain result is:

$$S = \frac{\partial u}{\partial x} \qquad (2.3)$$

where $\vec{u} = f(x, y, z)$ is the position.

- Making time-derivative and the second derivative on the equations (2.1) and (2.2):

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial t^2}\right) = \eta \frac{\partial^2 T}{\partial t^2} + d \frac{\partial^2 H}{\partial t^2} \qquad (2.4)$$

- Consider the second Newton's law:

$$F = ma \qquad (2.5)$$

Then obtaining:

$$\frac{\partial T}{\partial x} = \frac{\partial\left(\frac{F}{A}\right)}{\partial x} = \frac{\partial F}{\partial V} = \frac{\partial(m \cdot a)}{\partial V} = \rho \cdot a = \rho \frac{\partial^2 u}{\partial t^2} \qquad (2.6)$$

- The second step comes from Maxwell's equation:

$$\begin{cases} \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & (2.7) \\[2mm] \vec{\nabla} \times \vec{B} = \mu \vec{J} + \mu\varepsilon \dfrac{\partial \vec{E}}{\partial t} & (2.8) \end{cases}$$

where: J is the electric current density, E the electric filed, ε the permittivity of free space and μ the permeability.

- By using the Ohm's law:

$$J = \sigma E \qquad (2.9)$$

where: J is the current density, σ is the conductivity and E is the electric field.
Consider the fact $\nabla \cdot B = 0$.
It can be shown that the results of all the mathematical calculating are the follows:

$$\begin{cases} \nabla^2 H = \sigma d \dfrac{\partial T}{\partial t} + \dfrac{d}{\mu}\left(\mu\varepsilon - \rho\eta\right)\dfrac{\partial^2 T}{\partial t^2} + \mu\sigma\dfrac{\partial H}{\partial t} + \dfrac{\left(\mu^2\varepsilon - \rho d^2\right)}{\mu}\dfrac{\partial^2 H}{\partial t^2} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.10) \\[2mm] \dfrac{\partial^2 T}{\partial x^2} = \rho\eta\dfrac{\partial^2 T}{\partial t^2} + \rho d\dfrac{\partial^2 H}{\partial t^2} \qquad\qquad (2.11) \end{cases}$$

Considering harmonic analysis, the system must be exposed by the harmonic excitation and the follow equation will be simplified in this manner:

$$\begin{cases} \nabla^2 H = \omega^2 \tilde{k}_1 T + \omega^2 \tilde{k}_2 H \qquad\qquad (2.12) \\[2mm] \dfrac{\partial^2 T}{\partial x^2} = \omega^2 \tilde{G}_1 T + \omega^2 \tilde{G}_2 H \qquad\qquad (2.13) \end{cases}$$

where:

$$\tilde{k}_1 = \frac{\left(d\left(\rho\eta - \mu\varepsilon\right) + \dfrac{i\mu\sigma d}{\omega}\right)}{\mu} \qquad \tilde{G}_1 = -\rho\sigma$$

$$\tilde{G}_2 = -\rho d$$

$$\tilde{k}_2 = \frac{\left(\left(\rho d^2 - \mu^2\varepsilon\right) + \dfrac{i\mu^2\sigma}{\omega}\right)}{\mu}$$

Now we can consider the parameters used above $(\eta \to \eta_{(H)} \;\; \mu \to \mu_{(T)})$ with some iterative evaluation with H and T.
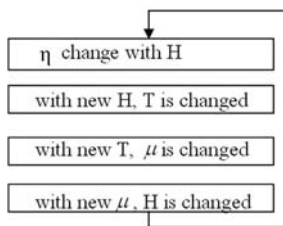
Figure 2.1 Iterative evaluations

4. Physics Model

It is necessary to define a physical model related to the mathematical one just done above. To simulate the model in the right way, it has to provide the magnetic field inside the magnetostrictive rod by a coil around it. The coil is made of copper wires. When the current flows, its generates the magnetic field H inside the material.

To create a closed magnetic circuit for the magnetic flux, the model is provided with a part of ferromagnetic with high permeability (iron μ =2000).
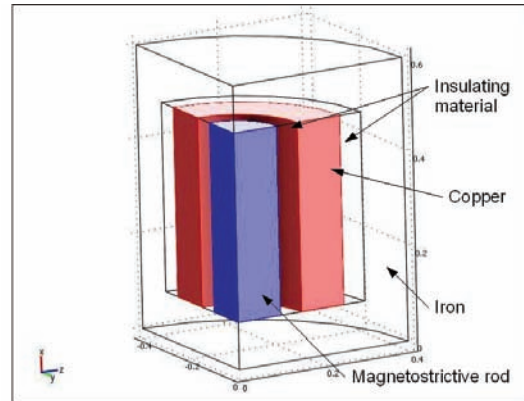
Figure 4.1 Structure of the model

Between the coil and the magnetostrictive rod there is a little space constitute of plastic isolation, the same between the coil and the iron.
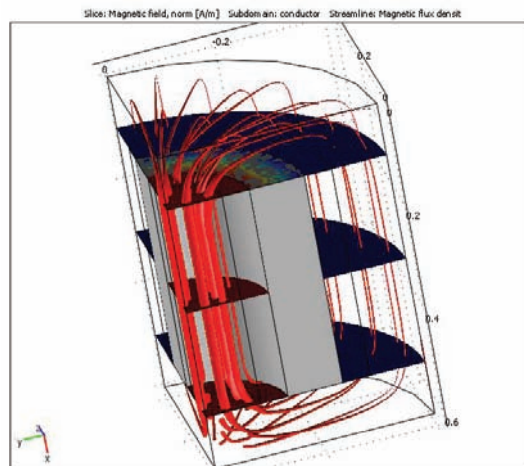
Figure 4.2 Induction flux lines

5. Modelling in Comsol Multiphysics

By using Comsol© Multiphysics it is possible to describe the behaviour of the magnetostrictive material and capturing it on Comsol to have a predictive and detailed analysis of that material. Moreover one advantage of the simulation

program is: that it can be programmed to know the values of the variables inside the magnetostrictive material in each condition are needed, for example the response under harmonic excitation.

To implement the equations is required the Partial Differential Equation's, called PDEs mode General form in Comsol.

The subdomain settings are very important and the simulation depends on these, in particular the equations in the PDEs mode have to be active only in the subdomain constitute of the rod.
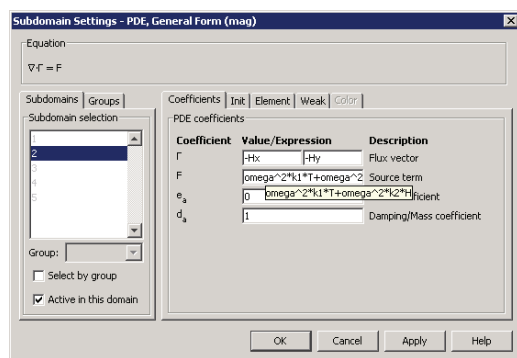

Figure 5.1 PDE Subdomain settings

The 3D draw is useful to set up the value and the direction of the current in the coil.
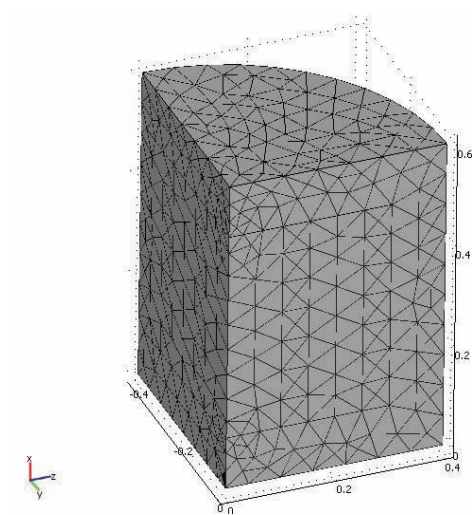

Figure 5.1 Meshing for 3D geometry

An additional auxiliary 2D geometry allows one to solve the PDEs in a more quickly way than the 3D ones because the meshing is more simple and

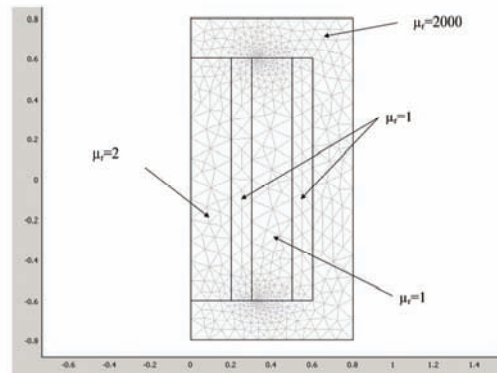it is possible to save more time during the simulation.

Figure 5.2 Meshing for 2D geometry

It is also necessary set up the mesh parameters to obtain a good convergence and a good compromise with the time required for the simulation.

6. Results

The stress under harmonic excitation is time dependent and it is possible to see the stress shown below in the post-processing figures. The direction of the stress is dependent of the direction of the magnetic field. The input into this model is the sinusoidal coil current. The frequency in the follow examples is 50[Hz].
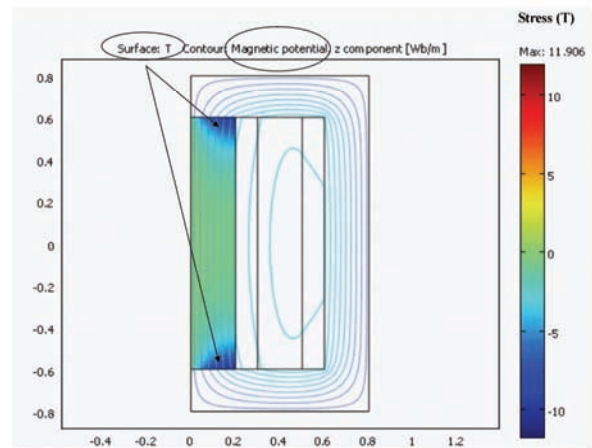

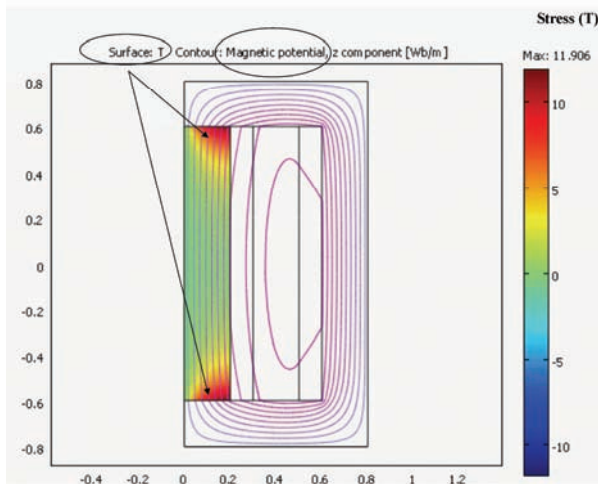**Figure 6.2** T and H at the peak negative value, time 1.

Figure 6.3 T and H at the positive peak value, time 1.

It can be note that the stress (represented by the colors surfaces) is maximum when the magnetic potential is at the max value.

7. Conclusions

Comsol is able to solve the magnetostrictive material modelling with the inputs given by PDE General Mode. From the model, it can be observed that the magnetic potential is dependent on the applied magnetic field, its value and its frequency. In order to show the stress, for the moment one has to use only the surface post-plotting setting and it means that is not possible to understand the direction of this type of vector. As a result the stress (T) is increasing in relation of H. The tractive or compressive efforts are located on the top and on the bottom of the rod. This happens because the magnetostrictive ones is fixed between the iron. Further research is required to obtain a complete and detailed development in every aspect of the behavior about the magnetostrictive effects and its surrounding.

8. References

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