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Research Article

Some Similarity Measures for Triangular Fuzzy Number and Their Applications in Multiple Criteria Group Decision-Making

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We propose some similarity measures between two triangular fuzzy numbers (TFNs) based on the vector similarity measures in vector space, which can be used to aggregate the decision information with TFNs. A methodology for multiple criteria group decision-making (MCGDM) problems with triangular fuzzy information is proposed; the criteria values take the form of linguistic values, which can be converts to TFNs. According to the weighted similarity measures between each alternative and ideal alternative, it is easy to rank alternatives and select the most desirable alternative. Finally, we apply the proposed methods to an illustrative example of MCGDM; the numerical results show that our method is effective and practical. For comparison, we also apply our similarity measures method to solve the fuzzy decision-making problem in Wei (2011); our method has simpler computation and gets the same results more rapidly than the FLOWHM method.

1. Introduction

Fuzzy multiple criteria decision-making (FMCDM) is the process of ranking the feasible alternatives and selecting the best one by considering multiple criteria, in which the alternatives and criteria values are carried by fuzzy sets [1], intuitionistic fuzzy set [2, 3], triangular fuzzy number [4], triangular intuitionistic fuzzy number [5], interval-valued triangular fuzzy number [6], trapezoidal fuzzy number [7, 8], intuitionistic trapezoidal fuzzy number [9], internal-valued trapezoidal fuzzy number [10], and so on. As an important part of the modern decision science, some related methods have been successfully applied to fuzzy decision-making problems, for example, ordered weighted aggregation operators [11], weighted geometric aggregation operators [10], TOPSIS method [12–14], analytic hierarchy process method [15], grey relational analysis method [16, 17], and similarity measures [3, 18], and so forth. Over the last decades, many studies have been done on the concepts of similarity measures between two intuitionistic fuzzy sets. On the one hand, the similarity measures were defined based on distance models, such as the Hamming distance similarity method [19], the Hausdorff distance similarity measure [20], and the Euclidean distance similarity measure [3]. On the other hand, the intuitionistic fuzzy set was seen as a vector containing some elements, by using the vector similarity measures to define the similarity measures between two intuitionistic fuzzy sets, for example, the cosine similarity measures [21] and so forth. However, as an effective method and a wide range of application in various fields, very few researchers worked on similarity measures between two triangular fuzzy numbers and applied them to triangular fuzzy multiple criteria group decision-making.

Because the triangular fuzzy number (TFN) is intuitive, easy to use, computationally simple, and useful in promoting representation and information processing in a fuzzy environment, it was usefully applied to solve FMCDM problems, in which the criteria values are carried by the TFNs. Many methodologys have been proposed to solve FMCDM with TFNs in the literature; Fu [22] presented a fuzzy optimization method based on the concept of ideal and anti-ideal points to solve FMCDM problems with TFNs. A case study of

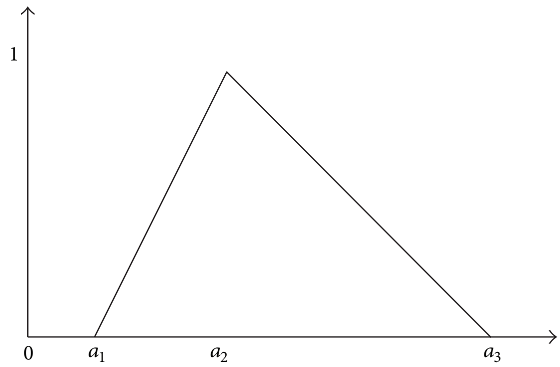

Figure 1: Membership functions of TFNs.

reservoir flood control operation was given to demonstrate the proposed method's effectiveness. Wei [23] proposed fuzzy induced ordered weighted harmonic mean operator to solve fuzzy group decision-making problems. In [24], Wei et al. investigated the multiple attribute decision-making problems with triangular fuzzy information and developed an generalized triangular fuzzy correlated averaging operator; numerical results showed the method was applicable. The above literatures mainly focus on the group decision-making with unknown information on criterion weight and on group member weight. However, in some actual decision-making process, the information about criterion weight and on group member weight are important. Therefore, with the defects of the methods in the literatures, we develop some similarity measures to solve FMCGDM problems with known information on criterion weight and on group member weight. In order to do so, the rest of this paper is set out as follows. In the next section, we introduce some basic concepts and operational laws of TFNs; three similarity measures between two TFNs are also defined. In Section 3, we present a triangular FMCGDM method based on similarity measures. Section 4 gives two illustrated examples to verify the developed approach; all numerical results show that our method is feasible and applicable. The paper ends with conclusion in Section 5.

2. Preliminaries

We consider the following well-know description of a triangular fuzzy number α.

Definition 1 (see [25]). A triangular fuzzy number α can be defined by a triplet (a_1, a_2, and a_3). The membership function $\mu_\alpha(x)$ is

$$\mu_\alpha(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ \dfrac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3, \\ 0, & a_3 < x, \end{cases} \tag{1}$$

where $0 \le a_1 \le a_2 \le a_3 \le 1$, a_1 and a_3 stand for the lower and upper values of the support of α, respectively, and a_2 stands for the modal values. The membership functions can be seen in Figure 1.

Let $\alpha = (a_1, a_2, a_3)$ and $\beta = (b_1, b_2, b_3)$ be two TFNs and γ a positive scalar number; the basic operational laws related to TFNs are shown as

$$\begin{aligned} \alpha \oplus \beta &= (a_1 + b_1, a_2 + b_2, a_3 + b_3), \\ \alpha \otimes \beta &= (a_1 b_1, a_2 b_2, a_3 b_3), \\ \gamma \otimes \alpha &= (\gamma a_1, \gamma a_2, \gamma a_3), \\ \frac{1}{\alpha} &= \left(\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}\right). \end{aligned} \tag{2}$$

In the vector space, there are some similarity measures between two vectors, which successfully apply to various fields, such as pattern recognition, description and classification of complex structured objects, faculty recruitment, and fuzzy assignment problems. In this section, we introduce three important vector similarity measures.

Let $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$ be two vectors of length n, where all the coordinates are positive; three important similarity measures are defined as

$$\begin{aligned} J(Y, Z) &= \frac{XY}{\|X\|_2^2 + \|Y\|_2^2 - XY} \\ &= \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2 - \sum_{i=1}^n x_i y_i}, \\ E(Y, Z) &= \frac{2XY}{\|X\|_2^2 + \|Y\|_2^2} = \frac{2\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2}, \\ C(Y, Z) &= \frac{XY}{\|X\|_2 \|Y\|_2} = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2}\sqrt{\sum_{i=1}^n y_i^2}}. \end{aligned} \tag{3}$$

The three parameters a_i ($i = 1, 2$, and 3) in TFN $\alpha = (a_1, a_2$, and $a_3)$ can be considered as one vector representation with three elements. Based on the extension of the similarity measures in vector space, the similarity measures between two TFNs are shown in Definition 2.

Definition 2. Let $\alpha = (a_1, a_2, a_3)$ and $\beta = (b_1, b_2, b_3)$ be two TFNs, where $0 \le a_1 \le a_2 \le a_3 \le 1$, $0 \le b_1 \le b_2 \le b_3 \le 1$; the similarity measures between two TFNs can be defined as follows:

$$S^J(\alpha, \beta) = \frac{\sum_{i=1}^3 a_i b_i}{\sum_{i=1}^3 a_i^2 + \sum_{i=1}^3 b_i^2 - \sum_{i=1}^3 a_i b_i}, \tag{4}$$

$$S^E(\alpha, \beta) = \frac{2\sum_{i=1}^3 a_i b_i}{\sum_{i=1}^3 a_i^2 + \sum_{i=1}^3 b_i^2}, \tag{5}$$

$$S^C(\alpha, \beta) = \frac{\sum_{i=1}^3 a_i b_i}{\sqrt{\sum_{i=1}^3 a_i^2}\sqrt{\sum_{i=1}^3 b_i^2}}. \tag{6}$$

The three similarity measures between two TFNs in Definition 2 satisfies the following properties, as in Theorem 3.

Theorem 3. *The similarity satisfies the following properties:*

(1) $0 \leq S(\alpha, \beta) \leq 1$,

(2) $S(\alpha, \beta) = S(\beta, \alpha)$,

(3) *if* $\alpha = \beta$, *that is*, $a_i = b_i$, $i = 1, 2$, *and* 3. $S(\alpha, \beta) = 1$.

Proof. Firstly, we prove $S^J(\alpha, \beta)$ satisfies the above properties, as follows.

(P1) It is obvious that $S^J(\alpha, \beta) \geq 0$. Thus, we only need to prove $S^J(\alpha, \beta) \leq 1$.

By using the basic mathematical equations,

$$2a_i b_i \leq a_i^2 + b_i^2, \tag{7}$$

we get

$$\sum_{i=1}^{3} a_i^2 + \sum_{i=1}^{3} b_i^2 - \sum_{i=1}^{3} a_i b_i \geq 2a_i b_i - a_i b_i \geq a_i b_i. \tag{8}$$

Taking (8) in (4), we get $S^J(\alpha, \beta) \leq 1$.

(P2) It is obvious that the equation is true.

(P3) When $\alpha = \beta$, that is, $a_i = b_i$, for $i = 1, 2$, and 3,

$$S^J(\alpha, \beta) \frac{\sum_{i=1}^{3} a_i a_i}{\sum_{i=1}^{3} a_i^2 + \sum_{i=1}^{3} a_i^2 - \sum_{i=1}^{3} a_i a_i} = 1. \tag{9}$$

In the same way, we can prove $S^E(\alpha, \beta)$ and $S^C(\alpha, \beta)$ satisfy the properties in Theorem 3.

Therefore, we have finished the proofs. □

3. Triangular Fuzzy Multiple Criteria Group Decision-Making Method Based on Similarity Measures

In this section, we present a handling method for TFNs multiple criteria group decision-making problems.

Let $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria and $A = \{A_1, A_2, \ldots, A_m\}$ a set of alternatives. Suppose we invite p experts to make the judgement, and let $G = \{G_1, G_2, \ldots, G_p\}$ be a set of experts. They are expected to give the linguistic value of TFNs (see Table 1). The vectors of alternative A_i given by the expert G_k are represented by following TFNs:

$$\begin{aligned} v_i^k = \{ & \langle C_1, (a_{i11}^k, a_{i12}^k, a_{i13}^k) \rangle, \\ & \langle C_2, (a_{i21}^k, a_{i22}^k, a_{i23}^k) \rangle, \ldots, \\ & \langle C_n, (a_{in1}^k, a_{in2}^k, a_{in3}^k) \rangle \}, \end{aligned} \tag{10}$$

for $i = 1, 2, \ldots, m$, $k = 1, 2, \ldots, p$.

The weight vector of the experts is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$; for each alternative A_i, the group preference vector can be calculated by

$$\begin{aligned} V_i = \Bigg\{ & \left\langle C_1, \left(\sum_{k=1}^{p} \lambda_k a_{i11}^k, \sum_{k=1}^{p} \lambda_k a_{i12}^k, \sum_{k=1}^{p} \lambda_k a_{i13}^k \right) \right\rangle, \\ & \left\langle C_2, \left(\sum_{k=1}^{p} \lambda_k a_{i21}^k, \sum_{k=1}^{p} \lambda_k a_{i22}^k, \sum_{k=1}^{p} \lambda_k a_{i23}^k \right) \right\rangle, \\ & \ldots, \left\langle C_n, \left(\sum_{k=1}^{p} \lambda_k a_{in1}^k, \sum_{k=1}^{p} \lambda_k a_{in2}^k, \sum_{k=1}^{p} \lambda_k a_{in3}^k \right) \right\rangle \Bigg\}. \end{aligned} \tag{11}$$

For the different critera C_j $(j = 1, 2, \ldots, n)$, the weight of a criterion is a triangular fuzzy weight, which is obtained by the decision maker, as $w_j = (a_{j1}, a_{j2}, a_{j3})$ $(j = 1, 2, \ldots, n)$. The expected weight value w_j $(j = 1, 2, \ldots, n)$ for a triangular fuzzy weight is obtained by the following equation:

$$EV(w_j) = \frac{a_{j1} + a_{j2} + a_{j3}}{3}. \tag{12}$$

Then we normalize the expected weight value w_j $(j = 1, 2, \ldots, n)$ by the following formula:

$$\widetilde{w}_j = \frac{EV(w_j)}{\sum_{i=1}^{m} EV(w_j)}. \tag{13}$$

In FMCGDM environments, the concept of ideal point has been used to determine the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct to evaluate alternatives (Ye [26]).

Thus three weighted similarity measures between an alternative A_i and the ideal alternative A_P represented by the TFNs are defined as follows:

$$\begin{aligned} S^J(A_i, A_P) &= \sum_{j=1}^{n} w_j \frac{\sum_{l=1}^{3} a_{ijl} b_{pjl}}{\sum_{l=1}^{3} a_{ijl}^2 + \sum_{l=1}^{3} b_{pjl}^2 - \sum_{l=1}^{3} a_{ijl} b_{pjl}}, \\ S^E(A_i, A_P) &= \sum_{j=1}^{n} w_j \frac{2\sum_{l=1}^{3} a_{ijl} b_{pjl}}{\sum_{l=1}^{3} a_{ijl}^2 + \sum_{l=1}^{3} b_{pjl}^2}, \\ S^C(A_i, A_P) &= \sum_{j=1}^{n} w_j \frac{\sum_{l=1}^{3} a_{ijl} b_{pjl}}{\sqrt{\sum_{l=1}^{3} a_{ijl}^2} \sqrt{\sum_{l=1}^{3} b_{pjl}^2}}. \end{aligned} \tag{14}$$

The decision procedure for the proposed method can be summarized as follows.

Step 1. Give the characteristic of each alternative and criteria by the linguistic values of TFNs.

Step 2. Using (11), we can obtain the group preference vector for each alternative A_i $(i = 1, 2, \ldots, m)$.

Table 1: Linguistic values of TFNs.

Linguistic term	Linguistic values of TFN
Absolutely poor (AP)	$(0.0, 0.0, 0.1)$
Very poor (VP)	$(0.0, 0.1, 0.2)$
Poor (P)	$(0.1, 0.2, 0.3)$
Medium poor (MP)	$(0.3, 0.4, 0.5)$
Medium (M)	$(0.4, 0.5, 0.6)$
Medium good (MG)	$(0.5, 0.6, 0.7)$
Good (G)	$(0.7, 0.8, 0.9)$
Very good (VG)	$(0.8, 0.9, 1.0)$
Absolutely good (AG)	$(0.9, 1.0, 1.0)$

Step 3. Calculate the expected weight value w_j for a criterion C_j $(j = 1, 2, \ldots, n)$ by using (12) and (13).

Step 4. From formula (14), we can get three weighted similarity measures for an alternative A_i $(i = 1, 2, \ldots, m)$.

Step 5. Rank the alternatives, and select the best one(s) in accordance with the weighted similarity measure.

4. Illustrative Example

4.1. Example 1: Personnel Selection. In this section, a numerical example is presented to illustrate the application of the proposed method. Suppose that the human resources department of a company desires to hire a competent. After initial screening, four candidates (i.e., alternatives) A_1, A_2, A_3, and A_4 remain for further evaluation. In order to select the most suitable candidate, the decision maker invite five experts, who take into account the following five criteria: (1) emotional steadiness (C_1); (2) oral communication skill (C_2); (3) education experience (C_3); (4) work experience (C_4); (5) personality and self-confidence (C_5). Assume that the five experts provide his/her preference information on candidates with regard to criteria by using a linguistic variable, as listed in Table 1.

The decision procedure for the above decision-making problem is as follows.

Step 1. Five experts provide his/her preference information on candidates with linguistic terms; we convert the linguistic variables to TFNs, as depicted in Table 2.

Step 2. The weight vector of five experts is $\lambda = (0.1, 0.2, 0.3, 0.2,$ and $0.2)$, and using (11), we can obtain the group preference vector for each alternative A_i $(i = 1, 2, 3, 4)$, as follows:

$$\begin{aligned} V_1 = \{ & \langle C_1, (0.13, 0.17, 0.22) \rangle, \langle C_2, (0.36, 0.46, 0.56) \rangle, \\ & \langle C_3, (0.39, 0.54, 0.69) \rangle, \langle C_4, (0.42, 0.50, 0.60) \rangle, \\ & \langle C_5, (0.52, 0.60, 0.60) \rangle \}; \\ V_2 = \{ & \langle C_1, (0.22, 0.27, 0.32) \rangle, \langle C_2, (0.54, 0.64, 0.74) \rangle, \\ & \langle C_3, (0.60, 0.75, 0.90) \rangle, \langle C_4, (0.44, 0.54, 0.62) \rangle, \\ & \langle C_5, (0.18, 0.26, 0.26) \rangle \}; \\ V_3 = \{ & \langle C_1, (0.18, 0.23, 0.27) \rangle, \langle C_2, (0.28, 0.36, 0.46) \rangle, \\ & \langle C_3, (0.69, 0.84, 0.99) \rangle, \langle C_4, (0.16, 0.24, 0.34) \rangle, \\ & \langle C_5, (0.48, 0.58, 0.58) \rangle \}; \\ V_4 = \{ & \langle C_1, (0.21, 0.26, 0.31) \rangle, \langle C_2, (0.34, 0.42, 0.52) \rangle, \\ & \langle C_3, (0.57, 0.72, 0.87) \rangle, \langle C_4, (0.28, 0.38, 0.48) \rangle, \\ & \langle C_5, (0.46, 0.56, 0.56) \rangle \}. \end{aligned} \tag{15}$$

Step 3. The weight value of criteria C_i is represented by the following TFNs:

$$\begin{aligned} w_1 &= \langle C_1, (0.3, 0.4, 0.5) \rangle, \\ w_2 &= \langle C_2, (0.5, 0.6, 0.7) \rangle, \\ w_3 &= \langle C_3, (0.1, 0.2, 0.3) \rangle, \\ w_4 &= \langle C_4, (0.9, 1.0, 1.0) \rangle, \\ w_5 &= \langle C_5, (0.4, 0.5, 0.7) \rangle. \end{aligned} \tag{16}$$

The expected weight values can be obtained by (12). Then we normalize the expected weight value by (13) and get the weight vector $W = (0.1481, 0.2222, 0.0741, 0.3580,$ and $0.1975)$.

Step 4. The ideal alternative is given by the decision maker as

$$\begin{aligned} A_p = \{ & \langle C_1, (0.40, 0.50, 0.60) \rangle, \langle C_2, (0.70, 0.80, 0.90) \rangle, \\ & \langle C_3, (0.80, 0.90, 1.00) \rangle, \langle C_4, (0.70, 0.80, 0.90) \rangle, \\ & \langle C_5, (0.50, 0.60, 0.70) \rangle \}. \end{aligned} \tag{17}$$

From formula (14), we can get three weighted similarity measure for an alternative A_i $(i = 1, 2, \ldots, 4)$, as listed in Table 3.

Step 5. From Table 3, we can see that all the proposed method have the same decision results. The decision results of different similarity measures demonstrate that the proposed method for FMCGDM problem is effective.

Let us discuss the different weight $W^* = (0.2, 0.1, 0.3, 0.20,$ and $0.2)$, and using the above 4 steps, we can get the decision results as follows.

In Table 4, it is easy to see that the three similarity measures get the same decision results; the best alternative

Table 2: Preference values of alternative given by 5 experts for TFNs.

A	M	C_1	C_2	C_3	C_4	C_5
	1	(0.0, 0.0, 0.1)	(0.4, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.8, 0.9, 1.0)	(0.5, 0.6, 0.7)
	2	(0.1, 0.2, 0.3)	(0.3, 0.4, 0.5)	(0.4, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.9, 1.0, 1.0)
A_1	3	(0.4, 0.5, 0.6)	(0.4, 0.5, 0.6)	(0.1, 0.2, 0.3)	(0.3, 0.4, 0.5)	(0.0, 0.0, 0.1)
	4	(0.3, 0.4, 0.5)	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.2)	(0.5, 0.6, 0.7)	(0.7, 0.8, 0.9)
	5	(0.5, 0.6, 0.7)	(0.7, 0.8, 0.9)	(0.2, 0.3, 0.4)	(0.0, 0.0, 0.1)	(0.5, 0.6, 0.7)
	1	(0.4, 0.5, 0.6)	(0.8, 0.9, 1.0)	(0.5, 0.6, 0.7)	(0.4, 0.5, 0.6)	(0.0, 0.1, 0.2)
	2	(0.1, 0.2, 0.3)	(0.4, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.8, 0.9, 1.0)	(0.1, 0.2, 0.3)
A_2	3	(0.7, 0.8, 0.9)	(0.5, 0.6, 0.7)	(0.3, 0.4, 0.5)	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.2)
	4	(0.5, 0.6, 0.7)	(0.7, 0.8, 0.9)	(0.3, 0.4, 0.5)	(0.9, 1.0, 1.0)	(0.0, 0.0, 0.1)
	5	(0.5, 0.6, 0.7)	(0.3, 0.4, 0.5)	(0.4, 0.5, 0.6)	(0.0, 0.1, 0.2)	(0.8, 0.9, 1.0)
	1	(0.3, 0.4, 0.5)	(0.1, 0.2, 0.3)	(0.7, 0.8, 0.9)	(0.4, 0.5, 0.6)	(0.5, 0.6, 0.7)
	2	(0.1, 0.2, 0.3)	(0.5, 0.6, 0.7)	(0.3, 0.4, 0.5)	(0.0, 0.0, 0.1)	(0.9, 1.0, 1.0)
A_3	3	(0.9, 1.0, 1.0)	(0.4, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.0, 0.1, 0.2)	(0.3, 0.4, 0.5)
	4	(0.1, 0.2, 0.3)	(0.0, 0.0, 0.1)	(0.5, 0.6, 0.7)	(0.4, 0.5, 0.6)	(0.0, 0.1, 0.2)
	5	(0.4, 0.5, 0.6)	(0.4, 0.5, 0.6)	(0.3, 0.4, 0.5)	(0.0, 0.1, 0.2)	(0.7, 0.8, 0.9)
	1	(0.5, 0.6, 0.7)	(0.5, 0.6, 0.7)	(0.4, 0.5, 0.6)	(0.3, 0.4, 0.5)	(0.8, 0.9, 1.0)
	2	(0.4, 0.5, 0.6)	(0.3, 0.4, 0.5)	(0.5, 0.6, 0.7)	(0.0, 0.1, 0.2)	(0.4, 0.5, 0.6)
A_4	3	(0.3, 0.4, 0.5)	(0.3, 0.4, 0.5)	(0.4, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.7, 0.8, 0.9)
	4	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.2)	(0.5, 0.6, 0.7)	(0.3, 0.4, 0.5)	(0.1, 0.2, 0.3)
	5	(0.7, 0.8, 0.9)	(0.4, 0.5, 0.6)	(0.1, 0.2, 0.3)	(0.3, 0.4, 0.5)	(0.3, 0.4, 0.5)

Table 3: Decision results of three weighted similarity measures for TFNs.

	S^J	S^E	S^C
A_1	0.7869	0.8707	0.9974
A_2	0.7976	0.8775	0.9988
A_3	0.6367	0.7546	0.9918
A_4	0.7528	0.8525	0.9964
Ranking	$A_2 \succ A_1 \succ A_4 \succ A_3$	$A_2 \succ A_1 \succ A_4 \succ A_3$	$A_2 \succ A_1 \succ A_4 \succ A_3$
Best	A_2	A_2	A_2

Table 4: Decision results for different weights in the same numerical example.

	S^J	S^E	S^C
A_1	0.7674	0.8561	0.9959
A_2	0.8022	0.8793	0.9971
A_3	0.7567	0.8381	0.9950
A_4	0.8180	0.8928	0.9984
Ranking	$A_4 \succ A_2 \succ A_1 \succ A_3$	$A_4 \succ A_2 \succ A_1 \succ A_3$	$A_4 \succ A_2 \succ A_1 \succ A_3$
Best	A_4	A_4	A_4

is A_4. From Tables 3 and 4, for the different weights of the criteria, the decision results are different.

4.2. Example 2: Investment Case Selection. Using the illustrative example in [23], an investment company wants to invest a sum of money in the best option. There is a panel with five possible alternatives to invest the money: A_1 is a car company; A_2 is a food company; A_3 is a computer company; A_4 is an arms company; A_5 is a TV company. The three experts (G_1, G_2, and G_3) must take a decision according to the following four attributes: C_1 is the risk analysis; C_2 is the growth analysis; C_3 is the social-political impact analysis; C_4 is the environmental impact analysis.

Using our methods in Section 3, the decision procedure is as follows.

Step 1. The TFNs preference vector of the three experts is shown in Table 5.

Step 2. The weight vector of three experts is $\lambda = (0.2, 0.5, \text{and } 0.3)$, and using (11), we can obtain the group preference

Table 5: The TFNs preference vector of the three experts.

A	M	C_1	C_2	C_3	C_4
	1	(0.80, 0.85, 0.90)	(0.72, 0.76, 0.80)	(0.91, 0.93, 0.96)	(0.62, 0.65, 0.68)
A_1	2	(0.65, 0.70, 0.75)	(0.57, 0.61, 0.65)	(0.76, 0.78, 0.81)	(0.47, 0.50, 0.53)
	3	(0.42, 0.47, 0.52)	(0.34, 0.38, 0.42)	(0.53, 0.55, 0.58)	(0.24, 0.27, 0.30)
	1	(0.88, 0.90, 0.93)	(0.67, 0.77, 0.83)	(0.60, 0.67, 0.70)	(0.69, 0.72, 0.75)
A_2	2	(0.73, 0.75, 0.78)	(0.52, 0.62, 0.68)	(0.42, 0.52, 0.50)	(0.54, 0.57, 0.60)
	3	(0.50, 0.52, 0.55)	(0.29, 0.39, 0.45)	(0.22, 0.39, 0.32)	(0.31, 0.34, 0.37)
	1	(0.95, 0.97, 0.98)	(0.90, 0.93, 0.95)	(0.77, 0.79, 0.82)	(0.93, 0.95, 0.96)
A_3	2	(0.80, 0.82, 0.83)	(0.75, 0.78, 0.80)	(0.62, 0.64, 0.67)	(0.78, 0.80, 0.81)
	3	(0.57, 0.59, 0.60)	(0.52, 0.55, 0.57)	(0.39, 0.41, 0.44)	(0.55, 0.57, 0.58)
	1	(0.82, 0.85, 0.88)	(0.97, 0.98, 1.00)	(0.98, 0.99, 1.00)	(0.97, 0.99, 1.00)
A_4	2	(0.67, 0.70, 0.73)	(0.82, 0.83, 0.85)	(0.83, 0.84, 0.85)	(0.82, 0.84, 0.85)
	3	(0.44, 0.47, 0.50)	(0.59, 0.60, 0.62)	(0.60, 0.61, 0.62)	(0.59, 0.61, 0.62)
	1	(0.78, 0.79, 0.81)	(0.78, 0.79, 0.81)	(0.83, 0.85, 0.88)	(0.94, 0.97, 0.99)
A_5	2	(0.63, 0.64, 0.66)	(0.63, 0.64, 0.60)	(0.68, 0.70, 0.76)	(0.79, 0.82, 0.80)
	3	(0.40, 0.41, 0.43)	(0.40, 0.41, 0.43)	(0.45, 0.47, 0.50)	(0.56, 0.59, 0.61)

Table 6: Decision results of different methods in the same numerical example.

	S^J	S^E	S^C	FIOWHM operator
A_1	0.6486	0.7141	0.7997	1.3310
A_2	0.6465	0.7136	0.7988	0.6690
A_3	0.7329	0.7630	0.7998	2.5000
A_4	0.7730	0.7859	0.7999	4.5000
A_5	0.7624	0.7803	0.7998	3.5000
Ranking	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$
Best	A_4	A_4	A_4	A_4

vector for each alternative A_i $(i = 1, 2, 3, 4, 5)$ as follows:

$$\begin{aligned} V_1 = \{ & \langle C_1, (0.611, 0.661, 0.711) \rangle, \langle C_2, (0.531, 0.571, 0.611) \rangle, \\ & \langle C_3, (0.721, 0.741, 0.771) \rangle, \langle C_4, (0.431, 0.461, 0.491) \rangle \}; \\ V_2 = \{ & \langle C_1, (0.691, 0.711, 0.741) \rangle, \langle C_2, (0.481, 0.581, 0.641) \rangle, \\ & \langle C_3, (0.396, 0.511, 0.486) \rangle, \langle C_4, (0.501, 0.531, 0.561) \rangle \}; \\ V_3 = \{ & \langle C_1, (0.761, 0.781, 0.791) \rangle, \langle C_2, (0.711, 0.741, 0.761) \rangle, \\ & \langle C_3, (0.581, 0.601, 0.631) \rangle, \langle C_4, (0.741, 0.761, 0.771) \rangle \}; \\ V_4 = \{ & \langle C_1, (0.631, 0.661, 0.691) \rangle, \langle C_2, (0.781, 0.791, 0.811) \rangle, \\ & \langle C_3, (0.791, 0.801, 0.811) \rangle, \langle C_4, (0.781, 0.801, 0.811) \rangle \}; \\ V_5 = \{ & \langle C_1, (0.591, 0.601, 0.621) \rangle, \langle C_2, (0.591, 0.601, 0.591) \rangle, \\ & \langle C_3, (0.641, 0.661, 0.706) \rangle, \langle C_4, (0.751, 0.781, 0.781) \rangle \}. \end{aligned} \tag{18}$$

Step 3. From [23], the weight value of criteria vector C_i is $W = (0.2, 0.1, 0.1, 0.4)$.

Step 4. The ideal alternative is given by the decision maker as

$$\begin{aligned} A_p = \{ & \langle C_1, (0.44, 0.47, 0.50) \rangle, \langle C_2, (0.79, 0.82, 0.80) \rangle, \\ & \langle C_3, (0.97, 0.98, 1.00) \rangle, \langle C_4, (0.83, 0.85, 0.88) \rangle \}. \end{aligned} \tag{19}$$

From formula (14), we can get three weighted similarity measures for an alternative A_i $(i = 1, 2, \ldots, 5)$, as listed in Table 6. For comparison, we also list the decision results of FIOWHM operator method in Table 6.

Step 5. From Table 6, we can see that all the proposed methods have the same decision results, but our method has simpler computation and gets the same results more rapidly than the FLOWHM method.

The decision results of different methods demonstrate that the proposed method for FMCGDM problem is effective.

5. Conclusion

The similarity measure is an effective method to solve the FMCDM problem, but it rarely applies to triangular FMCDM problem. In this paper, we propose three similarity measures and three weighted similarity measures between two TFNs based on the vector similarity measures. Using the similarity measures model, we establish a method for FMCGDM with known information on criterion weight and group weight, in which the alternatives and criteria are given by triangular fuzzy information. Finally, a practical example is given to select the most suitable candidate in human resource evaluation system, by the weighted similarity measures between each alternative and ideal alternative; the ranking order of all alternatives can be determined, and the best one(s) can be easily identified as well. For comparison, we also apply our method to solve the triangular FMCGDM problem in [23]. The different methods have the same decision results, which show that our proposed method in this paper is applicable and effective.

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