5.46

ALGORITHM FOR THE PREDICTION OF FUNCTIONAL DELAYS IN THE BEHAVIOUR OF CONCRETE DAMS

by

Dr Eng. Giovanni Lombardi

Consulting Engineer, Minusio (Switzerland)

Eng. Francesco Amberg

Head Section for Analysis by Lombardi Ltd., Minusio (Switzerland)

LOMBARDI ENGINEERING LIMITED

Dr Eng. Georges R. Darbre

Head Federal Authority for Dam Safety, Bern (Switzerland)

Our Ref.: 102.2-R-165C

Minusio, April-August 2006

TABLE OF CONTENTS

KEYWORDS: dams, delays, Laplace, transform, numerical analysis, temperature, deformations, drain flow rate.

1. INTRODUCTION

Concrete dams respond with some delays to external solicitations. This may be the temperature within the dam concrete as a response to the external one, drainage flow as a response to reservoir level, uplift forces as a response to reservoir level, and so on. These internal states in turn influence dam behaviour, the continuous assessment of which is part of every dam surveillance concept (in particular the comparison between the actual behaviour – say deformation - and the expected one).

In practice, instantaneous relationships between external solicitations and internal ones or between external solicitations and dam behaviour are often used. This is consistent with the theory of elasticity, which assumes an instantaneous response of a structure to the loads. The delays due to viscosity, diffusion of water pressure in rock masses, porosity of the materials and so on are thus neglected. Experience however shows that their account is sometimes necessary to obtain a meaningful assessment of dam behaviour or dam internal state.

The aim of the paper is to present a simple algorithm for the calculation of delayed responses. A first application is made to the calculation of the internal temperature field in a concrete dam, for which the algorithm was originally developed. Further applications include the calculation of drainage flows and uplift pressures, as well as the deformation resulting from the latter. These applications demonstrate the versatility of the algorithm and its wide range of applicability.

At this point, it may already be mentioned that delayed responses are often also damped (as compared to a hypothetical instantaneous elastic response). In some cases though, the delayed response may be amplified, for example when dealing with viscosity.

2. DEVELOPMENT OF THE ALGORITHM

2.1 Basic formulation

The fundamental idea is to recognise that the response at a time *t* is not only related to the values of external variables at the same time *t*, but also to their values at earlier times τ. This relation is assumed to be described by an appropriate weighting function chosen by analogy to Laplace transforms /1/. This results in the following fundamental expression

$$F(t) = \frac{1}{T_0} \cdot \int_{-\infty}^{t} e^{-\frac{(t-\tau)}{T_0}} \cdot \varphi(\tau) \cdot d\tau \qquad [1]$$

Where F(t) is the (delayed) response at time *t*, $\varphi(\tau)$ are the (past) values of external variables at times τ and $\frac{1}{T_0} e^{-\frac{(t-\tau)}{T_0}}$ is the weighting function whose shape is illustrated in **Figure 1**. The decay of the weighting function with time is controlled by the value of the characteristic time T_0. This is the only controlling parameter of the algorithm. It will have to be selected according to the problem at hand as will be illustrated later. For the sake of simplicity, *F(t)* will be called the weighted function and $\varphi(\tau)$ the original function.

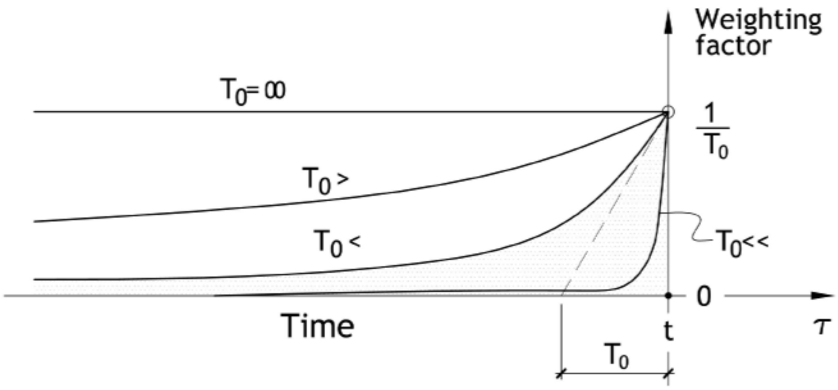

Figure 1: Shape of weighting function for different values of characteristic time T_0 .

Applying equation 1 leads to a response that is smoother than the original function. This "smoothing effect" depends on T_0 (smoother for larger values of T_0) as illustrated in **Figure 2**

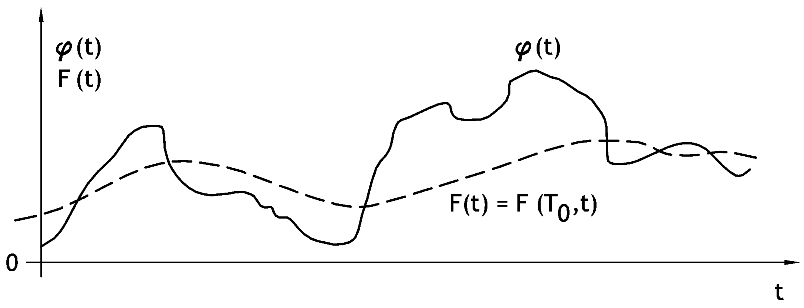

Figure 2: Original function φ(t) and weighted function F(t).

Three distinct ways of numerically integrating equation 1 are shown hereafter.

2.2 Time-stepping algorithm

2.2.1 Case 1: Discrete integration at regular time steps

It is postulated that the original function *φ(t)* is known at regular time steps *Δt* apart (e.g. 1 day or 1 week). As a first approximation, the integral of equation [1] is written as the sum of discrete values as follows

$$F(t) \cong \frac{1}{T_0} \cdot \sum_{n=-\infty}^{i} e^{-\frac{(i\Delta t - n\Delta t)}{T_0}} \cdot \varphi(n\Delta t) \cdot \Delta t = \frac{1}{T_0} \cdot \sum_{n=-\infty}^{i-1} e^{-\frac{(i\Delta t - n\Delta t)}{T_0}} \varphi(n\Delta t) \cdot \Delta t + \frac{1}{T_0} \cdot \varphi(i\Delta t) \cdot \Delta t \qquad [2]$$

Similarly, the same weighted function takes the following value one time step earlier

$$F(t - \Delta t) \cong \frac{1}{T_0} \cdot \sum_{n=-\infty}^{i-1} e^{-\frac{(i\Delta t - n\Delta t - \Delta t)}{T_0}} \cdot \varphi(n\Delta t) \cdot \Delta t = \frac{1}{T_0} \cdot e^{\frac{\Delta t}{T_0}} \cdot \sum_{n=-\infty}^{i-1} e^{-\frac{(i\Delta t - n\Delta t)}{T_0}} \cdot \varphi(n\Delta t) \cdot \Delta t \qquad [3]$$

Substituting equation [3] in equation [2] results in the following, simple recursive formula

$$F(t) \cong e^{-\frac{\Delta t}{T_0}} \cdot F(t - \Delta t) + \frac{\Delta t}{T_0} \cdot \varphi(t) \qquad [4a]$$

Or

$$F(t) \cong \alpha \cdot F(t - \Delta t) + \frac{\Delta t}{T_0} \cdot \varphi(t) \qquad [4b]$$

With $\alpha = e^{-\Delta t / T_0}$

Thus, the value of the delayed function at time *t* is a linear combination of its value at time *t-Δt* and of the value of the original function at time *t*.

Note that if the time interval *Δt* is very small with respect to T_0 (let say $\Delta t<0.1\cdot T_0$), then

$$\alpha = e^{-\frac{\Delta t}{T_0}} \cong 1-\frac{\Delta t}{T_0} \qquad [5]$$

and equation [4] simplifies to

$$F(t) \cong \alpha \cdot F(t-\Delta t)+(1-\alpha)\cdot\varphi(t) \qquad [6]$$

2.2.2 Case 2: Linear integration at regular time steps

It is again postulated that the original function $\varphi(t)$ is known at regular time steps *Δt* apart. Equation [1] is now calculated based on a linear interpolation of these discrete values. This results in

$$F(t) \cong \alpha \cdot F(t-\Delta t)+\left(1+\frac{\alpha}{\beta}-\frac{1}{\beta}\right)\cdot\varphi(t)+\left(\frac{1}{\beta}-\frac{\alpha}{\beta}-\alpha\right)\cdot\varphi(t-\Delta t) \qquad [7]$$

with

$$\alpha = e^{-\frac{\Delta t}{T_0}} \qquad [8]$$

and

$$\beta = \frac{\Delta t}{T_0} \qquad [9]$$

In this more precise integration, the value of the delayed function at time *t* now also depends on the value of the original function at time *t-Δt*.

Note that if *Δt* is very small with respect to T_0 and thus the approximation of equation [5] applies, then equation [7] simplifies to

$$F(t) \cong \alpha \cdot F(t-\Delta t)+(1-\alpha)\cdot\varphi(t-\Delta t) \qquad [10]$$

That is, the delayed response at time *t* depends now on the values of the response and of the original function at time *t-Δt*.

2.2.3 *Case 3: Linear integration at irregular time steps*

For values of the origin function $\varphi(t)$ known at irregular time steps, equation [7] or [10] (together with equations [8] and [9]) still apply, whereby Δt is now the last time step considered, i.e. the time lapse between the discrete time *t* and the previous discrete time.

3. TEMPERATURE IN CONCRETE DAMS

3.1 General

When analysing the behaviour of a concrete dam, it is generally sufficient to assume an instantaneous response of the dam to the temperature field measured at a number of discrete locations in the dam and outside. Concrete dams are however known to "respond" primarily to changes to the average temperatures across the sections and to the first thermal moments. In an analysis, it is thus preferable to first calculate these averages and moments from the individual temperature values. They obviously vary less than the temperatures at the surface or near to it ("damped" response) and also with some delay with respect to them.

The equations governing the temperature distribution in a body subjected to temperature changes at its boundaries are known. They can be solved precisely either with the help of Fourier transforms in the frequency domain or directly in the time domain through convolution integrals. Then, obtaining the average temperature and the thermal moments across a section is fairly straightforward. Still, this is not done in every day practice because of the complexity of the associated procedures. The proposed time-stepping algorithm was originally developed to overcome this complexity. This is presented here. The theoretical basis and solution of the temperature problem are recalled in the following Section, followed by applications of the proposed algorithm.

3.2 One dimensional thermal problem

For « thin » structures without internal heat sources and with a fairly uniform spatial distribution of the surface temperatures, it can often be assumed that the internal temperature distribution is governed by the one-dimensional heat-conduction equation in which the spatial variable is the position across the thickness of the structure. The aforementioned conditions hold true for concrete arch dams. The governing differential equation is then (*T* being the temperature, *t* the time and *x* the abscissa)

$$\frac{\partial T(x,t)}{\partial t} = a\frac{\partial^2 T(x,t)}{\partial x^2} \qquad \text{(a: coefficient of thermal diffusivity)} \qquad [11]$$

3.3 Analytical solution by way of a convolution integral

Only the analytical solution in the time domain is presented here, because of its similitude with the proposed method.

Equation [11] can, in principle, be directly solved by way of a convolution integral. For the case of a semi-infinite body, the solution simplifies to

$$T(x,t) = \int_{-\infty}^{t} g(x,t-\tau)\cdot T(x=0,\tau)\cdot d\tau \qquad [12]$$

i.e. the temperature *T* at any location *x* and instant *t* depends on the temperature at the wall surface (*x=0*) at the past instants τ. The analytical expression for the impulse response function *g* is (Carslaw, /2/)

$$g(x,t) = \frac{x}{2\sqrt{\pi a}}\cdot\frac{e^{-\frac{x^2}{4at}}}{t^{1.5}} \qquad [13]$$

Physically, solving the governing equation in the time domain corresponds to decomposing the given temperature history at *x=0* in a series of heat impulses, with the associated temperature development across the section being calculated for each one of them. The actual temperature distribution follows by superposition. This way of proceeding is however extremely challenging and hardly a practical option.

Equation [12] can be viewed as the expression of the internal temperature at any location *x* and time *t* as a weighted average of the temperature at the surface *T(x=0)* at past times τ, with g(x,t) being the weighting function. This is quite similar to the basic formulation of the delay algorithm of equation [1], where the weighting function, identified as *l* in Figure 3, is

$$l(x,t) = \frac{1}{T_0} \cdot e^{-\frac{t}{T_0}} \qquad [14]$$

The comparison of both weighting functions is shown in **Figure 3.**

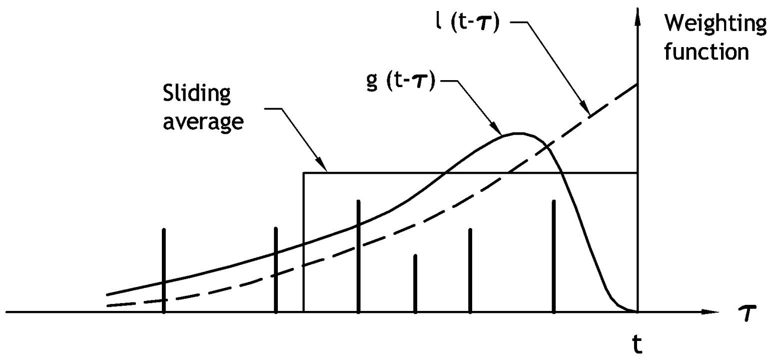

Figure 3: Comparison between the impulse response function (g(t-τ)), the weighting function of the delay algorithm (l(t-τ)) and a possible sliding average. The integral of each of these functions equals unity.

The idea is now to use the form of equation [14] rather than that of equation [13] in the convolution integral of equation [12] as

$$l(x,t) = \frac{B}{T_0} \cdot e^{-\frac{t}{T_0}} \qquad [15]$$

Where *B* is a constant to be defined.
This obviously leads to an approximate solution of the temperature problem at hand. As has been illustrated in Section 2, this is however also a very simple integration procedure! To what extent the resulting solution is acceptable is investigated in Section 3.4 for a harmonic temperature.

3.4 Harmonic temperature variation

The exact temperature distribution inside a concrete body of semi-infinite extent subjected to a harmonic variation of the surface temperature of frequency ω is

$$T(x,t) = A \cdot e^{-\frac{x}{d}} \cdot \sin\left(\omega \cdot t - \frac{x}{d}\right) \qquad [16]$$

It can be shown that this exact analytical solution is recovered form the proposed algorithm when T_0 and B are selected as follows

$$T_0 = \frac{\tan\left(x/d\right)}{\omega} \qquad [17]$$

$$B = \frac{e^{-x/d}}{\cos\left(x/d\right)} \qquad [18]$$

with

$$d = \sqrt{\frac{2a}{\omega}} \qquad [19]$$

and

$$a = \frac{\lambda}{c \cdot \gamma} \qquad \text{(a=coefficient of thermal diffusivity)} \qquad [20]$$

λ being the thermal conductivity, c the specific heat and γ the density of the concrete. Note that because of the factor B appearing in equation [15], equation [7] now becomes (similar for equation [10] as well as for [4a] and [4b])

$$F(t) \cong \alpha \cdot F(t - \Delta t) + \left(1 + \frac{\alpha}{\beta} - \frac{1}{\beta}\right) \cdot B \cdot \varphi(t) + \left(\frac{1}{\beta} - \frac{\alpha}{\beta} - \alpha\right) \cdot B \cdot \varphi(t - \Delta t) \qquad [21]$$

Having established the relations that reproduce the exact solution in the case of a harmonic temperature variation at the surface, it remains to be seen how well the algorithm performs for transient temperatures at the wall surface.

3.5 Transient surface temperature

It is recalled that the parameters of the recursive formula of Section 3.4 have been determined for a harmonic excitation of circular frequency ω. Since external temperature changes are in most cases nearly periodic with a period of one year, the circular frequency corresponding to this period of one year is selected to calculate the parameters T_0 and B.

A transient surface temperature however also contains higher frequencies. Keeping the values of the parameters T_0 and B fixed (basis: period of one year), the internal temperature field is calculated with some error by the algorithm. **Table 1** shows the decrease of the amplitude of a harmonic oscillation as a function of the period and the distance x for a = 0.1 m^2/day.

Period of harmonic oscillation	Heat propagation ("exact") at x=1 m, 2 m, 3 m, 4 m	Recursive formulae at x=1 m, 2 m, 3 m, 4 m
T = 1 year	75% 56% 41% 31%	75% 56% 41% 31%
T = 30 days	36% 13% 5% 2%	21% 8% 4% 3%
T = 7 days	12% 1% - -	5% 2% 1% 1%

Table 1: Decrease in amplitude of a harmonic oscillation (100%) as a function of the distance *x* from the surface and the period *T* (with a=0.1 m^2/day and $\omega=2\pi$ (1/year)).

It appears clearly that the higher frequencies are calculated with some imprecision by way of the recursive formula. These high frequencies are however heavily damped so that the absolute error remains small. In addition, the amplitude of the higher frequencies at the surface is very often smaller than that of the reference yearly harmonic, whose propagation is exactly reproduced.

3.6 Application to dams

3.6.1 Wall of finite thickness

In the presence of a wall of finite thickness, the algorithm is applied starting independently from both sides. The total temperature is obtained from superposition of the two components.

Furthermore, the reflection of the thermal wave at the surface has to be considered. When the temperature within the wall due to the temperature variations on one of the surfaces (say the downstream one) is calculated, a temperature on the opposite surface is thus obtained. This calculated temperature is then introduced with the opposite sign together with the actual one as "surface temperature" in the calculation from upstream; and so on. This is illustrated in **Figure 4** in which the upstream surface temperature is equal to the average downstream one in this example.

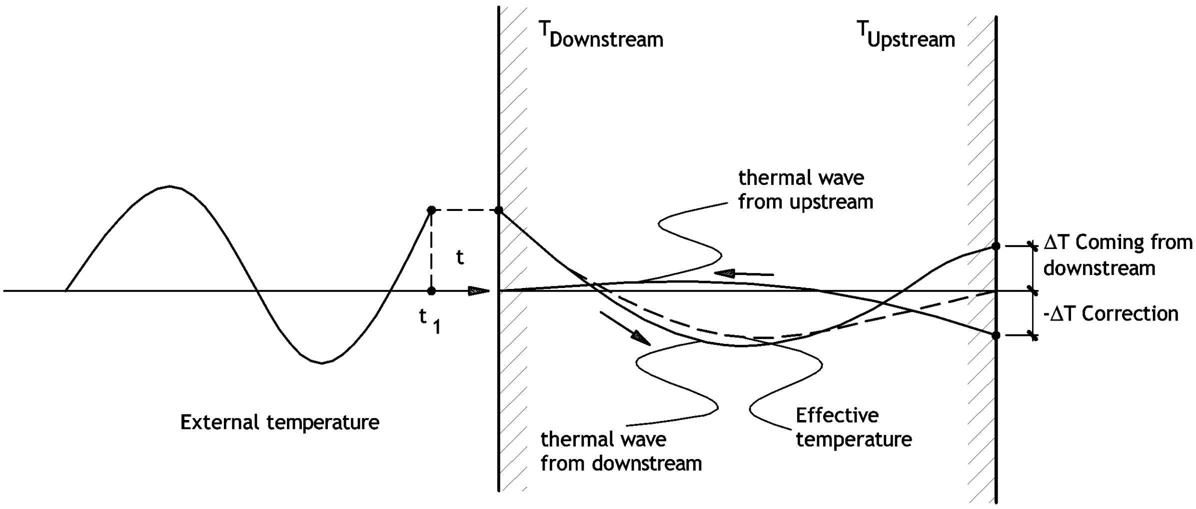

Figure 4: Correction of the error resulting on the opposite face due to the reflection of the thermal wave.

3.6.2 Implementation issues

From the equations [17] and [18] it appears that the application of the recursive formula should remain limited to a depth of $x_{max}<d\cdot\pi/2$ to avoid singularities (division by 0 or tan(90°)=∞). For a concrete body this corresponds to a maximal distance x from the surface of 2.5-4.0 m. For larger thicknesses, the wall can simply be divided in successive slices and the heat propagation simulated by applying the weighting, recursive formula in series.

At the same time, the formula should be used only in situations of nearly periodic variations of surface temperatures. This implies that it cannot be applied to the average yearly temperatures, which are different on either side of a wall. These averages must be treated separately by a linear distribution across the wall. The outside temperatures to be introduced in the calculation are then the differences between the measured surface temperatures, corrected with the temperature coming from inside, and the average ones. For all practical purposes, ω will be selected to correspond to a period of 1 year (i.e. $\omega=2\pi$ [1/year]). The recursive formula will then reproduce very well the heat propagation of annual to semi-annual period.

3.7 Example

A wall of 16 m thickness is looked at. The surface temperatures are reported in **Figure 5**. This corresponds to a real case, in which the upstream face is under water for some periods of the year and above water for others. Temperatures have been measured at time intervals varying between 2 and 18 days (average of 7 days).

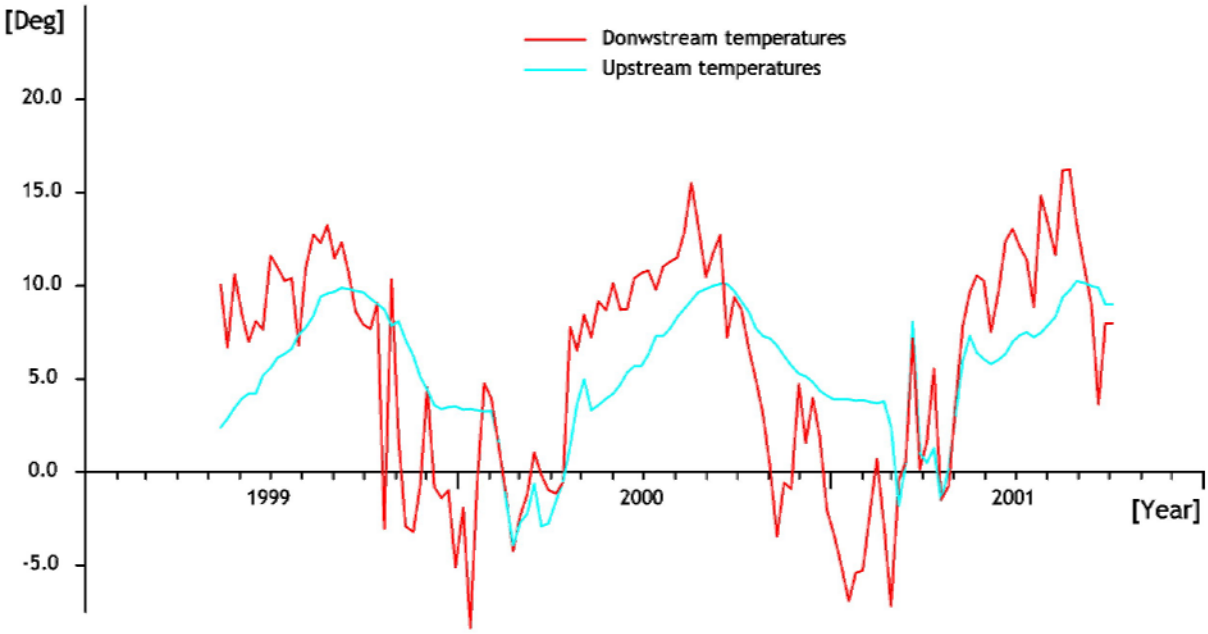

Figure 5: Readings of external temperatures.

The model and the calculation results are presented in **Figure 6**. The comparison shows a good agreement between the results obtained by way of the recursive formula and by way of a direct, "exact" integration. The maximum difference (as soon the influence of the initial conditions has disappeared) is less than 0.4°C. One notes some smoothening and delay with respect to the "exact" solution whenever sudden variations and irregularities occur, they are however of minor importance.

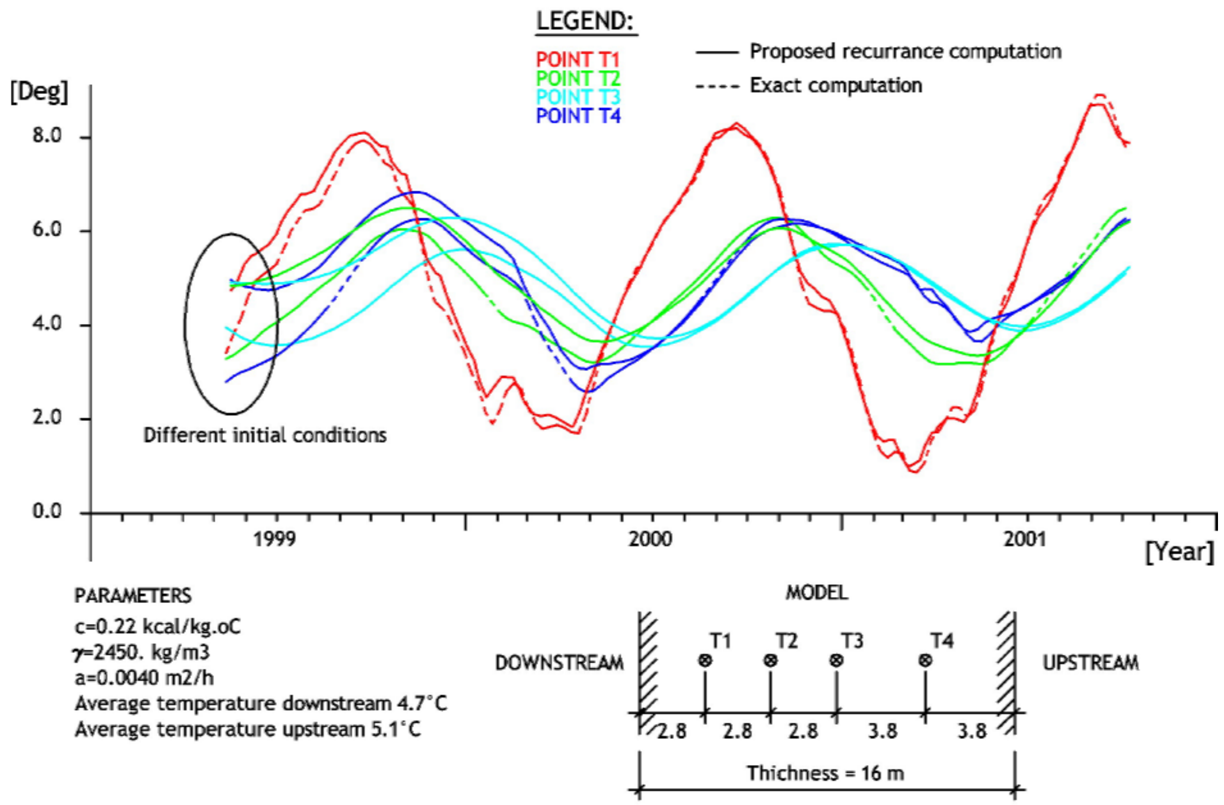

Figure 6: Comparison between results of time steps simulation and finite difference computation. Input according to figure 5.

This example shows a very good agreement between the results obtained by application of the recursive formula and the "exact" integration. The difference of 0.4°C can be considered to be a maximum value for the error as in this example (selected on purpose) the surface temperatures vary strongly. In the case that temperatures are measured with thermometers placed at shallow depths from the surface, this difference will be even more reduced. The recursive formula is thus, for all practical purposes, equivalent to a direct integration … but much simpler to handle.

3.8 Other applications for thermal problems

The recursive formula is a substitute to the numerical integration of the governing differential equations. The associated calculation of the temperature at any location in a wall is a very simple one. Air and water temperatures are usually taken as surface (or "outside") temperatures (possibly modified for solar radiation and convection). Temperatures measured at internal thermometers (for example placed near both surfaces) can however also be advantageously used. These thermometers define in fact a fictitious wall of reduced thickness between them, on which the recursive formula can be applied. The advantage of placing thermometers below the wall surface is that they filter out the higher frequencies (for example the daily variations), but retains the lower ones (for example the monthly variations) that affect the dam behaviour.

A further use of the time-step algorithm can be the interpolation of the temperature between two existing thermometers. This question may rise for example when an intermediate thermometer falls out of service

Taking advantage of the good results obtained for the example presented here above, where the parameters are defined by the thermal properties of the concrete, the same algorithms can be integrated in a statistical model (a posteriori) to explain dam displacements due to the external temperatures. The algorithm takes care of the delay between internal and external temperature. In this case the parameters T_0 and B need, in no way, to correspond to an annual frequency, but are optimised by a statistical analysis similarly as presented in Section 4.2 for the flow rates of drains and the uplift pressures.

4. OTHER APPLICATIONS

4.1 Viscous-elastic displacements of concrete dam

The displacements of a concrete dam are obviously function of the water level in the reservoir and of the temperature field in the concrete. It is generally assumed that the governing relationships are linear elastic, justified by the fact that the stress level in a dam is moderate. This approach tends to give satisfactory results.

In a recent analysis of dam behaviour, the viscous response of a 220 m high arch dam to the variations of the impounding level could be clearly observed. This viscous behaviour might stem both from the properties of the concrete as well as from the effect of the drainage of the rock foundation.

The total displacement of any point due to the water level is composed of an instantaneous elastic part and of a delayed viscous elastic part. This is illustrated in **Figure 7** for a sudden change of an external load. In the absence of an appropriate constitutive law, this viscous behaviour was accounted for by means of the proposed algorithm.

The displacements measured in a selected point of the dam are shown in **Figure 8**. The difference between measured and computed displacements (response to both the variations of water level and of temperatures) is reported in **Figure 9,** considering no delay in the calculated displacements.

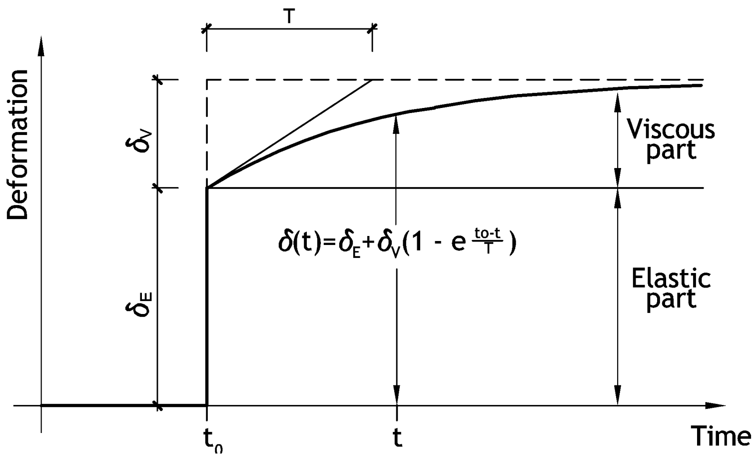

Figure 7: Elastic-viscous behaviour of the concrete modeled by the proposed algorithm.

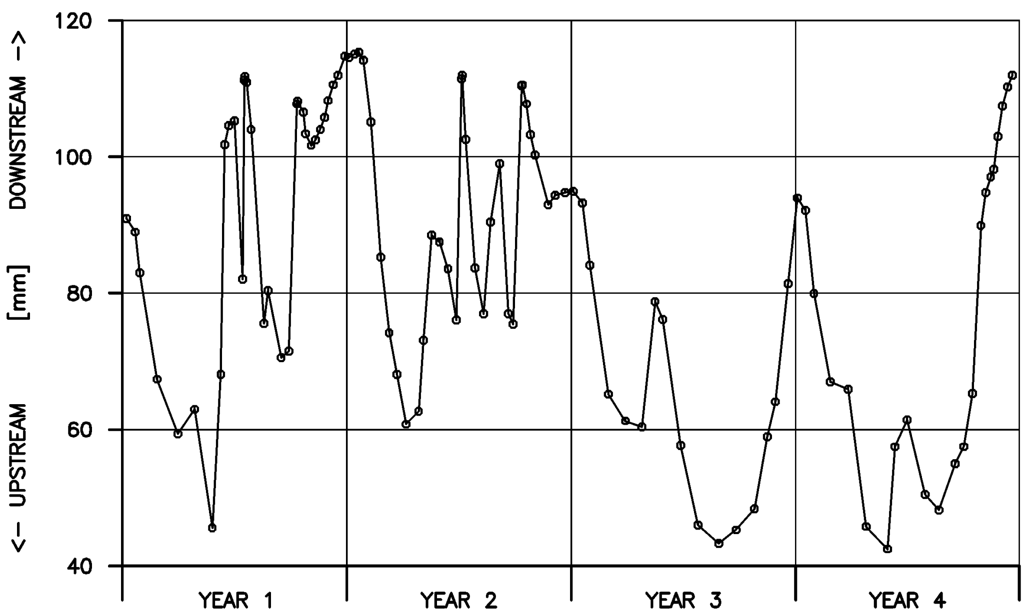

Figure 8: Radial displacements measured in a selected point of the dam.

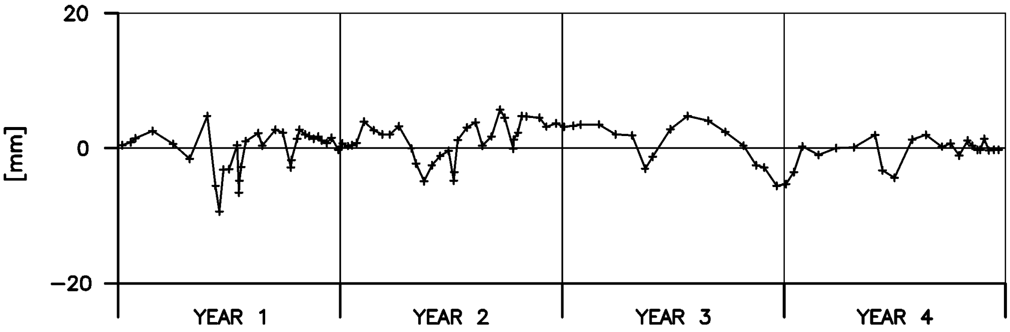

Figure 9: Case 1, elastic model without delay: difference between measured and computed displacements.

The residues shown in Figure 9 seem to reflect, in some reverse way, the rapid changes of the water level, while the relatively slow seasonal fluctuations of the lake are quite well simulated. This trend is typical of a viscous behaviour. The simulation of the observed displacement could be improved only after introducing a delay in the analysis for the part of the displacements due to the water level. The corresponding results are presented in **Figure 10**. The standard deviation of the residues could be reduced from 2.6 mm to 1.6 mm.

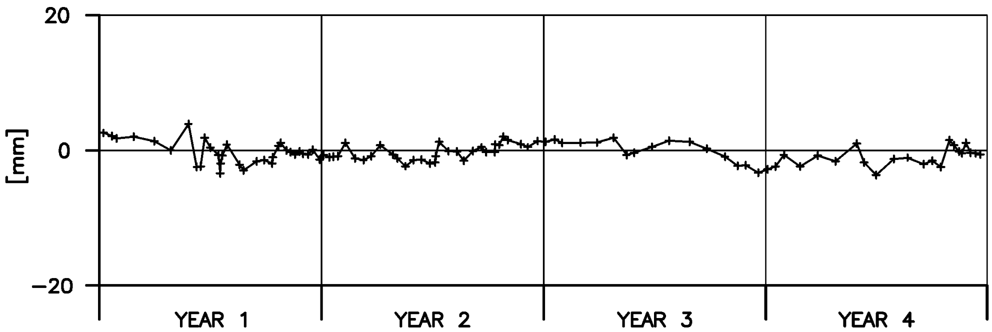

Figure 10: Case 2, elastic viscous model considering a delay: difference between measured and computed displacements.

In this particular example, the analysis results indicate that the total displacement due to the water pressure is composed per 80% of the instantaneous linear elastic part and per 20% of the delayed viscous one. The latter is calculated according to equation [7] with a characteristic time of 60 days.

4.2 Drain flow rates and uplift pressures

The use of predictive models for the flow rate of drains and for the uplift pressures is less frequent than for the deformations and displacements of a dam. One reason may be that a greater importance is attached to the deformations than to other quantities when assessing the dam behaviour for safety purposes. An other reason may also be that it is difficult to set up a model for predicting, say, flow rates or uplift pressures.

There are however situations where these variables are of decisive importance, for example when assessing the safety of rock abutments. It may be recalled that many dam accidents were due to hydraulic phenomena in the rock foundation as well as in the fill of embankment dams.

Interstitial water pressures and drainage rates are primarily function of the water level in the reservoir and in some cases also of the external temperature. The season may also play a role due to external inflows.
The influence of the impounding levels is only approximately instantaneous due to a number of aspects like the presence of pores and cracks or joints, which need to be filled or emptied before the pressure changes can be transferred to downstream. A larger or smaller delay will thus occur between cause and effects. Due to the complexity of the water circulation in rock masses, there is no hope to set up an "a priori deterministic model" that is able to explain the phenomena. The only way is to define a "statistical a posteriori model" based on historical data.

The algorithm of Section 2 is used to do so. The results are illustrated in **Figure 11** for the flow rate of drains in the foundation rock of an arch dam as a function of the water level. A characteristic time of about 6 days was used, a value obtained by an optimisation process.

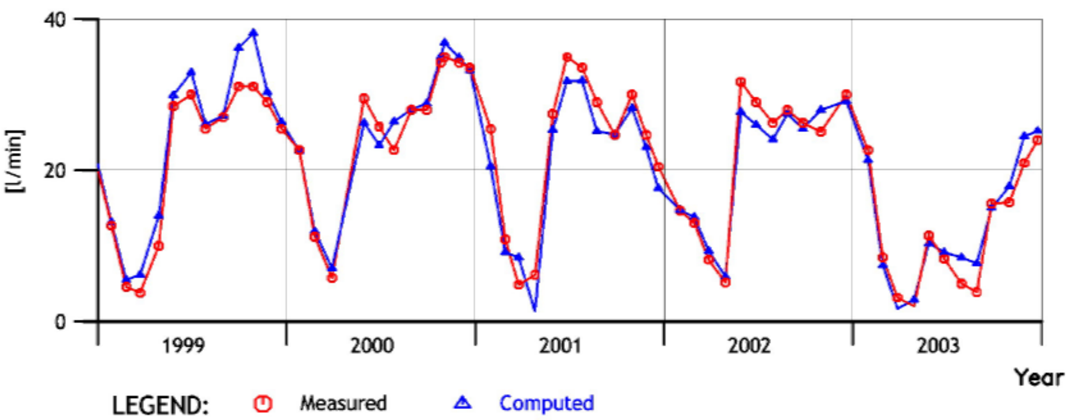

Figure 11: Measured flow rate of drain from the rock foundation of an arch dam.

5. CONCLUSION

An algorithm – similar in its form to the Laplace Transform – was developed in order to analyse the functional delays often observed in the behaviour of concrete dams. It is seen to provide very accurate results in the analysis of the thermal field in a concrete mass, which was the primary incentive for its development.
The analysis of the viscous behaviour of concrete dams appears to be also possible. The algorithm was successfully applied to the calculation of the flow rate of drains and to that of the uplift pressures both in the dam and in the foundation rock. Extension of its applicability to the field of fill dams would be a further interesting development.
The use of the algorithm developed may improve the quality of the interpretation of the data provided by dam monitoring.

6. BIBLIOGRAPHY

/1/ Istituto Editoriale Internazionale (1971), *Enciclopedia dell'ingegneria*, Arnoldo Mondatori Editore

/2/ Carslaw, H.S., and Jaeger J.C. (1959), *Conduction of Heat in Solids*, Oxford at the Clarendon Press

/3/ Lombardi, G. (2004), Advanced data interpretation for diagnosis of concrete dams, CISM – Udine 14th December 2004