3-D Surface Moment Invariants

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Abstract

3-D surface moments and surface moment invariants under similarity transformation are defined in this paper. This variation of traditional moments and moment invariants can handle the situation where the object is unclosed. 3-D surface moment invariants can be used as shape descriptors for the representation of free-form surfaces. Some explicit surface moment invariants are illustrated in the experiment to describe the partial meaningful polygonal patches.

1. Introduction

With the rapid development of the acquisition of three-dimensional information, it is possible for us to recognize the shapes of 3-D objects directly. 3-D shape models have become more and more common now, part of which are made by some softwares, others come directly from special devices, like 3-D laser scanners. Applications such as object tracking and shape retrieval require us to consider how to choose the feature descriptors of 3-D shapes and how to measure the similarities between 3-D objects.

2-D moment invariants were firstly proposed by Hu [1] in 1962 for character recognition. The property that they are independent of orientations has attracted many researchers' interest and been widely used in various applications. In 1980, Sadjadi and Hall [2] first extended moment invariants from 2-D to 3-D. Lo and Don [3] constructed 3-D moment invariants with complex moments and group-theoretic technique. They also mentioned moments of a surface patch in [4].

Some 3-D object file formats (.obj, .off et al) now are commonly used representations of 3-D models, which include coordinates of vertexes and polygonal patches. Li [5] used Gaussian theorem to convert a volume integral into a surface one and decreased the computational complexity of polyhedra moments. Moments can also be approximately computed by accumulating the Cartesian coordinates of the volume pixels sampled in the inner of objects.

Sometimes, these object files may have some irregularities, such as absence of several patches. Some surface models only have partial unclosed surfaces, like 3-D face model. Traditionally defined moments can not be computed in these situations. In this paper, we introduce 3-D surface moments which are defined on the surface of the 3-D models directly, which can solve the above problems.

3-D surface moments are defined in Part 2. We give some 3-D surface moment invariants under similarity transformation and discuss shape descriptors for the representation of free-form surfaces in Part 3. Experimental evaluation of the given surface moment invariants for partial surfaces is conducted in Part 4. We conclude the paper and open perspectives for future work in Part 5.

2. 3-D Surface Moments and Similarity Transformation

2.1 3-D Surface Moments Definition

Suppose $P(u,v)=(x(u,v),y(u,v),z(u,v))$ is a parametric surface in R^3, D is definition domain of (u,v) in R^2. Three-dimensional surface moments of order $l+m+n$ are defined by the surface integrals defined on the surface area S of $P(u,v)$:

$$\begin{aligned} M_{lmn} &= \iint_S x^l y^m z^n \rho(x,y,z) ds \\ &= \iint_D x^l(u,v) y^m(u,v) z^n(u,v) \\ &\quad \rho(x(u,v),y(u,v),z(u,v))\sqrt{EG-F^2}\,dudv \end{aligned} \tag{1}$$

where $E=x_u^{\ 2}+y_u^{\ 2}+z_u^{\ 2}$, $G=x_v^{\ 2}+y_v^{\ 2}+z_v^{\ 2}$ and $F=x_u x_v+y_u y_v+z_u z_v$ are the coefficients of the first fundamental form and $\rho(x,y,z)$ is the density function defined on the surface.

The centroid of the 3-D surface can be determined from the zeroth and the first-order moments by

$$\bar{x} = \frac{M_{100}}{M_{000}},\ \bar{y} = \frac{M_{010}}{M_{000}},\ \bar{z} = \frac{M_{001}}{M_{000}} \tag{2}$$

Then central moments are defined as

$$M_{lmn} = \iint_S (x-\bar{x})^l (y-\bar{y})^m (z-\bar{z})^n \rho(x,y,z) ds \tag{3}$$

The central surface moments are invariants under translation.

Assume that the surface is scaled by factor λ, we have

$$\begin{aligned} P'(u,v) &= (x'(u,v), y'(u,v), z'(u,v)) \\ &= (\lambda x(u,v), \lambda y(u,v), \lambda z(u,v)) \end{aligned} \tag{4}$$

$$E' = {x'_u}^2 + {y'_u}^2 + {z'_u}^2 = \lambda^2 E \tag{5}$$

$$G' = {x'_v}^2 + {y'_v}^2 + {z'_v}^2 = \lambda^2 G \tag{6}$$

$$F' = x'_u\, x'_v + y'_u\, y'_v + z'_u\, z'_v = \lambda^2 F \tag{7}$$

Then the surface moment after scaling becomes:

$$\begin{aligned} M'_{lmn} &= \iint_D x'^l(u,v) y'^m(u,v) z'^n(u,v) \\ &\rho'(x'(u,v), y'(u,v), z'(u,v)) \sqrt{E'G' - F'^2}\, du dv \\ &= \lambda^{l+m+n+2} M_{lmn} \end{aligned} \tag{8}$$

So $\mu_{lmn} = \frac{M_{lmn}}{M_{000}^{1+(l+m+n)/2}}$ is an invariant surface moment under scaling.

2.2 Similarity Transformation of Parametric Surface

Moment invariants are expressions of moments which are invariant under a kind of transformation group. In this paper, we only discuss 3-D surface moment invariants under similarity transformation. This transformation can be decomposed into translation, scaling and rotation parts.

Suppose $P'(u,v) = (x'(u,v), y'(u,v), z'(u,v))$ is the new surface of P after similarity transformation. The relationship between P and P' can be expressed as

$$\begin{bmatrix} x'(u,v) \\ y'(u,v) \\ z'(u,v) \end{bmatrix} = \lambda R \begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \tag{9}$$

Here R is an orthogonal matrix which has the property that $RR^T = R^T R = I$ where R^T is the transpose of R.

Since translation and scaling invariance has been achieved in section 2.1, we only consider the rotation under orthogonal matrix R. The new coefficients of the first fundamental form are the same as the original ones.

$$\begin{aligned} E' &= {x'_u}^2 + {y'_u}^2 + {z'_u}^2 \\ &= (x_u, y_u, z_u) R^T R (x_u, y_u, z_u)^T \\ &= x_u^2 + y_u^2 + z_u^2 = E \end{aligned} \tag{10}$$

$$\begin{aligned} G' &= {x'_v}^2 + {y'_v}^2 + {z'_v}^2 \\ &= (x_v, y_v, z_v) R^T R (x_v, y_v, z_v)^T \\ &= x_v^2 + y_v^2 + z_v^2 = G \end{aligned} \tag{11}$$

$$\begin{aligned} F' &= x'_u\, x'_v + y'_u\, y'_v + z'_u\, z'_v \\ &= (x_u, y_u, z_u) R^T R (x_v, y_v, z_v)^T \\ &= x_u x_v + y_u y_v + z_u z_v = F \end{aligned} \tag{12}$$

The analysis of those coefficients can help us understand the relationship between surface moments before and after similarity transformation. For instance, we can put them into the multiple integral framework in the following section to get rotation invariance like the generation of traditional moment invariants.

3. 3-D Surface Moment Invariants and Free-form Surface Representation

3.1 3-D Surface Moment Invariants under Rotation

We focus on the derivation of surface moment invariants under rotation here, since translation and scaling have been solved in section 2.1. Suppose (x_i, y_i, z_i), (x_j, y_j, z_j),… (x_k, y_k, z_k) etc. are arbitrary points on the surface of the model. The below four geometric primitives are distance between a point and the origin, area of a triangle between two points and the origin, inner product of two vectors between two points and the origin and volume of a tetrahedron between three points and the origin. They are invariant measurements under rotation.

$$R(i) = (x_i^2 + y_i^2 + z_i^2)^{\frac{1}{2}} \tag{13}$$

$$A(O,i,j) = \frac{1}{2} \| (x_i, y_i, z_i) \times (x_j, y_j, z_j) \|_2 \tag{14}$$

$$An(O,i,j) = (x_i, y_i, z_i) \cdot (x_j, y_j, z_j) \tag{15}$$

$$V(O,i,j,k) = \frac{1}{6} [(x_i, y_i, z_i) \times (x_j, y_j, z_j)] \cdot (x_k, y_k, z_k) \tag{16}$$

Then, we set core(1,2,…,n) to be the multiplication of the above four primitives, which involves n participating points. The four primitives in core(1,2,…,n) can be of different powers. The multiple integral of core(1,2,…,n) is just the moment invariants if we expand core(1,2,…,n) by the polynomial form $core(1,2...n) = \sum_i a_i \prod_{j=1}^{n} x_j^{i_{j1}} y_j^{i_{j2}} z_j^{i_{j3}}$. That is, surface moment invariants of different orders can be constructed by the following integral formula:

$$\iint_S \iint_S \cdots \iint_S core(1,2,...,n)\rho(x_1,y_1,z_1) \rho(x_2,y_2,z_2)...\rho(x_n,y_n,z_n)ds_1 ds_2 ...ds_n \tag{17}$$

Here $\rho(x_i,y_i,z_i), 1 \le i \le n$ are density functions defined on the same surface of the model.

We give six surface moment invariants below by the multiple integrals of primitives $R^4(1)$, $A^4(O,1,2)$, $An^4(O,1,2)$, $An^3(O,1,2)$, $An(O,1,2)R^2(1)R^2(2)$ and $An^2(O,1,2)R^2(1)$. They are then divided by certain powers of the zeroth order moments for normalization. These six invariants consist of 3 fourth order, 2 third order and 1 mixed order surface moment invariants.

$$I_1 = \frac{1}{M^3_{000}}(M_{400}+M_{040}+M_{004}+2M_{220}+2M_{202}+2M_{022}) \tag{18}$$

$$\begin{aligned} I_2 = \frac{1}{M^6_{000}}(&M_{400}M_{040}+M_{400}M_{004}+M_{004}M_{040} \\ &+3M^2_{220}+3M^2_{202}+3M^2_{022}-4M_{103}M_{301}-4M_{130}M_{310} \\ &-4M_{013}M_{031}+2M_{022}M_{202}+2M_{022}M_{220}+2M_{220}M_{202} \\ &+2M_{022}M_{400}+2M_{004}M_{220}+2M_{040}M_{202}-4M_{103}M_{121} \\ &-4M_{130}M_{112}-4M_{013}M_{211}-4M_{121}M_{301}-4M_{112}M_{310} \\ &-4M_{211}M_{031}+4M^2_{211}+4M^2_{112}+4M^2_{121}) \end{aligned} \tag{19}$$

$$\begin{aligned} I_3 = \frac{1}{M^6_{000}}(&M^2_{400}+M^2_{040}+M^2_{004}+4M^2_{130}+4M^2_{103} \\ &+4M^2_{013}+4M^2_{031}+4M^2_{310}+4M^2_{301}+6M^2_{220}+6M^2_{202} \\ &+6M^2_{022}+12M^2_{112}+12M^2_{121}+12M^2_{211}) \end{aligned} \tag{20}$$

$$\begin{aligned} I_4 = \frac{1}{M^5_{000}}(&M^2_{300}+M^2_{030}+M^2_{003}+3M^2_{120}+3M^2_{102} \\ &+3M^2_{012}+3M^2_{021}+3M^2_{210}+3M^2_{201}+6M^2_{111}) \end{aligned} \tag{21}$$

$$\begin{aligned} I_5 = \frac{1}{M^5_{000}}(&M^2_{300}+M^2_{030}+M^2_{003}+M^2_{120}+M^2_{012} \\ &+M^2_{102}+M^2_{210}+M^2_{021}+M^2_{201}+2M_{300}M_{120} \\ &+2M_{300}M_{102}+2M_{120}M_{102}+2M_{003}M_{201}+2M_{003}M_{021} \\ &+2M_{021}M_{201}+2M_{030}M_{012}+2M_{030}M_{210}+2M_{012}M_{210}) \end{aligned} \tag{22}$$

$$\begin{aligned} I_6 = \frac{1}{M^5_{000}}[&M_{200}(M_{400}+M_{220}+M_{202}) \\ &+M_{020}(M_{220}+M_{040}+M_{022})+M_{002}(M_{202}+M_{022}+M_{004}) \\ &+2M_{110}(M_{310}+M_{130}+M_{112})+2M_{101}(M_{301}+M_{121}+M_{103}) \\ &+2M_{011}(M_{211}+M_{031}+M_{013})] \end{aligned} \tag{23}$$

Besides, three second order moment invariants under rotation can be seen in [2], and another nine moment invariants under fourth order were shown in [3]. All of them can be used to construct surface moment invariants. Numerators of formulas (21) and (22) have already appeared in [3], but theirs are not explicit expressions.

3.2 3-D Surface Moment Invariants for Free-form Surface Representation

Free-form surface representation claims that shape descriptors of the surface we get should be independent of orientations. Some works concern the features of points on the surface. Iterative Closest Points algorithms in [6] find the correspondence of two points sets and transform one points set to the position and orientation of the other one's. Chua and Jarvis [7] use point signature to describe the structural neighborhood of a point on the surface. It is invariant to rotation and translation, and can recognize partial-overlapping objects. Other works describe free-form surfaces using the surface information. Extended Gaussian Images [8] map surface normal vectors onto a unit sphere, called the Gaussian Sphere. Yamany and Farag [9] use surface curvature information from certain points and produce images, called "surface signatures" for accurate surface registration and matching. Spin images in [10] is a data level shape descriptor that is used to match surfaces represented as surface meshes. Adan and Adan [11] present a flexible similarity measure based on Modeling Wave (MV) topology in spherical models. Different partial information of the model has been carried out to recognize a mesh model.

Similarly, 3-D surface moment invariants in this paper can be used as shape descriptors for free-form surfaces, since they are also independent of the orientations in 3-D space. In next part, we illustrate the usage of surface moment invariants for representation of partial meaningful surfaces.

4. Experimental Evaluation

A 3-D cow model is selected for the experiment, whose surfaces are polygonal patches. Four meaningful parts of the cow can be seen in Figure 1. These four partial surfaces are not closed, but we can compute the surface moment invariants of them. We transform the four parts into 125 new ones under different similarity transformations.

We uniformly sample the points on the surface patches to approximately compute each surface moment of each part. It is easier than accurate computation of surface moments, but will result in small degree of deviation under rotation.

The means and standard deviations of the six surface moment invariants (18)-(23) of the four parts are shown in Table 1. The relatively low ratios of standard deviation/mean testify the invariance of these six surface moment invariants experimentally. By comparing the means of the six invariants of the four parts, we can find that surface moment invariants have the ability to distinguish these parts and can be used to represent the characteristics of the meaningful parts.

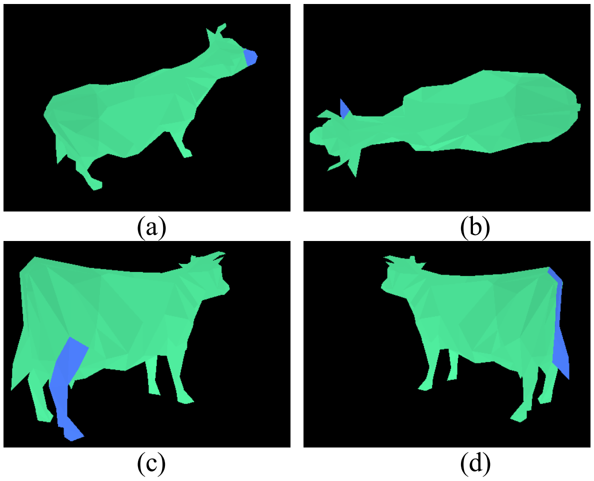

Figure 1. Partial surfaces of mouth (a), ear (b), leg (c) and tail (d) in a cow model

5. Conclusions and Future Work

In this paper, we introduce a kind of moment—surface moment, and give some explicit surface moment invariants under similarity transformation. They can be treated as a new kind of shape descriptors of free-form surfaces and can handle the situation where 3-D surface objects are not closed.

The next things we can do are the automatic meaningful segmentation of 3-D surfaces, and the selection of a suitable set of surface moment invariants for shape representation. Fast and accurate computation of surface moment invariants can also be investigated. Surface moment invariants can be applied to 3-D shape retrieval and 3-D face recognition which require the independence of rotation.

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Table 1. Invariance test of the six 3-D surface moment invariants of the partial surfaces

3-D Surface Moment Invariants		I_1 (10^{-2})	I_2 (10^{-5})	I_3 (10^{-3})	I_4 (10^{-4})	I_5 (10^{-4})	I_6 (10^{-3})
Mouth	Mean	1.760	6.081	0.1099	0.6635	0.1453	0.9210
	Stand. Dev.	0.01939	0.1169	0.001530	0.001832	0.002138	0.01799
Ear	Mean	2.313	2.786	0.3674	0.5736	0.05839	1.875
	Stand. Dev.	0.008552	0.01978	0.002753	0.005604	0.001102	0.01088
Leg	Mean	8.609	10.46	5.959	8.154	4.061	16.54
	Stand. Dev.	0.007399	0.02967	0.009559	0.01942	0.02694	0.02450
Tail	Mean	28.28	5.555	76.13	102.0	97.33	109.5
	Stand. Dev.	0.1460	0.05293	0.7923	1.896	1.832	0.8204