Proceedings of the ASME 2009 International Design Engineering Technical Conferences &
Computers and Information in Engineering Conference
IDETC/CIE 2009
August 30 - September 2, 2009, San Diego, California, USA

DETC2009-87585

LOGARITHMIC COMPLEXITY SENSITIVITY ANALYSIS OF FLEXIBLE MULTIBODY SYSTEMS

Kishor D. Bhalerao*
Mohammad Poursina
Kurt S. Anderson

Department of Mechanical, Nuclear and Aerospace Engineering
Rensselaer Polytechnic Institute
Troy, NY 12180
Email: bhalek@rpi.edu

ABSTRACT

This paper presents a recursive direct differentiation method for sensitivity analysis of flexible multibody systems. Large rotations and translations in the system are modeled as rigid body degrees of freedom while the deformation field within each body is approximated by superposition of modal shape functions. The equations of motion for the flexible members are differentiated at body level and the sensitivity information is generated via a recursive divide and conquer scheme. The number of differentiations required in this method is minimal. The method works concurrently with the forward dynamics simulation of the system and requires minimum data storage. The use of divide and conquer framework makes the method linear and logarithmic in complexity for serial and parallel implementation, respectively, and ideally suited for general topologies. The method is applied to a flexible two arm robotic manipulator to calculate sensitivity information and the results are compared with the finite difference approach.

INTRODUCTION

Development of efficient, computationally low cost and highly parallelizable methods in the multibody systems has greatly expanded the realms into which these tools may and have been effectively applied. As evidence is the degree to which articulated multibody methods now enjoy use in the areas ranging from the dynamics and control of large, complex, highly flexible spacecraft, to the modeling, simulation and analysis of molecular systems at the nano-scale. In the area of modern multibody systems dynamics, design of highly complex systems, which is iterative and computationally taxing in nature, can still be challenging. Sensitivity analysis, can play an important role associated with multibody computational problems such as implicit integration schemes, linearized dynamics, optimal control, and design optimization.

Although easy implementation and simplicity makes finite difference approximation perhaps the most broadly adopted approach to generate sensitivity information, it suffers from critical shortcomings. This procedure is time-consuming due to the fact that it requires one additional simulation for each of the perturbed design parameters. Furthermore, selecting the optimal perturbation size of a set of design variables [1] and sensitivity of the numerical solution to the perturbation size [2–4] are important (at times critical) issues, and may significantly influence the success of this technique. Analytical sensitivity analysis methods such as adjoint variable, direct differentiation and automatic differentiation are capable of overcoming most of the problems which exist in finite difference techniques.

In the adjoint variable methods, based on variation prin-

*Address all correspondence to this author.

ciples, explicit calculation of the state sensitivities is avoided through the introduction of a set of adjoint variables [2,5–9]. Manipulating these adjoint variables, variations of the system equations, and the variations of design criteria, produces the required adjoint relationships. Solving sequence of adjoint relationships provides the design sensitivity vector. This vector directly corresponds to the variation (sensitivity) of design criteria in terms of design variable variations. Implementation of these methods can be complex, particularly when dealing with variable step size or multirate integrations methods, and a large amount of data (the complete state of the system at each function valuation, for the duration of the simulation over which the sensitivity is being evaluated) has to be stored for the forward problem. The need to access this complete set of state data as the adjoint equations are integrated backwards in time can require a large number of I/O operations which greatly slows the rate at which the sensitivities may be determined [10, 11]. Another source of error is the backward temporal integration necessary for the calculation of adjoint variables. Additionally, numerical stability for the adjoint variable methods remains an open question as indicated in [2] and [12].

Automatic differentiation [13, 14] is a computer science based approach in which the variables are identified within existing code, and derivative expressions are determined by direct application of the chain rule of differentiation. This method can provide results which are numerically unstable and a blind application of the chain rule of differentiation can lead to erroneous results, particularly with regard to *DAE* systems.

The other popular methods for sensitivity analysis are those based on direct differentiation [10, 15–21]. In these techniques, direct application of the chain rule of differentiation is used to explicitly form the states sensitivities. Mathematically easy to understand, high numerical stability and relative insensitivity of the solution accuracy to parameter perturbations have distinguished direct differentiation algorithms among many competitive approaches. Besides that, if the number of design variables is small and the number of design constraints is large, the direct differentiation method becomes more attractive than adjoint variable techniques. Direct differentiation however, can be computationally expensive if not performed intelligently when dealing with large systems as is clear from the following example.

There are several ways to describe the dynamics of a flexible body and here we limit our discussion to modal superposition method (FDCA) as described in [22]. In sensitivity analysis, it is desired to determine the sensitivity of a specific cost function to the variation of the particular design or control variables. In FDCA, large rotations or translations are modeled as n rigid body (relative and/or absolute) degrees of freedom associated with the interconnecting kinematic joint free modes of motion. These are fully described by introducing sets of generalized coordinates $\{q_i\}_{i=1}^n$ and generalized speeds $\{u_i\}_{i=1}^n$. The elastic deformation of the body k in the system may be represented by sets of modal coordinates $\{q_i^k\}_{i=1}^{\nu_k}$ and their time derivatives $\{u_i^k\}_{i=1}^{\nu_k}$. The objective function J is often an explicit function of the design and state variables, while states of the system, themselves, are implicitly dependent on the values of the design parameters. Therefore, the sensitivity equation of the objective function J with respect to design variable p can be written as

$$\nabla J = \frac{\partial J}{\partial p} + \sum_{r=1}^{n}\left(\frac{\partial J}{\partial q_r}\frac{dq_r}{dp_j} + \frac{\partial J}{\partial u_r}\frac{du_r}{dp_j} + \frac{\partial J}{\partial \dot{u}_r}\frac{d\dot{u}_r}{dp_j}\right) + \sum_{k=1}^{nb}\sum_{r=1}^{\nu_k}\left(\frac{\partial J}{\partial q_r^k}\frac{dq_r^k}{dp_j} + \frac{\partial J}{\partial u_r^k}\frac{du_r^k}{dp_j} + \frac{\partial J}{\partial \dot{u}_r^k}\frac{d\dot{u}_r^k}{dp_j}\right). \quad (1)$$

Here nb represents the number of bodies. In multi-flexible-body systems, generating the dependencies of highly coupled states and states derivatives on design parameters is computationally expensive. Fortunately, the state variable sensitivities need not be solved for directly, but can be determined from temporal integration of the system state time derivatives as

$$\left.\frac{dq_r}{dp_j}\right|_{t=\tau+dt} = \int_{t=\tau}^{t=\tau+dt} \left.\frac{d\dot{q}_r}{dp_j}\right|_{t=\tau} dt + \left.\frac{dq_r}{dp_j}\right|_{t=\tau}, \quad (2a)$$

$$\left.\frac{du_r}{dp_j}\right|_{t=\tau+dt} = \int_{t=\tau}^{t=\tau+dt} \left.\frac{d\dot{u}_r}{dp_j}\right|_{t=\tau} dt + \left.\frac{du_r}{dp_j}\right|_{t=\tau}. \quad (2b)$$

In the above equations, q_r contains the generalized coordinates for all the kinematical joints and the modal coordinates in the system. Similarly, all the generalized and modal speeds are buried in the vector u_r, while the vector $\dot{u}_r$ involves the time derivatives of the generalized speeds. Based on equation (2), the main task in sensitivity analysis reduces to that of finding efficiently the sensitivity of $\dot{u}_r$ with respect to the design variable(s). Integrating $\frac{d\dot{u}_r}{dp_j}$ over the time domain of interest and substituting back the results in equation (1) will provide the sensitivity of the objective function with respect to the desired design parameter(s).

The governing equations of motion of a general multi-flexible-body system, in the state space form, is represented as,

$$\mathcal{M}_{m\times m}\dot{u}_{m\times 1} = \mathcal{K}_{m\times 1}. \quad (3)$$

In this coupled set of equations, m defines the total number of rigid and flexible degrees of freedom of the system. $\mathcal{M}$ is the known mass matrix and $\mathcal{K}$ is a known vector of the applied and state-dependent inertia forces on the system. Direct differentiation of equation (3) with respect to the desired parameter gives an expression for $\frac{d\dot{u}}{dp_j}$ as

$$\Rightarrow [\mathcal{M}_{\,m\times m}]\frac{d\dot{u}}{dp_j}_{m\times 1} = \frac{\partial \mathcal{K}}{\partial p_j} + \frac{\partial \mathcal{K}}{\partial q}\frac{dq}{dp_j} + \frac{\partial \mathcal{K}}{\partial u}\frac{du}{dp_j} - [\frac{\partial \mathcal{M}}{\partial p_j} + \frac{\partial \mathcal{M}}{\partial q}\frac{dq}{dp_j} + \frac{\partial \mathcal{M}}{\partial u}\frac{du}{dp_j}]\dot{u}. \quad (4)$$

Application of the direct method in this manner incurs large computational expenses in generating the differentiations which ranges from $O(m^2)$ to $O(m^3)$. Also, solution for the state derivative sensitivities by direct methods is of $O(m^3)$ expense. These costs can quickly become prohibitive for larger values of m. Therefore, in generating sensitivity information for such highly complex systems, it is necessary to introduce quick and efficient algorithms.

In this paper, an efficient logarithmic complexity (for parallel implementation) direct differentiation method is presented for the determination of first order design sensitivities of flexible multibody systems. The governing equations which define the states and state derivatives of the system are derived based on Flexible Divide and Conquer Algorithm (FDCA) as described in [22]. The use of divide and conquer framework makes the method suitable for systems with tree or ladder like topologies. In such complex topologies FDCA-based sensitivity analysis is expected to be more efficient than the other methods [17, 23] in which the constraint equations and their sensitivities to the design variables in forward dynamics and sensitivity equations should be solved for. Since this methodology works concurrently with the forward dynamics problem as opposed to the adjoint variable methods, there is no need for the backward temporal integration of the equations and consequently requires minimal data storage. The sensitivity equations for the entire system are generated via a hierarchic *assembly* process and as a result the actual differentiation required in this method is minimum as compared to traditional methods.

REFERENCES

[1] Anderson, K. S., and Hsu, Y. H., 2000. “Analytic full-recursive sensitivity analysis for multibody systems design”. In Proceedings of the International Congress of Theoretical and Applied Mechanics (ICTAM 2000), no. Paper-1874.

[2] Bestle, D., and Eberhard, P., 1992. “Analysing and optimizing multibody systems”. *Structures and Machines*, **20**, pp. 67–92.

[3] Haftka, R. T., and Guerdal, Z., 1992. *Elements of Structural Optimization*. Kluwer Academic Publishers.

[4] Anderson, K. S., and Hsu, Y. H., 2001. “Low operational order analytic sensitivity analysis for tree-type multibody dynamic systems”. *Journal of Guidance, Control and Dynamics*, **24**(6), Nov.–Dec., pp. 1133–1143.

[5] Haug, E., and Ehle, P. H., 1982. “Second-order design sensitivity analysis of mechanical system dynamics”. *Int J Numer Methods Eng*, **18**, pp. 1699–1717.

[6] Bestle, D., and Seybold, J., 1992. “Sensativity analysiz of constrained optimization in dynamic systems”. *Archive of Applied Mechanics*, **62**, pp. 181–190.

[7] Eberhard, P., 1996. “Analysis and optimization of complex multibody systems using advanced sensitivity methods”. *Mathematical Mechanics*, **76**, pp. 40–43.

[8] Haug, E., Wehage, R. A., and Mani, N. K., 1984. “Design sensitivity analysis of large-scaled constrained dynamic mechanicsl systems”. *Transactions of the ASME*, **106**, June, pp. 156–162.

[9] Ding, J.-Y., Pan, Z.-K., and Chen, L.-Q., 2007. “Second order adjoint sensitivity analysis of multibody systems described by differential-algebraic equations”. *Multibody System Dynamics*, **18**, pp. 599–617.

[10] Chang, C. O., and Nikravesh, P. E., 1985. “Optimal design of mechanical systems with constaint violation stabilization method”. *Journal of Mechanisms, Transmissions, and Automation in Design*, **107**, Dec., pp. 493–498.

[11] Pagalday, J., Aranburu, I., Avello, A., and Jalon, J. D., 1995. “Multibody dynamics optimization by direct differentiation methods using object oriented programming”. In Proceedings of the *IUTAM Symposium on Optimization of Mechanical Systems*, pp. 213–220.

[12] Etman, L., 1997. “Optimization of multibody systems using approximation concepts”. PhD thesis, Technische Universiteit Eindhoven.

[13] Bischof, C. H., 1996. “On the automatic differentiation of computer programs and an application to multibody systems”. In Proceedings of the *IUTAM Symposium on Optimization of Mechanical Systems*, pp. 41–48.

[14] Barthelemy, J.-F., and Hall, L., 1995. “Automatic differentiation as a tool in engineering design”. *Structural Optimization*, **9**, pp. 76–82.

[15] Greene, W., and Haftka, R., 1989. “Computational aspects of sensitivity calculations in transient structural analysis”. *Comput. Struct.*, **32**(2), pp. 433–443.

[16] Tak, T., 1990. “A recursive approach to design sensitivity analysis of multibody systems using direct differentiation”. PhD thesis, University of Iowa, Iowa City.

[17] Dias, J. M. P., and Pereira, M., 1997. “Sensitivity analysis of rigid-flexible multibody systems”. pp. 303–322.

[18] Serban, R., and Haug, E. J., 1998. “Kinematic and kinetics derivatives for multibody system analyses”. *Mechanics of Structures and Machines*, **26**(2), pp. 145–173.

[19] Jain, A., and Rodrigues, G., 2000. “Sensitivity analysis of multibody systems using spatial operators”. In Proceedings of the *International Conference on Method and Models in*

Automation and Robotics (MMAR 2000).

[20] Hsu, Y. H., and Anderson, K. S., 2001. "Efficient direct differentiation sensitivity analysis for general multi-rigid-body systems". In Proceedings of the Third Symposium on Multibody Dynamics and Vibrations, ASME Design Engineering Technical Conference 2001, (DETC01), no. DETC2001/VIB-21335.

[21] Mukherjee, R. M., Bhalerao, K. D., and Anderson, K. S., 2007. "A divide-and-conquer direct differentiation approach for multibody system sensitivity analysis". *Structural and Multidisciplinary Optimization*, **35**, pp. 413–429.

[22] Mukherjee, R., and Anderson, K. S., 2007. "A logarithmic complexity divide-and-conquer algorithm for multi-flexible articulated body systems". *Computational and Nonlinear Dynamics*, **2**(1), pp. 10–21.

[23] Neto, M. A., Ambrosio, J. A. C., and Leal, R. P., 2008. "Sensitivity analysis of flexible multibody systems using composite materials components". *Int. J. Numer. Meth. Engng*.