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# Numerical solution on thermal radiation with unsteady MHD flow

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**Abstract** The numerical study on unsteady MHD flow in a porous medium through past a vertical porous plate. The governing equation of flow field used by Crank-Nicolson finite difference method. Under thermal radiation only,showing the figures temperature for different values of function, parameter ($v_0$) and prandtl number ($P_r$).

**Key Words** MHD, porous medium, heat and mass transfer, Crank-Nicolsan method, thermal radiation
**MSC 2010** 76W05, 76R50

## 1 Introduction

The accelerated flow of a viscous incompressible fluid past an infinite vertical porous plate with suction has many important technological applications in the astrophysical, geophysical and engineering problems are in. heat losses from hot pipes, ovens etc surrounded by cooler air, are at least in part, due to free convection.

The problem of heat transfer in a vertical channel has been studied in recent years as a model for the re-entry problem to the significant role of thermal radiation in surface heat transfer when convection heat transfer, Soundalgekar and Takhar (1981) studied radiation effects on free convection flow of a gas past a semi-infinite flat plate. Hossain and Takhar (1996) studied the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature.

Yamamoto et al. (1976) investigated the acceleration of convection in a porous permeable medium along an arbitrary but smooth surface. Raptis (1983) studied free convection in a porous medium bounded by an infinite plate. Sattar (1992) studied the same problem and obtained analytical solution by the perturbation technique adopted by Singh and Dikshit (1988). Sattar et al. (2000) studied unsteady free convection flow along a vertical porous plate embedded in a porous medium numerically the thermal radiation interaction with convection in a boundary layer flow at a vertical plate with variable suction.

In the present paper we investigate the thermal radiation interaction on an absorbing emitting fluid permitted by a transversely applied magnetic field past a moving vertical porous plate embedded in a porous medium with time dependent suction and temperature.

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## 2 Mathematical Formulation

Let us consider the problem of an unsteady MHD free convection flow of a viscous, a vertical porous flat plate under the influence of a uniform magnetic field. The flow is assumed to be in the $x-$direction, which is taken along the plate in the upward direction and $y-$axis normal to the plate. Initially it is assumed that the plate and the fluid are at a constant temperature $T_\infty$ at all points. At time $t > 0$ the plate is assumed to be moving in the upward direction with the velocity $U(t)$ and there is a suction velocity $v_0(t)$ taken to be a function of time, the temperature of the plate raised to $T(t)$ where $T(t) > T_\infty$. The plate is considered to be of infinite length, all derivatives with respect to $x$ vanish and so the physical variables are functions of $y$ and $t$ only.

The fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium and the Roseland approximation is used to describe the radioactive heat flux in the $x-$direction is considered negligible in comparison to the $y-$direction.

Assuming that the Boussinesq and boundary-layer approximations hold and using the Darcy-Forchhemier model, the governing equations for the problem are as follows: Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0(constant) \tag{1}$$

Momentum equation

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T_\infty{}'\right)\cos(\alpha) + g\beta*\left(T' - T_\infty{}'\right)\cos(\alpha) - \frac{\sigma B_0^2 u'}{\rho} - \frac{vu'}{K'} \tag{2}$$

Energy equation:

$$\rho C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = k\frac{\partial^2 T'}{\partial T'^2} - \frac{\partial q_r}{\partial y'} + \frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{3}$$

Equation of continuity for mass transfer:

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

where $(u, v)$ are the components of velocity along the $x-$ and $y-$ directions respectively, $t$ is the time, $v$ is the kinematic viscosity, $\rho$ is the density of the fluid, $g_0$ is the acceleration due to gravity, $\beta$ is the coefficient of volume expansion, $B_0$ is the magnetic induction, $T$ and $T_\infty$ are the temperature of the fluid within the boundary layer and in the free stream respectively, $\sigma$ is the electric conductivity, $\alpha$ is the thermal diffusivity and $c_p$ is the specific heat at constant pressure, $k$ is the permeability of the porous medium.

The corresponding boundary conditions for the above problem are given by

$$\begin{array}{lllll}
T' \leqslant 0 & u' = 0 & T' - T_\infty{}' & C' - C_\infty{}' & \forall y' \\
T' > 0 & u' = u_0 & v' = v_0 & T' = T' + (T_w{}' - T_\infty{}')e^{At'}, & \\
C' = C' + (C_w{}' - C_\infty{}')e^{At'} & & at \quad y' = 0 & & \\
U' = 0 & T' \to \infty & C' \to \infty & y' \to \infty &
\end{array} \tag{5}$$

by using Rosseland approximation $q_r$ takes the form

$$q_r = -\frac{4\sigma}{3k_1}\frac{\partial T'^4}{\partial y'} \tag{6}$$

where $\sigma_1$, the Stefan-Boltzamann constant and $k_1$, the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that $T_4$ may be expressed as a linear function of temperature. This is accomplished by expanding $T_4$ in a Taylor series about $T_\infty$ and neglecting higher-order terms, thus

$$T'^4 \cong 4T_\infty^{'3}T' - 3T_\infty^{'4} \tag{7}$$

Using (5) and (6) in equation (3) we have

$$\rho C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial}{\partial y'}\right) = k\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_\infty^{'3}}{3k_1}\frac{\partial^2 T'}{\partial y'^2} + \frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{8}$$

In order to obtain a similarity solution in time of the problem, we introduce a similarity parameter $\delta$ as $\delta = \delta(t)$, such that $\delta$ is a length scale. then introduced as

$$u = \frac{u'}{u_0}, t = \frac{t'v_0^2}{v}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gm = \frac{vg\beta^*\left(C'_w - C'_\infty\right)}{u_0 v_0^2}$$

$$Gr = \frac{vg\beta\left(T'_w - T'_\infty\right)}{u_0 v_0^2}, Dm = \frac{D_m K_T\left(C'_w - C'_\infty\right)}{c_s c_p v\left(T'_w - T'_\infty\right)}, Sr = \frac{D_m K_T\left(T'_w - T'_\infty\right)}{T_m v\left(C'_w - C'_\infty\right)}$$

$$K = \frac{v_0^2 K'}{v^2}, \Pr = \frac{\mu c_p}{k}, M = \frac{\rho B_0^2 v'}{\rho v_0^2}, R = \frac{4\sigma T_\infty^{'3'}}{k_l k}, Sc = \frac{v}{D_m}, Ec = \frac{u_0^2}{c_p\left(T'_w - T'_\infty\right)}, y = \frac{y' v_0}{v} \tag{9}$$

In terms of this length scale, a convenient solution of the equation (1) can be taken as $v = v\left(t\right) = -\frac{v}{\delta}v_0$, where $v_0$ is the mass transfer parameter, which is $+ve$ for suction and $-ve$ for injection.

Following Sattar and Hossain (1992) $U(t)$ and $T(t)$ are now consider to have the following form:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\cos\left(\alpha\right) + Gm\cos\left(\alpha\right) - \left(M + \frac{1}{K}\right)u \tag{10}$$

where $n$ is a non-negative integer and $U_0, T_0$ are respectively the free stream velocity and mean temperature. Here $\delta_* = \frac{\delta}{\delta_0}$, where $\delta_0$ is the value of $\delta$ at $t = t_0$.

Now to make the equations (2) and (7) dimensionless, we introduce the following transformations:

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1 + \frac{4R}{3}\right)\frac{\partial^2 \theta}{\partial y^2} + Du\frac{\partial^2 C}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

Using equations (8), (9), and (11) the equations (2) and (7) are become [using the analysis of Hashimoto (1957), Sattar et al. (2000) and Sattar and Maleque (2000)]

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{12}$$

$$\begin{array}{lllll} t \leqslant 0 & u = 0 & \theta = 0 & C = 0 & \forall y \\ t > 0 & u = 1 & \theta = e^t & at & y = 0 \\ u = 0 & u \to 0 & C \to 0 & y \to 0 & \end{array} \tag{13}$$

where $Gr = \frac{g_0\beta(T_0 - T_\infty)\delta_0^2}{\upsilon U_0}$ is the local Grashof number, $M = \frac{\sigma B_0^2\delta^2}{\rho\upsilon}$ is the local magnetic parameter, $\Pr = \frac{\upsilon}{\alpha}$ is the Forchhemier number and $Fs_1 = \frac{b}{\delta}\left(\frac{\delta}{\delta_0}\right)^{2n+2} Re$ is the modified Forchhemier number, $N = \frac{kk_1}{4\sigma_1 T_\infty^3}$ is the radiation number.

## 3 Numerical Computation

The numerical solutions of the nonlinear differential Equations (12) – (13) under the boundary conditions (14) have been implicit on crank Nicolson finite difference method is a second order .We have chosen a step size of $\Delta\eta = 0.01$ to satisfy the convergence criterion of $10^{-6}$ in all cases. The value of $\eta_\infty$ was found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of $\eta_\infty$ to each group of parameters $v_0, M, n, Pr, Gr, Da$, and $Fs_1$ determined when the value of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than $10^{-6}$.

In order to verify the effects of the step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta = 0.01, \Delta\eta = 0.005, \Delta\eta = 0.001$ and in each case we found excellent agreement among them.

## 4 Results and Discussion

For the purpose of discussing the results, the numerical calculations are presented in the form of non-dimensional velocity and temperature profiles. Numerical computations have been carried out for different values of the parameters entering into the problem. The values of Grashof number ($Gr$) are taken to be large from the physical point of view. The large Grashof number values correspond to free convection problem. The effects of suction parameter $v_0$ on the velocity and temperature. that the velocity decreases with the increase of suction for cooling of the plate and increases for the heating of the plate. It is also clear that suction stabilizes the boundary layer growth.

Effects of Prandtl number ($Pr$) on the velocity as well as temperature profiles. decrease with the increase of $Pr$ whereas these profiles increase with the increase of $Pr$ for a heating plate. For cooling plate $Pr$ has decreasing effect on the temperature profiles.

The effects of radiation parameter ($N$) on the velocity for both cooling and heating plates. This figure shows that velocity decreases with the increase of the radiation parameter. This parameter has reverse effects on the heating plate. of $N$ on the temperature profiles. For large $N$, it is clear that temperature decreases more rapidly with the increase of $N$ . Therefore using radiation we can control the flow characteristic and temperature distribution. The effect of magnetic field parameter on the velocity It is observed from this figure that the magnetic field has decreasing effect on the velocity field for cooling plate and increasing effect for heating plate. Magnetic field lines act as a string to retard the motion of the fluid as consequence the rather heat transfer increases.the effect of non-negative integer n on the velocity and temperature profiles. we see that velocity profiles decrease for the cooling plate while it increases for the heating plate with the increase of $n$ . Here $n = 0$ case represents the velocity as well as temperature is time independent. The nonzero values of $n$ represents the case of time dependent velocity and temperature.

## 5 Conclusions

The thermal radiation interaction with unsteady MHD boundary layer flow past a continuously moving vertical porous plate immersed in a porous medium. From the present study we can make the following conclusions:

($i$) The suction stabilizes the boundary layer growth.

($ii$) The velocity profiles increase whereas temperature profiles decrease with an increase of the free convection currents.

($iii$) Using magnetic filed we can control the flow characteristics and heat transfer.

($iv$) Radiation has significant effects on the velocity as well as temperature distributions.

($v$) Flow characteristics strongly depend on the nonnegative integer $n$.

($vi$) Large Darcy number leads to the increase of the velocity profiles.

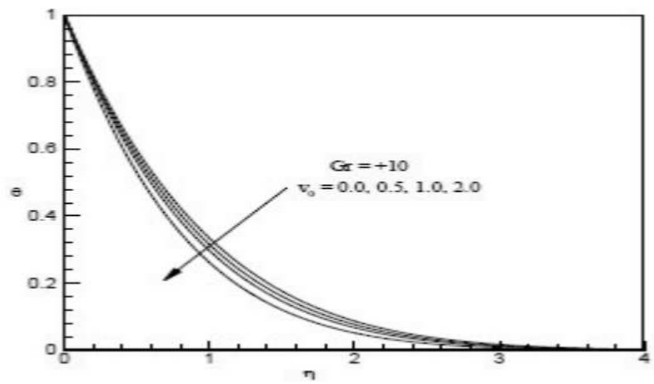


Figure 1. Temperature profiles for different values of suction parameter ($v_0$)

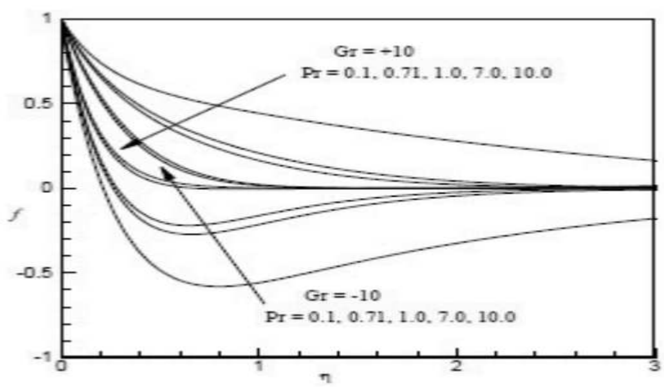


Figure 2. Velocity profiles for different values of Prandtl number ($Pr$)

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