Robust Relay Beamforming for Two-Way Relay Networks

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***Abstract*—We study the design of robust relay beamforming for two-way relay networks by considering channel feedback errors. Our objective is to maximize the minimum worst-case signal-to-noise ratios (SNRs) of two sources subject to a total relay power budget. We first decompose this non-convex problem into a series of relay power minimization problems under minimum SNR constraints by using bisection search. Then the relay power minimization problem is recast to a semidefinite programming relaxation (SDR) problem. A suboptimal but efficient solution is finally obtained for the original design. A necessary condition for the power minimization problem to be feasible is also given. For the special case with perfect channel state information, an alternative algorithm is introduced to find the optimal beamformer, which has lower complexity than the existing method.**

***Index Terms*—Two-way relaying, beamforming, channel uncertainty, robust optimization.**

I. INTRODUCTION

RECENTLY proposed two-way relaying appears as a new transmission paradigm to achieve high spectral efficiency in wireless networks. The main idea of two-way relaying is to apply physical layer network coding at the relay to assist two source nodes to exchange information with each other. Compared to one-way relaying, half of channel usages can be saved to complete one round of information exchange.

To realize more reliable transmission in two-way relay systems, multi-antenna based relay beamforming can be exploited to achieve spatial diversity [1]. Employing multiple antennas at the relay node, however, may not be feasible in certain resource-constrained networks, such as sensor network, due to hardware size limitation. An alternative way to increase the diversity is using multiple single-antenna relay nodes to collaboratively form a virtual beam as in [2], [3]. In particular, in [2], the authors consider the joint power allocation and beamforming design. In [3], the authors study the achievable rate region through beamforming.

In general, perfect and global channel state information (CSI) is needed for collaborative relay beamforming in two-way relay networks (TWRNs). This can be done by first estimating the channel coefficient at each receiving node with the help of training sequence or pilot symbols and then feeding back to a central processor via a feedback channel. In practice, the CSI information collected at the central processor may not be perfect. One reason is due to the feedback error. In specific, due to the limited capacity of feedback channel, the channel coefficients need to be quantized before sending to the central processor and, moreover, the feedback may be delayed, thereby, causing quantization error and delay error. Another reason is that channel estimation at each node is not accurate due to insufficient training or low signal-to-noise ratio (SNR).

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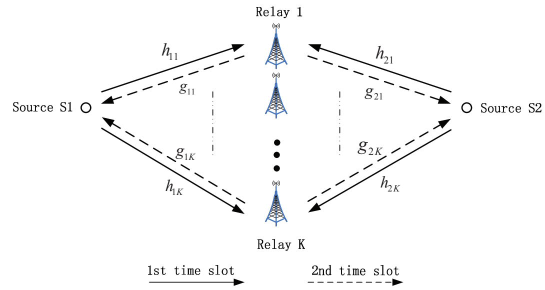

Fig. 1. Illustration of the two-way relay system.

The design of robust relay beamforming in two-way relay networks by taking CSI uncertainty into account is crucial from the practical perspective. This is by no means an easy task compared with that in point-to-point multiple-input multiple-output (MIMO) channel (e.g., [4]) or one-way MIMO relay channel (e.g., [5]–[8]). In this letter, we only consider the CSI uncertainty due to channel feedback errors in TWRN and investigate the design of robust beamforming. Our objective is to maximize the minimum worst-case SNRs of the two destinations subject to a total relay power budget. To solve this non-convex problem, we first decompose it into a series of robust relay power minimization problems by using bisection search. Then, after applying S-procedure, rank relaxation and some other transformations, the robust relay power minimization problem is further recast into a semidefinite programming relaxation (SDR) problem, for which a suboptimal but efficient solution of original problem is finally obtained. The feasibility of the robust power minimization problem is also analyzed. For the special case where the channel feedback is ideal, we introduce an alternative method to find the optimal relay beamforming. This method has lower complexity than the existing method [1], [3] and is easier for practical implementation.

Notations: $\odot$ denotes the Hadamard product. $\mathrm{Tr}(\mathbf{A})$ is the trace of $\mathbf{A}$. $\mathrm{Diag}(\mathbf{a})$ denotes a diagonal matrix with $\mathbf{a}$ being its diagonal entries. Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote transpose, conjugate and conjugate transpose, respectively. $\|\cdot\|_2^2$ denotes the squared Euclidean norm. The distribution of a circular symmetric complex Gaussian vector with mean vector $\mathbf{x}$ and covariance matrix $\mathbf{\Sigma}$ is denoted by $\mathcal{CN}(\mathbf{x}, \mathbf{\Sigma})$.

II. SYSTEM MODEL

Consider a TWRN where two source nodes, denoted as S_1 and S_2, intend to exchange information via K relay nodes, denoted as R_k, $k = 1, 2, \cdots, K$, as shown in Fig. 1. Each node is equipped with single antenna and subject to half-duplex constraint. The bidirectional communications take two time slots. In the first times slot, also called multiple access (MAC) phase, both S_1 and S_2 transmit their signals to relay nodes simultaneously. The received signals at all the relay nodes can be written in vector form as

$$\mathbf{y}_R = \mathbf{h}_1 s_1 + \mathbf{h}_2 s_2 + \mathbf{n}_R,$$

where s_i, for $i = 1, 2$, is the transmit signal from S_i with $\mathcal{E}(|s_i|^2) = P_i$, $\mathbf{n}_R = [n_1, n_2, \cdots, n_K]^T$ with n_k denoting the additive noise at R_k, and modeled as $\mathcal{CN}(0, \sigma_R^2)$, $\mathbf{y}_R = [y_{R,1}, y_{R,2}, \cdots, y_{R,K}]^T$ with $y_{R,k}$ being the received signal at R_k, $\mathbf{h}_i = [h_{i,1}, h_{i,2}, \cdots, h_{i,K}]^T$ with $h_{i,k}$ being the complex-valued channel coefficient from S_i to R_k. Upon receiving the superimposed signal, each R_k multiplies it with a complex scalar w_k to change its magnitude and phase. Let $x_{R,k}$ denote the resulting transmit signal from R_k. The transmit signals from the relay nodes can be written in vector form as

$$\mathbf{x}_R = \mathbf{w} \odot \mathbf{y}_R = \mathbf{w} \odot \mathbf{h}_1 s_1 + \mathbf{w} \odot \mathbf{h}_2 s_2 + \mathbf{w} \odot \mathbf{n}_R,$$

where $\mathbf{x}_R = [x_{R,1}, x_{R,2}, \cdots, x_{R,K}]^T$ and $\mathbf{w}$ is the relay beamformer, given by $\mathbf{w} = [w_1, w_2, \cdots, w_K]^T$. Here we assume that the relay nodes are in close proximity to each other such that they can form a cluster and share the power. Then the total consumed power at the relay nodes is constrained as

$$\mathrm{Tr}\left\{P_1 \mathbf{H}_1 \mathbf{w}\mathbf{w}^H \mathbf{H}_1^H + P_2 \mathbf{H}_2 \mathbf{w}\mathbf{w}^H \mathbf{H}_2^H + \sigma_R^2 \mathbf{w}\mathbf{w}^H\right\} \leq P_R, \tag{1}$$

where $\mathbf{H}_1 = \mathrm{Diag}(\mathbf{h}_1)$ and P_R denotes the total relay power budget.

During the second time slot, also called broadcast (BC) phase, the received signal at each destination is denoted as

$$\bar{\mathbf{y}}_i = \mathbf{g}_i^T(\mathbf{w} \odot \mathbf{h}_{\bar{i}}) s_{\bar{i}} + \mathbf{g}_i^T(\mathbf{w} \odot \mathbf{h}_i) s_i + \mathbf{g}_i^T(\mathbf{w} \odot \mathbf{n}_R) + n_i, \tag{2}$$

where $\bar{i} = 2$ if $i = 1$ and $\bar{i} = 1$ if $i = 2$, $\mathbf{g}_i = [g_{i,1}, g_{i,2}, \cdots, g_{i,K}]^T$ with $g_{i,k}$ being the complex-valued channel coefficient from R_k to S_i. Unlike [2], we do not assume reciprocal channels so that the BC phase channel $g_{i,k}$ is independent from $h_{i,k}$ in the MAC phase.

Now, let us introduce the following assumptions on the knowledge of CSI:

A1) Each receiving node can estimate the local receive CSI perfectly. This assumption is valid when the SNR during training is high enough. Thus, each relay node R_k, for $k = 1, 2, \cdots, K$, can get perfect $h_{1,k}$ and $h_{2,k}$ in the MAC phase, and S_i, for $i = 1, 2$, can acquire perfect $\mathbf{g}_i$ in the BC phase. Moreover, each source node S_i can also obtain the perfect combined channel and beamformer coefficients $\mathbf{g}_i^T(\mathbf{w} \odot \mathbf{h}_{\bar{i}})$ and $\mathbf{g}_i^T(\mathbf{w} \odot \mathbf{h}_i)$.

A2) There is a central processor that collects all the CSI $\{h_{i,k}, g_{i,k}\}, \forall i, k$, and conducts the beamforming design. This central processor can be either embedded in one of the relay nodes or be placed near the relay cluster.

A3) The information about $\mathbf{g}_i$, for $i = 1, 2$, collected at the central processor is not perfect, due to the feedback errors from each source node. Specifically, each S_i quantizes the channel vector $\mathbf{g}_i$ and then sends the quantized version to the central processor through a feedback channel with limited capacity. In addition, the feedback may not be timely compared with the channel time variation. Therefore, both quantization error and delay error can occur.

A4) The central processor is aware of the perfect $h_{1,k}$ and $h_{2,k}$ for all k through high rate auxiliary channels or wired backhauls. This assumption can be justified since the central processor is very close to the relay cluster or is one of the nodes in the cluster as assumed in A2) .

Based on assumption A1), the self-interference can be completely subtracted from (2), which yields

$$\mathbf{y}_i = \mathbf{g}_i^T(\mathbf{w} \odot \mathbf{h}_{\bar{i}}) s_{\bar{i}} + \mathbf{g}_i^T(\mathbf{w} \odot \mathbf{n}_R) + n_i. \tag{3}$$

Then, the received SNR at S_i is denoted as

$$\mathrm{SNR}_i = \frac{P_{\bar{i}} |\mathbf{g}_i^T(\mathbf{w} \odot \mathbf{h}_{\bar{i}})|^2}{\sigma_R^2 ||\mathbf{g}_i \odot \mathbf{w}||_2^2 + \sigma_i^2}. \tag{4}$$

Based on assumption A3), we model the uncertainty in $\mathbf{g}_i$ as

$$\mathbf{g}_i = \hat{\mathbf{g}}_i + \triangle\mathbf{g}_i, \ i = 1, 2 \tag{5}$$

where $\hat{\mathbf{g}}_i$ denotes the observed channel vector and the error vector $\triangle\mathbf{g}_i$ is bounded, given by $\triangle\mathbf{g}_i \in \mathbb{S}_i$ with $\mathbb{S}_i = \left\{\mathbf{a} \in \mathbb{C}^K : ||\mathbf{a}||_2^2 \leq \rho_i\right\}$. Note that such bounded error model has been widely adopted in the literature [7]–[9].

III. Robust Beamforming Design

Our design objective of the beamformer is to maximize the minimum worst-case SNRs of the two destinations in order to ensure user fairness. Based on (4) and (5), the worst-case based max-min optimization can be formulated as

$$\max_{\mathbf{w}} \min_{i=1,2} \min_{\triangle\mathbf{g}_i \in \mathbb{S}_i, i=1,2} \{\mathrm{SNR}_1, \mathrm{SNR}_2\} \tag{6a}$$

$$s.t. \ \mathrm{Tr}\left\{\mathbf{A}_0 \mathbf{w}\mathbf{w}^H\right\} \leq P_R \tag{6b}$$

where the power constraint in (6b) is rewritten from (1) with $\mathbf{A}_0 = P_1 \mathbf{H}_1^H \mathbf{H}_1 + P_2 \mathbf{H}_2^H \mathbf{H}_2 + \sigma_R^2 \mathbf{I}_K$. By introducing an auxiliary variable t, the max-min problem in (6) can be equivalently written as:

$$\begin{aligned} &\max_{\mathbf{w},t} \quad t \\ s.t. \quad &\mathrm{Tr}\{\mathbf{A}_0 \mathbf{w}\mathbf{w}^H\} \leq P_R \\ &\min_{\triangle\mathbf{g}_i \in \mathbb{S}_i} \mathrm{SNR}_i \geq t, \ i = 1, 2 \end{aligned} \tag{7}$$

The problem (7) can be decomposed into a series of solvable subproblems by using bisection search. More specifically, for a given t, we define the following relay power minimization problem subject to a minimum SNR threshold

$$P^*(t) = \min_{\mathbf{w}} \ \mathrm{Tr}\left\{\mathbf{A}_0 \mathbf{w}\mathbf{w}^H\right\} \tag{8a}$$

$$s.t. \ \min_{\triangle\mathbf{g}_i \in \mathbb{S}_i} \mathrm{SNR}_i \geq t, \ i = 1, 2 \tag{8b}$$

If the optimal $P^*(t)$ in (8) for the given t is larger than P_R, we need to reduce t, otherwise, the system can support higher t. Therefore, solving the max-min problem (6) finally turns to solving the relay power minimization problem (8). As a result, we focus on solving problem (8) in the following.

According to (4), the constraint (8b) can be rewritten as

$$\min_{\triangle \mathbf{g}_i \in \mathbb{S}_i} \frac{P_{\bar{i}} \mathbf{g}_i^T (\mathbf{w} \odot \mathbf{h}_{\bar{i}})(\mathbf{w} \odot \mathbf{h}_{\bar{i}})^H \mathbf{g}_i^*}{\sigma_R^2 \mathrm{Tr}\left\{(\mathbf{g}_i \odot \mathbf{w})(\mathbf{g}_i \odot \mathbf{w})^H\right\} + \sigma_i^2} \geq t, \quad (9)$$

with $\mathbf{g}_i = \hat{\mathbf{g}}_i + \triangle \mathbf{g}_i$ for $i = 1, 2$. For notation convenience, we denote $\bar{\mathbf{g}}_i = \mathbf{g}_i^*$. Since $\mathrm{Tr}\left\{(\mathbf{g}_i \odot \mathbf{w})(\mathbf{g}_i \odot \mathbf{w})^H\right\} = \mathrm{Tr}\left\{(\bar{\mathbf{g}}_i \odot \mathbf{w})(\bar{\mathbf{g}}_i \odot \mathbf{w})^H\right\}$, the constraint (9) is equivalent to

$$\min_{\triangle \bar{\mathbf{g}}_i \in \mathbb{S}_i} \frac{P_{\bar{i}} \bar{\mathbf{g}}_i^H (\mathbf{w} \odot \mathbf{h}_{\bar{i}})(\mathbf{w} \odot \mathbf{h}_{\bar{i}})^H \bar{\mathbf{g}}_i}{\sigma_R^2 \mathrm{Tr}\left\{(\bar{\mathbf{g}}_i \odot \mathbf{w})(\bar{\mathbf{g}}_i \odot \mathbf{w})^H\right\} + \sigma_i^2} \geq t, \quad (10)$$

with $\bar{\mathbf{g}}_i = \hat{\bar{\mathbf{g}}}_i + \triangle \bar{\mathbf{g}}_i$ for $i = 1, 2$. Then, by using the rule $(\mathbf{a} \odot \mathbf{b})(\mathbf{a} \odot \mathbf{b})^H = (\mathbf{a}\mathbf{a}^H) \odot (\mathbf{b}\mathbf{b}^H)$, we have $\bar{\mathbf{g}}_i^H (\mathbf{w} \odot \mathbf{h}_{\bar{i}})(\mathbf{w} \odot \mathbf{h}_{\bar{i}})^H \bar{\mathbf{g}}_i = \bar{\mathbf{g}}_i^H \left[(\mathbf{h}_{\bar{i}} \mathbf{h}_{\bar{i}}^H) \odot (\mathbf{w}\mathbf{w}^H)\right] \bar{\mathbf{g}}_i$ and $\mathrm{Tr}\left\{(\bar{\mathbf{g}}_i \odot \mathbf{w})(\bar{\mathbf{g}}_i \odot \mathbf{w})^H\right\} = \mathrm{Tr}\left\{(\bar{\mathbf{g}}_i \bar{\mathbf{g}}_i^H) \odot (\mathbf{w}\mathbf{w}^H)\right\} = \mathrm{Tr}\left\{(\bar{\mathbf{g}}_i \bar{\mathbf{g}}_i^H)(\mathbf{I}_K \odot \mathbf{w}\mathbf{w}^H)\right\} = \bar{\mathbf{g}}_i^H (\mathbf{I}_K \odot \mathbf{w}\mathbf{w}^H) \bar{\mathbf{g}}_i$. Thus, the constraint (9) can be reexpressed as

$$\min_{\triangle \bar{\mathbf{g}}_i \in \mathbb{S}_i} \bar{\mathbf{g}}_i^H [(P_{\bar{i}} \mathbf{h}_{\bar{i}} \mathbf{h}_{\bar{i}}^H - t\sigma_R^2 \mathbf{I}_K) \odot (\mathbf{w}\mathbf{w}^H)] \bar{\mathbf{g}}_i \geq t\sigma_i^2. \quad (11)$$

Since $\triangle \bar{\mathbf{g}}_i$ is continuous over $\mathbb{S}_i$, there are infinite realizations of $\triangle \bar{\mathbf{g}}_i$ which make the problem unsolvable. To proceed, we transform (8) into the following form by applying the S-procedure on (11) as in [7], [9]

$$\begin{aligned} &\min_{\mathbf{W}, s_1, s_2} \mathrm{Tr}\left\{\mathbf{A}_0 \mathbf{W}\right\} && (12)\\ s.t. \quad &\begin{pmatrix} \hat{\bar{\mathbf{g}}}_i^H \mathbf{Q}_i \hat{\bar{\mathbf{g}}}_i - t\sigma_i^2 - s_i \rho_i & \hat{\bar{\mathbf{g}}}_i^H \mathbf{Q}_i \\ \mathbf{Q}_i \hat{\bar{\mathbf{g}}}_i & \mathbf{Q}_i + s_i \mathbf{I}_K \end{pmatrix} \succeq 0, \ i = 1, 2 \\ &\mathrm{Rank}(\mathbf{W}) = 1, \ \mathbf{W} \succeq 0, \ s_1 \geq 0, s_2 \geq 0 \end{aligned}$$

where $\mathbf{Q}_i = (P_{\bar{i}} \mathbf{h}_{\bar{i}} \mathbf{h}_{\bar{i}}^H - t\sigma_R^2 \mathbf{I}_K) \odot \mathbf{W}$.

Proposition 1: A necessary condition for the optimization problem (12) to be feasible is $P_1 ||\mathbf{h}_1||_2^2 > t\sigma_R^2$ and $P_2 ||\mathbf{h}_2||_2^2 > t\sigma_R^2$.

Proof: Based on *lemma 1* in Appendix A, we find that if $P_{\bar{i}} ||\mathbf{h}_{\bar{i}}||^2 \leq t\sigma_R^2$, the term $P_{\bar{i}} \mathbf{h}_{\bar{i}} \mathbf{h}_{\bar{i}}^H - t\sigma_R^2 \mathbf{I}_K$ must be negative semidefinite, which also makes the matrix $\mathbf{Q}_i = (P_{\bar{i}} \mathbf{h}_{\bar{i}} \mathbf{h}_{\bar{i}}^H - t\sigma_R^2 \mathbf{I}_K) \odot \mathbf{W}$ negative semidefinite. Then we have $\hat{\bar{\mathbf{g}}}_i^H \mathbf{Q}_i \hat{\bar{\mathbf{g}}}_i \leq 0$, which further leads to $\hat{\bar{\mathbf{g}}}_i^H \mathbf{Q}_i \hat{\bar{\mathbf{g}}}_i - t\sigma_i^2 - s_i \rho_i < 0$. Thus the problem (12) is infeasible and hence the Proposition 1 is proved. ■

Due to the rank-one constraint, the optimal solution of (12) is not easily tractable. We therefore resort to relaxing it by deleting the rank-one constraint, namely,

$$\begin{aligned} &\min_{\mathbf{W} \succeq 0, s_1 \geq 0, s_2 \geq 0} \mathrm{Tr}\left\{\mathbf{A}_0 \mathbf{W}\right\} && (13)\\ s.t. \quad &\begin{pmatrix} \hat{\bar{\mathbf{g}}}_i^H \mathbf{Q}_i \hat{\bar{\mathbf{g}}}_i - t\sigma_i^2 - s_i \rho_i & \hat{\bar{\mathbf{g}}}_i^H \mathbf{Q}_i \\ \mathbf{Q}_i \hat{\bar{\mathbf{g}}}_i & \mathbf{Q}_i + s_i \mathbf{I}_K \end{pmatrix} \succeq 0, \ i = 1, 2 \end{aligned}$$

We can verify that (13) is an semidefinite programming (SDP) problem and the optimal solution can be easily obtained.

After termination of bisection search, if the optimal solution of (13), $\bar{\mathbf{W}}$, is rank-one, then the optimal solution of (6) can be obtained by using eigenvalue decomposition. Otherwise, some other ways, for example, randomization and eigenvector approximation [10], should be used to find a suitable solution. Although the randomization method can obtain a near-optimal solution if the number of generated samples is large enough [10], it also results in high computational complexity. In this work, we apply the eigenvector approximation to obtain a suboptimal but practical solution, given as $\tilde{\mathbf{w}} = \sqrt{\lambda}\mathbf{q}$, where λ is the maximum eigenvalue of $\bar{\mathbf{W}}$ and $\mathbf{q}$ is the corresponding eigenvector. Then the final solution of (6) is obtained as $\mathbf{w} = \alpha \tilde{\mathbf{w}}$ where α is a scaling parameter to ensure that $\mathbf{w}$ consumes all the relay power.

In the rest of this section, we discuss the nonrobust beamforming design by considering the limiting case $\triangle \mathbf{g}_i = 0, \ i = 1, 2$. The power minimization problem (8) reduces to

$$\min_{\mathbf{w}} \ \mathrm{Tr}\left\{\mathbf{A}_0 \mathbf{w}\mathbf{w}^H\right\} \quad (14a)$$

$$s.t. \frac{\mathbf{w}^H \mathbf{A}_{i,1} \mathbf{w}}{\sigma_R^2 \mathbf{w}^H \mathbf{A}_{i,2} \mathbf{w} + \sigma_i^2} \geq t, \ i = 1, 2 \quad (14b)$$

where $\mathbf{A}_{i,1} = P_{\bar{i}} \mathbf{H}_{\bar{i}}^H \mathbf{g}_i^* \mathbf{g}_i^T \mathbf{H}_{\bar{i}}$ and $\mathbf{A}_{i,2} = \mathbf{G}_i^H \mathbf{G}_i$ with $\mathbf{G}_i = \mathrm{Diag}(\mathbf{g}_i)$. To obtain the constraint (14b), the circular property of trace operator has been used for SNR expression (4). Clearly, problem (14) can also be converted into a relaxed SDP problem as follows by introducing a new matrix $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ as in (12)

$$\begin{aligned} &\min\nolimits_{\mathbf{W} \succeq 0} \ \mathrm{Tr}\left\{\mathbf{A}_0 \mathbf{W}\right\} && (15)\\ s.t. \quad &\mathrm{Tr}\left\{\mathbf{A}_i \mathbf{W}\right\} \geq \sigma_i^2 t, \ i = 1, 2 \end{aligned}$$

where $\mathbf{A}_i = \mathbf{A}_{i,1} - t\sigma_R^2 \mathbf{A}_{i,2}$.

In fact, the nonrobust beamforming design for total relay power minimization based on perfect CSI has been considered in [3], wherein the optimal beamformer is obtained by using the method proposed in [1]. In this work, we introduce an alternative approach to obtain the optimal beamformer of (14) if the optimal solution of (15) is not rank-one.

Theorem 1: If the rank of the optimal solution $\bar{\mathbf{W}}$ of problem (15) , denoted as r, is higher than one, the optimal solution of (14) can be obtained using the following procedures.

- **Repeat**
  - Decompose $\bar{\mathbf{W}}$ as $\bar{\mathbf{W}} = \mathbf{V}\mathbf{V}^H$ with $\mathbf{V} \in \mathbb{C}^{K \times r}$;
  - Find the nonzero $r \times r$ Hermitian matrix $\mathbf{M}$ to satisfy the following linear equations
  $$\mathbf{Tr}\left(\mathbf{V}^H \mathbf{A}_i \mathbf{V}\mathbf{M}\right) = 0, \ i = 0, 1, 2. \quad (16)$$
  - Evaluate the eigenvalues $\varrho_1, \varrho_2, \cdots, \varrho_R$ of $\mathbf{M}$ and set $|\varrho| = \max\{|\varrho_i|, \forall i\}$;
  - Generate a new matrix as $\bar{\mathbf{W}}' = \mathbf{V}\left(\mathbf{I}_R - (1/\varrho)\mathbf{M}\right)\mathbf{V}^H$ and set $\bar{\mathbf{W}} = \bar{\mathbf{W}}'$;
- **Until** the rank $r = \mathrm{Rank}(\bar{\mathbf{W}})$ is equal to 1.

Proof: The proof is similar to [11]. Since $\mathbf{M}$ has r^2 real elements, if $r^2 > 3$, we can always find a nonzero $\mathbf{M}$ to satisfy (16), which further leads to that the rank of $\bar{\mathbf{W}}'$ reduces at least one compared to the original $\bar{\mathbf{W}}$. Besides that, we have $\mathrm{Tr}(\mathbf{A}_i \bar{\mathbf{W}}') = \mathrm{Tr}(\mathbf{A}_i \bar{\mathbf{W}} - \frac{1}{\varrho}\mathbf{A}_i \mathbf{V}\mathbf{M}\mathbf{V}^H) = \mathrm{Tr}(\mathbf{A}_i \bar{\mathbf{W}}), \ i = 1, 2$, which means that the new matrix $\bar{\mathbf{W}}'$ is also a feasible point of (15). We can also verify that $\mathrm{Tr}(\mathbf{A}_0 \bar{\mathbf{W}}') = \mathrm{Tr}(\mathbf{A}_0 \bar{\mathbf{W}})$, i.e., $\bar{\mathbf{W}}'$ achieves the same value of objective function as $\bar{\mathbf{W}}$. In other wards, $\bar{\mathbf{W}}'$ is also optimal for problem (15) but with lower rank. Repeating the above procedures, we can finally obtain the rank-one solution of (15) which is also the optimal solution of (14). ■

Remark: Note that in [1], [3], the authors proposed to extract the optimal rank-one solution from the SDP problem (15) by using a special matrix decomposition and solving a linear programming problem. In Theorem 1, we introduce an alternative way to obtain the rank-one solution by only solving some linear equations. Thus, the introduced method has lower complexity and is easier for practical implementation.

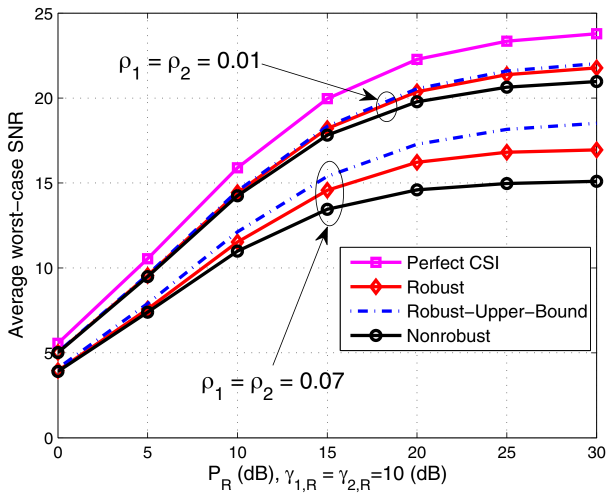

Fig. 2. Output worst-case SNR versus relay power.

IV. Simulation Results

The channels are set to be Rayleigh fading with all the channel coefficients $\{h_{i,k}, g_{i,k}\}$, $\forall i, k$ independent and following $\mathcal{CN}(0, 1)$. The CSI error $\triangle \mathbf{g}_i$, for $i = 1, 2$, is obtained as follows: we first generate a random vector following complex Gaussian distribution $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. If this vector is out of the bounded region, we then normalize it with the error bound. The number of relay nodes is $K = 4$. We assume that the noise power at all receive nodes are the same, i.e., $\sigma_R^2 = \sigma_1^2 = \sigma_2^2 = \sigma^2$. Let $\gamma_{i,R} = P_i/\sigma^2$ denote the average SNR from S_i to R.

In Fig 2, we illustrate the achieved average worst-cast SNR as the function of P_R by setting $\gamma_{1,R} = \gamma_{2,R} = 10$dB. Two error bounds $\rho_1 = \rho_2 = 0.01$ and $\rho_1 = \rho_2 = 0.07$ are considered. The ideal case with perfect CSI is also simulated as a benchmark. For the nonrobust design, the observed $\hat{\mathbf{g}}_i$ is treated as the perfect CSI. For comparison, we also simulate the upper bound of obtained worst-case SNR by only solving the relay power minimization (13) without extracting the rank-one solution. It is observed that the proposed robust design outperforms the non-robust scheme by a reasonable margin. Moreover, for the small error bound, the proposed design almost attains the upper bound. This indicates that the solution of (13) is rank-one in most cases for small error bounds.

V. Conclusions

In this letter, we proposed the robust relay beamforming for TWRN using worst-case optimization. We also introduced an alternative method to obtain the optimal relay beamforming with ideal channel knowledge, which has lower computational complexity than the prior work. This work only serves as the first step towards the global robust beamforming in two-way relay networks as only channel uncertainty at the broadcast phase is taken into account. Future work can consider channel errors in both multiple access phase and broadcast phase.

Appendix A

Lemma 1: If two $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ are positive semidefinite and negative semidefinite, respectively, then the matrix $\mathbf{A} \odot \mathbf{B}$ is negative semidefinite.

Proof: We first apply eigenvalue decomposition to decompose $\mathbf{A}$ and $\mathbf{B}$ as $\mathbf{A} = \sum_{i=1}^{p} \lambda_{A,i} \mathbf{u}_{A,i} \mathbf{u}_{A,i}^H$ and $\mathbf{B} = -\sum_{i=1}^{q} \lambda_{B,i} \mathbf{u}_{B,i} \mathbf{u}_{B,i}^H$ where p and q are the rank of $\mathbf{A}$ and $\mathbf{B}$, respectively. $\lambda_{A,i}$ and $-\lambda_{B,i}$ are the eigenvalues. Let $\mathbf{v}_i = \sqrt{\lambda_{A,i}} \mathbf{u}_{A,i}$ and $\mathbf{w}_i = \sqrt{\lambda_{B,i}} \mathbf{u}_{B,i}$, we have

$$\begin{aligned}\mathbf{A} \odot \mathbf{B} &= -\sum_{i=1}^{p} \sum_{j=1}^{q} (\mathbf{v}_i \mathbf{v}_i^H) \odot (\mathbf{w}_j \mathbf{w}_j^H) \\ &= -\sum_{i=1}^{p} \sum_{j=1}^{q} (\mathbf{v}_i \odot \mathbf{w}_j)(\mathbf{v}_i \odot \mathbf{w}_j)^H.\end{aligned}$$

We thus derive that the $\mathbf{A} \odot \mathbf{B}$ is negative semidefinite. ■

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