Vol. 36, No. 3, May–June 2013

Applications of Generalized Zero-Effort-Miss/Zero-Effort-Velocity Feedback Guidance Algorithm

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DOI: 10.2514/1.58099

The performance of the zero-effort-miss/zero-effort-velocity feedback guidance algorithm is evaluated through practical space application examples. The zero-effort-miss/zero-effort-velocity feedback guidance algorithm is, in general, not an optimal solution; however, it is an optimal solution in a uniform gravitational environment. It is also conceptually simple and easy to implement and, thus, has great potential for autonomous onboard implementation. It is shown that, for some classic ballistic missile intercept and asteroid intercept scenarios, the zero-effort-miss/zero-effort-velocity algorithm can even compete with corresponding open-loop optimal solutions, while its feedback characteristics make it more suitable to deal with uncertainties and perturbations. By employing the zero-effort-miss/zero-effort-velocity algorithm in the highly nonlinear orbital transfer and raising problems and comparing with corresponding open-loop optimal solutions, its simplicity and near-optimality are further verified.

I. Introduction

OPTIMAL control theory has been widely used for decades in many different applications; examples include spacecraft orbit control, missile guidance control, robot control, and flight vehicle trajectory control [1,2]. The problem of controlling the trajectory of aerospace vehicles from an arbitrary initial position and initial velocity to a desired target position with constrained, free or pointed terminal velocity in a specific time, or within a predefined time range, is of fundamental interest as an optimal control problem.

Bryson and Ho discussed optimal control laws for a simple rendezvous problem, considering both free terminal velocity and constrained terminal velocity [2]. They also discussed the relationship between optimal control and proportional navigation guidance. Battin also discussed an optimal terminal state vector control for the orbit control problem, directly compensating for the known disturbing gravitational acceleration [3]. D'Souza further examined an optimal control algorithm in a uniform gravitational field and developed a computational method to determine the optimal time-to-go [4]. Ebrahimi et al. proposed a robust optimal sliding mode guidance law for an exoatmospheric interceptor using fixed-interval propulsive maneuvers [5]. In this paper, gravity was considered to be an explicit function of time. One major contribution of Ebrahimi et al. was the new concept of the zero-effort-velocity (ZEV) error, analogous to the well-known zero-effort-miss (ZEM) distance. The ZEV is the velocity error at the end of the mission if no further control accelerations are imparted. Furfaro et al. later employed the ZEM/ZEV concept to construct two classes of nonlinear guidance algorithms for a lunar precision landing mission [6]. Guo et al. [7] showed that in a uniform gravitational field, the ZEM/ZEV logic is basically a generalized form of various well-known optimal feedback guidance solutions such as intercept or rendezvous [2], terminal guidance [3], and planetary landing [4]. The performance of the ZEM/ZEV logic for an asteroid intercept mission with precision targeting requirements was evaluated by Hawkins et al. [8] and compared with the performances of classical missile guidance methods like proportional navigation guidance (PNG) and augmented proportional navigation guidance (APNG).

For many practical missions, the gravitational acceleration is not constant nor an explicit function of time, but is instead a function of position. *The ZEM/ZEV algorithm is not an optimal solution when the gravitational acceleration is a function of position*. However, the ZEM and ZEV terms can be obtained by numerically propagating the dynamic equations, and the ZEM/ZEV algorithm can accomplish the control mission in a near-optimal manner. The objective of this paper is to show how to use the generalized ZEM/ZEV algorithm for a variety of practical applications. For highly nonlinear systems, numerical propagation of the states for the entire remaining mission time is not sufficient, as nonlinearities during the actual mission violate the assumptions of the ZEM/ZEV algorithm. For these highly nonlinear cases, a general way to improve the performance of the ZEM/ZEV feedback algorithm is to divide the total flight time into one or more segments and somehow determine optimal or near-optimal waypoints to connect the different segments. Such a waypoint concept was considered by Sharma et al. [9], and the computational method was provided to solve nonlinear optimal control problems with terminal constraints.

In the last decade, pseudospectral optimization methods have been used for a variety of optimal control applications [10–12]. NASA's Transition Region and Corona Explorer spacecraft successfully flight-tested time-optimal slews in the presence of various constraints [13], ushering in a new era of employing optimization techniques for spacecraft attitude control. A number of optimization software packages are now on the market, including SNOPT, DIDO, TOMLAB [14], and others. General pseudospectral optimal control software (GPOPS) is one of the most versatile open-source multiphase optimizers and is used in this paper [15]. GPOPS is used in this study to generate the open-loop optimal solution to compare the ZEM/ZEV algorithm against, and to obtain optimal waypoints to improve the performance of the generalized ZEM/ZEV algorithm for missions with highly nonlinear dynamics.

In this paper, the generalized optimal control problem is first briefly reviewed. Three different types of ZEM/ZEV optimal feedback control algorithms are obtained for different terminal requirements. Each algorithm is then investigated through an illustrative example, and the characteristics of each algorithm are discussed.

II. Optimal Feedback Guidance Algorithms

A. General Equations of Motion

The equations of motion of a space vehicle in a gravitational field are given by

Received 28 February 2012; accepted for publication 24 September 2012; published online 18 March 2013.

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$$\dot{\mathbf{r}} = \mathbf{v} \tag{1}$$

$$\dot{\mathbf{v}} = \mathbf{g}(\mathbf{r}) + \mathbf{a} \tag{2}$$

$$\mathbf{a} = \mathbf{T}/m \tag{3}$$

where $\mathbf{r}$ and $\mathbf{v}$ represent the position and velocity vectors, respectively; $\mathbf{a}$ is the control acceleration provided by the thrusting force $\mathbf{T}$; m is the vehicle mass; and $\mathbf{g}(\mathbf{r})$ denotes the gravitational acceleration acting on the vehicle, which is generally a function of position. In this paper, these vectors denote 3×1 column vectors expressed in a nonrotating inertial reference frame.

B. Optimal Feedback Guidance Algorithms for a Special Case of $g = g(t)$

The gravitational acceleration is, in general, a function of position, which will not lead to an analytical solution of the optimal control problem. However, if the gravitational acceleration is assumed to be an explicit function of only time, then the analytical optimal solution can be found.

For a mission from time t_0 to t_f, the optimal control acceleration needs to be determined by minimizing the classical performance index of the form

$$J = \frac{1}{2}\int_{t_0}^{t_f} \mathbf{a}^T \mathbf{a}\, \mathrm{d}t \tag{4}$$

subject to Eqs. (1–3) and the following given boundary conditions:

$$\mathbf{r}(t_0) = \mathbf{r}_0, \qquad \mathbf{r}(t_f) = \mathbf{r}_f \quad \mathbf{v}(t_0) = \mathbf{v}_0, \qquad \mathbf{v}(t_f) = \mathbf{v}_f \tag{5}$$

The Hamiltonian function for this problem is then defined as

$$H = \frac{1}{2}\mathbf{a}^T\mathbf{a} + \mathbf{p}_r^T\mathbf{v} + \mathbf{p}_v^T(\mathbf{g}(t) + \mathbf{a}) \tag{6}$$

where $\mathbf{p}_r$ and $\mathbf{p}_v$ are the costate vectors associated with the position and velocity vectors, respectively.

The costate equations provide the optimal control solution expressed as a linear combination of the terminal values of the costate vectors. The time-to-go is defined as: $t_{\mathrm{go}} = t_f - t$. The optimal acceleration at any time t is expressed as

$$\mathbf{a} = -t_{\mathrm{go}}\mathbf{p}_r(t_f) - \mathbf{p}_v(t_f) \tag{7}$$

By substituting Eq. (7) into the dynamic equations to solve for $\mathbf{p}_r(t_f)$ and $\mathbf{p}_v(t_f)$, the optimal control solution with the specified $\mathbf{r}_f$, $\mathbf{v}_f$, and t_{go} is finally obtained as

$$\mathbf{a} = \frac{6[\mathbf{r}_f - (\mathbf{r} + t_{\mathrm{go}}\mathbf{v})]}{t_{\mathrm{go}}^2} - \frac{2(\mathbf{v}_f - \mathbf{v})}{t_{\mathrm{go}}} + \frac{6\int_t^{t_f}(\tau - t)\mathbf{g}(\tau)\,\mathrm{d}\tau}{t_{\mathrm{go}}^2} - \frac{4\int_t^{t_f}\mathbf{g}(\tau)\,\mathrm{d}\tau}{t_{\mathrm{go}}} \tag{8}$$

The ZEM distance and ZEV error denote, respectively, the differences between the desired final position and velocity and the projected final position and velocity if no additional control is commanded after the current time. For the assumed gravitational acceleration $\mathbf{g}(t)$, the ZEM and ZEV have the following expressions [8,9]:

$$\mathbf{ZEM} = \mathbf{r}_f - \left[\mathbf{r} + t_{\mathrm{go}}\mathbf{v} + \int_t^{t_f}(t_f - \tau)\mathbf{g}(\tau)\,\mathrm{d}\tau\right] \tag{9}$$

$$\mathbf{ZEV} = \mathbf{v}_f - \left[\mathbf{v} + \int_t^{t_f}\mathbf{g}(\tau)\,\mathrm{d}\tau\right] \tag{10}$$

Then, the optimal control law Eq. (8) can be equivalently expressed as

$$\mathbf{a} = \frac{6}{t_{\mathrm{go}}^2}\mathbf{ZEM} - \frac{2}{t_{\mathrm{go}}}\mathbf{ZEV} \tag{11}$$

For certain missions where the terminal velocity is not specified, the optimal control law, in terms of ZEM only, can be obtained as

$$\mathbf{a} = \frac{3}{t_{\mathrm{go}}^2}\mathbf{ZEM} \tag{12}$$

Though of limited interest for most intercept and rendezvous missions, the optimal control to regulate only the terminal velocity, in terms of ZEV only, can also be obtained as

$$\mathbf{a} = \frac{1}{t_{\mathrm{go}}}\mathbf{ZEV} \tag{13}$$

A special case of uniform gravitational environment can be assumed for many planetary landing and asteroid terminal guidance problems. The three optimal algorithms described by Eqs. (11–13) then become the exact optimal solutions that achieve the optimal feedback performance and maintain robustness against uncertainties. For other missions, though, the gravitational acceleration cannot be simply modeled as a constant (or as a pure function of time), but must be considered a function of position. For this case, the ZEM and ZEV can be found by numerically integrating the dynamic equations. The same control expressions are used with the numerically propagated values of ZEM and ZEV. The predictions and controls are updated in real time, finally accomplishing the control mission at near-optimal levels with acceptable computational complexity.

For highly nonlinear systems, predicting the future states is prone to errors. Another alternative form of the ZEM/ZEV algorithm can be adopted for this situation. Rather than predicting the effect of the nonlinear terms, the effects of these terms are directly compensated for at all times. The algorithm thus approaches feedback linearization behavior. The control algorithm Eq. (8) then simply becomes the following form suggested by Battin in [3]:

$$\mathbf{a} = \frac{6[\mathbf{r}_f - (\mathbf{r} + t_{\mathrm{go}}\mathbf{v})]}{t_{\mathrm{go}}^2} - \frac{2(\mathbf{v}_f - \mathbf{v})}{t_{\mathrm{go}}} - \mathbf{g}(\mathbf{r}) \tag{14}$$

III. Optimization with General Pseudospectral Optimal Control Software

GPOPS uses hp-adaptive pseudospectral methods to solve optimal control problems of one or more phases. The user supplies the governing dynamic equations, cost function, and various constraints, and, if feasible, GPOPS returns the optimal control histories. The theoretical basis and implementation of GPOPS are now briefly described.

Pseudospectral numerical optimization methods approximate the states, costates, and controls using Lagrange interpolating polynomials. Because Lagrange polynomials and their derivatives can be evaluated for any time, the states and controls can be represented by a discrete set of points. If there is an integral in the cost functional, this is approximated with Gaussian quadrature. An hp-adaptive method can automatically adjust the mesh points (h-adaptivity) and the order of the approximating polynomial (p-adaptivity). The complete problem is thus posed as a nonlinear program (NLP), which can be solved using one of a variety of well-known NLP solvers.

The GPOPS user must supply the governing differential equations, minimum and maximum values for all of the states and controls, and minimum and maximum values for the initial and final times. Guesses for the initial and final values of all of the states, controls, and times are also supplied. Finally, the cost functional, a combination of the standard Mayer (endpoint) and Lagrange (running) costs, is specified.

The initial time is typically fixed, and the final time can be fixed or free. In a multiphase problem, each phase can have a minimum or maximum duration. For example, a three-stage rocket could be designed with the first two phases lasting a specified length of time, and the third phase and, thus, the final time, free.

In addition to these inputs, some optional constraints may be specified. Event constraints are equality or inequality constraints that are expressed as functions of initial and final time, as well as initial and final values of the states. A typical event constraint might be ensuring that the final position falls on some specified curve. Path constraints are equality or inequality constraints expressed as functions of the time, states, and controls. A typical path constraint might be ensuring that the norm of the controls always equals control magnitude for a constant-thrust mission. Parameter constraints are equality or inequality constraints on parameters, variables that are constant during the mission but not known ahead of time. A typical parameter constraint might be the mass ratio of a staged rocket.

Because the controls and states are approximated with polynomials, the controls and states (and costates) must be smooth within a given phase. The examples considered in this paper, and many other guidance problems with continuous thrust, are smooth in this sense. Known or suspected discontinuities and singularities, such as "bang–bang" control or problems with singular arcs, can be handled by breaking the problem into multiple phases. Even in these cases, GPOPS can often find a good approximation to the discontinuous solution in a single phase, which can then be refined into a multiphase problem. GPOPS can find numerical derivatives of the various constraints, but if available, analytical expressions should be used to enhance speed and accuracy. Certain constraints, such as control saturation, do not have continuous derivatives. For such cases, an approximation with continuous derivatives can be used.

IV. Ballistic Missile Intercept Example

Before introducing the full ZEM/ZEV algorithm, it is instructive to consider ZEM control for a well-known example problem. The ballistic missile intercept problem is considered to put considerations of the ZEM and the mission time in familiar terms. With this background, the full ZEM/ZEV algorithm will be introduced in the next example problem.

The objective of an example problem here is to examine the performance of guidance algorithms for a tactical missile to intercept a ballistic missile target. The dynamic models for the missile and target are represented as [16]

$$\begin{aligned}\ddot{x}_M &= \frac{-\mu x_M}{(x_M^2+y_M^2)^{1.5}} + a_{xM} = g_{xM} + a_{xM}\\ \ddot{y}_M &= \frac{-\mu y_M}{(x_M^2+y_M^2)^{1.5}} + a_{yM} = g_{yM} + a_{yM}\end{aligned} \tag{15}$$

and

$$\ddot{x}_T = \frac{-\mu x_T}{(x_T^2+y_T^2)^{1.5}} = g_{xT}; \qquad \ddot{y}_T = \frac{-\mu y_T}{(x_T^2+y_T^2)^{1.5}} = g_{yT} \tag{16}$$

where the subscript M is used for the missile, the subscript T is used for the target ballistic missile, (x, y) are position vector components in an Earth-centered inertial coordinate system, μ is the gravitational parameter of the Earth (3.986×10^{14} m^3/s^2), and a is the control acceleration.

The line-of-sight (LOS) is defined as the line from the missile to the target. The LOS angle λ is the angle between this line and a constant reference line in the inertial frame, and it is expressed as

$$\lambda = \tan^{-1}\frac{y_{TM}}{x_{TM}} \tag{17}$$

where (x_{TM}, y_{TM}) are the relative position components in the inertial frame defined as

$$x_{TM} = x_T - x_M \quad y_{TM} = y_T - y_M \tag{18}$$

The relative velocity components (v_x, v_y) are also defined as

$$v_x = \dot{x}_{TM} = \dot{x}_T - \dot{x}_M \quad v_y = \dot{y}_{TM} = \dot{y}_T - \dot{y}_M \tag{19}$$

The distance from the missile to the target is

$$r = \sqrt{x_{TM}^2 + y_{TM}^2} \tag{20}$$

A. Classical Guidance Algorithms

Before discussing an application of the ZEM/ZEV algorithms, first the classical PNG law will be described. The classical PNG law is simply given by

$$\begin{bmatrix} a_{xM} \\ a_{yM} \end{bmatrix} = N V_c \dot{\lambda} \begin{bmatrix} -\sin\lambda \\ \cos\lambda \end{bmatrix} \tag{21}$$

where N is the effective navigation ratio, a user-adjustable parameter, and the closing velocity V_c and LOS rate $\dot{\lambda}$ are determined as

$$V_c = -\dot{r} = \frac{-(x_{TM} v_x + y_{TM} v_y)}{r} \tag{22}$$

$$\dot{\lambda} = \frac{x_{TM} v_x - y_{TM} v_y}{r^2} \tag{23}$$

The classical PNG algorithm commands acceleration perpendicular to the instantaneous LOS direction. For a missile with aerodynamic actuators, the actual acceleration would be perpendicular to the missile's velocity vector. For small turn rates, the missile velocity vector and LOS direction are approximately aligned.

When the gravitational environment is known, the APNG algorithm can be used to improve the performance of the PNG. The APNG law is

$$\begin{bmatrix} a_{XM} \\ a_{YM} \end{bmatrix} = N\left(V_c\dot{\lambda} + \frac{1}{2}(g_{T\perp} - g_{M\perp})\right)\begin{bmatrix} -\sin\lambda \\ \cos\lambda \end{bmatrix} \tag{24}$$

where $g_{T\perp}$ and $g_{M\perp}$ are components of gravity acting on the missile and target, perpendicular to the LOS direction. The best PNG law, described in [16], is the so-called predictive guidance algorithm.

When accurate models of the missile and target dynamics are known, the positions of the missile and target at the planned intercept time can be found by numerical integration. The ZEM vector is determined as

$$\mathbf{ZEM} = \begin{bmatrix} \mathrm{ZEM}_x \\ \mathrm{ZEM}_y \end{bmatrix} = \begin{bmatrix} \tilde{x}_{TF} - \tilde{x}_{MF} \\ \tilde{y}_{TF} - \tilde{y}_{MF} \end{bmatrix} \tag{25}$$

where $(\tilde{x}_{TF}, \tilde{y}_{TF})$ are the predicted final position components of the target and $(\tilde{x}_{MF}, \tilde{y}_{MF})$ are the predicted final position components of the missile if no further accelerations are imparted.

The predictive guidance algorithm is based on PNG, so only components normal to the LOS are considered. The control acceleration is then given as

$$\begin{bmatrix} a_{XM} \\ a_{YM} \end{bmatrix} = N\frac{\mathrm{ZEM}_\perp}{t_{\mathrm{go}}^2}\begin{bmatrix} -\sin\lambda \\ \cos\lambda \end{bmatrix} \tag{26}$$

where

$$\mathrm{ZEM}_\perp = -\mathrm{ZEM}_x \sin\lambda + \mathrm{ZEM}_y \cos\lambda$$

B. Feedback Guidance Using Generalized Zero-Effort-Miss Algorithm

The optimal ZEM feedback algorithm given by Eq. (12) is then expressed as

$$\begin{bmatrix} a_{XM} \\ a_{YM} \end{bmatrix} = \frac{3}{t_{\mathrm{go}}^2}\begin{bmatrix} \mathrm{ZEM}_x \\ \mathrm{ZEM}_y \end{bmatrix} \tag{27}$$

The predictive guidance algorithm, Eq. (26), differs from the generalized ZEM algorithm [(Eq. 27)] in two main ways. First, the predictive algorithm restricts accelerations to be perpendicular to the LOS, while the ZEM algorithm can command accelerations in any direction. Second, the predictive algorithm leaves the navigation constant N as an adjustable parameter, while the ZEM algorithm has a fixed gain to ensure optimality.

Both algorithms are derived assuming that the total flight time is specified. There are some cases where enforcing a particular mission time may be desirable, such as intercepting a target missile before multiple payloads can be deployed or ensuring that intercept does not occur above a particular location on Earth. For most missions, though, the particular time of intercept is not crucial and is not specified beforehand. Although in principle a broad range of intercept times could be specified, missions that are too short or too long can experience problems. More control energy will be consumed, which may exceed the capacity of the actuators and result in a failure to intercept.

Recall that the ZEM is a function of the time-to-go, in addition to being a function of the current missile and target states. The simplest way to determine the time-to-go is to choose the time-to-go that corresponds to the minimum norm of the ZEM. The ZEM is found by numerically integrating the current states without control accelerations, so the ZEM is found at every time step in the integration. Finding the minimum norm of the ZEM is equivalent to simply predicting the time of closest approach. The goal of the control law is to reduce the ZEM to zero, so choosing the time of minimum ZEM corresponds to exploiting the dynamics to give the control algorithm the smallest error to overcome.

A simple line search can be used to find the best time-to-go. As the dynamic equations are integrated, there will be a ZEM associated with each time step. Due to the assumptions for PN guidance, the missile must be headed "toward" the target. When this is true, the predicted ZEM will monotonically decrease with increasing t_f, reach a minimum, then monotonically increase. Finding the time-to-go is just a matter of integrating until the predicted ZEM starts to increase. The smaller of the last two ZEM predictions corresponds to the best time-to-go.

C. Numerical Simulation Example

A ballistic intercept scenario, examined in [16], was used to evaluate both the predictive and ZEM algorithms. With "perfect" initial conditions, the predictive and ZEM laws will both achieve impact with no control accelerations (an ideal situation), while the PNG and APNG laws will still require some control accelerations. To be able to compare the predictive and ZEM laws, the example was modified appropriately. The initial position and initial velocity magnitude are the same, but the initial velocity direction is changed slightly. The initial conditions are $\mathbf{r}_{T0} = (0, 6378.245)$ km, $\mathbf{v}_{T0} = (6785, 2880)$ m/s, $\mathbf{r}_{M0} = (4510.1, 4510.1)$ km, and $\mathbf{v}_{M0} = (2006, 5954)$ m/s.

Solving the optimal control problem with GPOPS is straightforward for the ballistic missile intercept. There are no limits on the control inputs, and generous limits on the states can be used. For example, the x and y positions can be constrained to be from negative two to positive two Earth radii. GPOPS will converge more quickly if there are finite bounds. The flight time can be left free for the overall optimal solution, or specified to find the optimal intercept for a given mission time.

Figure 1 shows comparisons between the generalized ZEM algorithm with adaptive t_{go} and the open-loop optimal solution, as generated by GPOPS. The trajectories, control histories, and performance index histories are shown. The adaptive flight-time algorithm finds the best flight time to be 687 s, a little less than the optimal time of 700 s. The control histories, however, are similar, and the performance index value is only 2% larger.

The ZEM algorithm has a fixed gain to ensure optimality, while the gain is an adjustable parameter for the PNG-based methods. The optimal value for the navigation constant turns out to be three, but it is usually chosen in the range from three to five. Larger values cause the vehicle to turn onto a collision course more quickly, at the expense of more control effort. This increased control effort is manifest in two different ways. First, the performance index for the mission increases. Second, the maximum level of commanded acceleration increases, which can become important if there is an upper limit on available accelerations.

To study the effect of changing the navigation constant, the intercept mission example was simulated for both the PNG law and the APNG law for a variety of N ranging from 2 to 10. Figure 2 shows the performance index with varying N, as well as the maximum control magnitude required. For the test case shown, the optimal N for PNG is 5.3, while the optimal N for APNG is 3.4. Table 1 gives a detailed comparison of the different guidance laws. The ZEM law with adaptive flight time is shown, as well as the ZEM law using the known optimal flight time of 700 s.

The best PNG law performs better than the ZEM law with adaptive t_f, which in turn is better than the APNG law. The ZEM law with the known optimal flight time performs best of all, effectively identical to

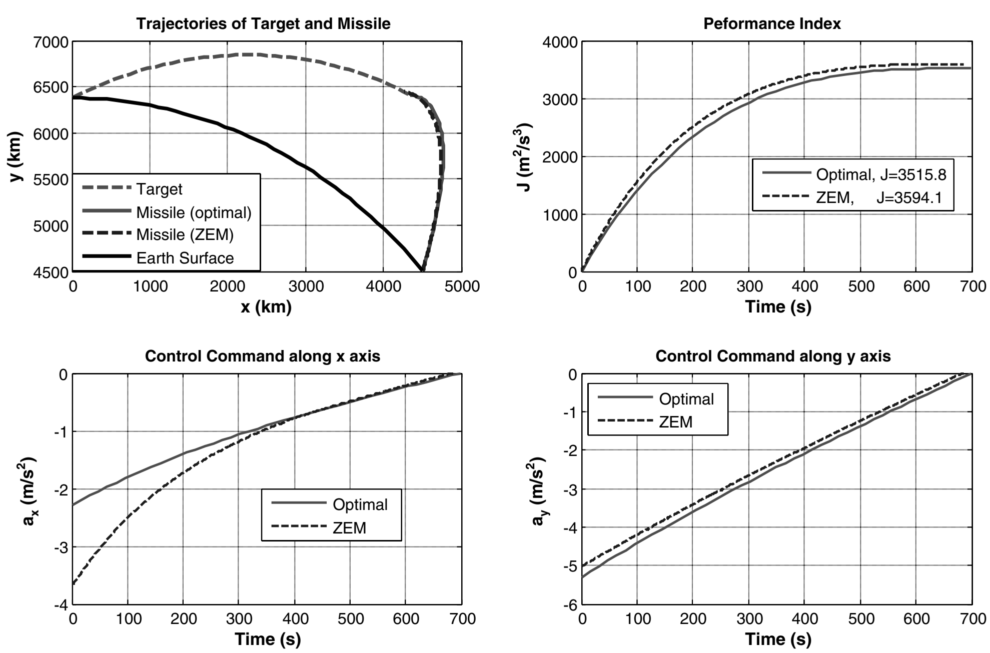
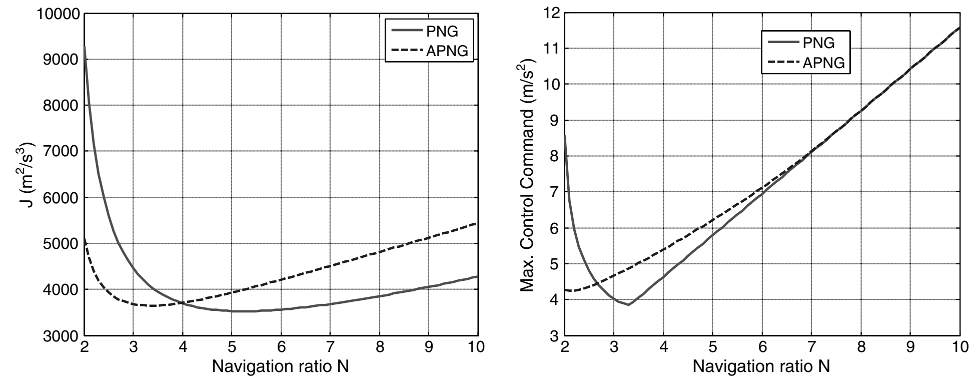


Fig. 1 Missile intercept using the generalized ZEM algorithm and an open-loop optimal approach.



a) Performance index

b) Maximum magnitude of control

Fig. 2 Missile intercept using PNG and APNG with different navigation ratios.

the optimal. The specific performance of the different algorithms depends on the setup of the problem. The optimal values for N are only found by simulating many cases, and the optimal flight time for the ZEM algorithm comes from first finding the open-loop optimal solution. If the maximum control magnitude is a concern, there is a limit to how large a navigation ratio is acceptable. And although the best case for the APNG does not perform as well as the best-case PNG law, other tradeoffs for J and control magnitude must be considered when the optimal N cannot be determined beforehand.

One advantage the ZEM law has over the PNG-based laws is the ability to control flight time. A mission with the same initial conditions was simulated with flight times ranging from 500 to 900 s. Figure 3 shows the trajectories for various cases, as well as the performance index J for both the ZEM law and the open-loop optimal solution. Not only is the ZEM law able to intercept at a variety of times, it does so with barely any discernible difference from the optimal.

V. Asteroid Proximity Operation Example

The problem of detecting a possibly threatening near-Earth object (NEO) and responding to that threat has been given much consideration in recent years. One major task of the planetary defense community is to find an optimal approach to avert a potential NEO–Earth collision [17]. Although there is no universally accepted definition of optimal mitigation for this problem, three broad categories of deflection missions have emerged. The first is a slow-push scheme to gradually change the NEO's orbit. The second is a high-speed intercept mission by a massive spacecraft, changing the NEO's orbit via a kinetic impact [18,19]. The third approach is a nuclear detonation for large NEOs, or when there is little mission lead time. A number of different guidance algorithms can be employed for such asteroid intercept or rendezvous missions.

A. Terminal Guidance for Asteroid Intercept

Consider both the target asteroid and the interceptor spacecraft as point masses in a heliocentric Keplerian orbit, with the equations of motion described by

$$\ddot{\mathbf{r}}_T = -\frac{\mu}{r_T^3}\mathbf{r}_T \tag{28}$$

$$\ddot{\mathbf{r}}_S = -\frac{\mu}{r_S^3}\mathbf{r}_S - \frac{\mu_\otimes}{r^3}\mathbf{r} + \mathbf{a} \tag{29}$$

where $\mathbf{r}_T$ and $\mathbf{r}_S$ are the position vectors of the target and the interceptor, respectively, with magnitudes r_T and r_S, μ is the gravitational parameter of the sun (1.32715×10^{20} m^3/s^2), $\mu_\otimes$ is the estimated gravitational parameter of the asteroid, $\mathbf{a}$ is the applied control acceleration, and $\mathbf{r}$ is the relative position vector of the interceptor (with magnitude r) with respect to the asteroid, defined as

$$\mathbf{r} = \mathbf{r}_S - \mathbf{r}_T \tag{30}$$

Neglecting the asteroid's gravitational parameter, the equations of motion are essentially the same as for missile intercept. The ZEM algorithm can be obtained as

$$\mathbf{a} = -\frac{3}{t_{go}^2}\tilde{\mathbf{r}} = \frac{3}{t_{go}^2}(\tilde{\mathbf{r}}_{TF} - \tilde{\mathbf{r}}_{SF}) \tag{31}$$

where $\tilde{\mathbf{r}}_{TF}$ and $\tilde{\mathbf{r}}_{SF}$ are the predicted final positions of the asteroid and interceptor if no further control accelerations are applied, and t_{go} can be based on a specified final time or adjusted as described in the previous section.

For the case where the gravitational effect is negligible, the optimal time-to-go can be calculated based on the current relative position $\mathbf{r}$ and relative velocity $\mathbf{v}$, as follows [7]:

$$t_{go} = \frac{2r}{v}\left(\cos\theta - \sqrt{\cos^2\theta - 3/4}\right); \qquad \theta \in (-30\ \text{deg}, 30\ \text{deg}) \tag{32}$$

where θ is the angle between $\mathbf{r}$ and $-\mathbf{v}$. There exists a solution for t_{go} that locally minimizes the optimal performance index only when θ is in the range shown in Eq. (32). Outside that range, the performance index decreases monotonically with increasing t_{go}, and some upper bound on the flight time must be selected.

B. Terminal Guidance for Asteroid Landing

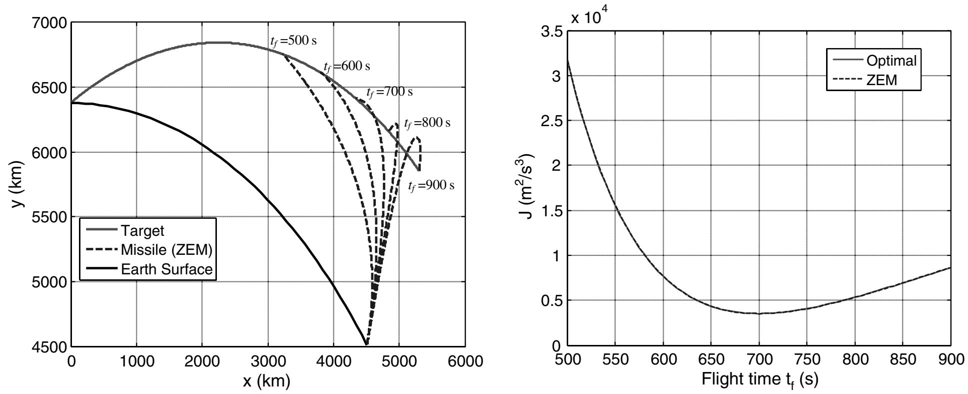
For an asteroid landing problem, we assume that the asteroid is a sphere with radius $R_\otimes$. For convenience, we also ignore the negligible gravitational acceleration of the asteroid, so the equations of motion become

$$\dot{\mathbf{r}} = \mathbf{v} \quad \dot{\mathbf{v}} = \mathbf{a} \tag{33}$$

The terminal velocity is by definition zero for the landing mission. If the landing site is specified, one of the algorithms from [8] can be

Table 1 Performance of various guidance methods for missile intercept

	PNG ($N = 5.3$)	APNG ($N = 3.4$)	ZEM (adaptive t_f)	ZEM ($t_f = 700$ s)	Open-loop optimal
Performance index J	3526.5	3648.2	3594.1	3515.9	3515.8
Max control, m/s^2	6.14	4.94	6.25	5.79	5.79
Flight time t_f, s	701	702	687	700	700



a) Vehicle trajectories using ZEM b) Performance comparison with the optimal

Fig. 3 ZEM algorithm with various specified flight times.

used. When the landing site is not specified, then the final position must meet the following constraint:

$$\mathbf{r}_f^T \mathbf{r}_f = R_\otimes^2 \tag{34}$$

Incorporating the terminal constraint, the performance index can be modified as follows:

$$J = \frac{1}{2}\sigma(\mathbf{r}_f^T \mathbf{r}_f - R_\otimes^2) + \frac{1}{2}\int_{t_0}^{t_f} \mathbf{a}^T \mathbf{a}\, \mathrm{d}t \tag{35}$$

where σ is the scalar multiplier for the terminal constraint. The costate vector has the following terminal constraint:

$$\mathbf{p}_r(t_f) = \frac{\partial J}{\partial \mathbf{r}_f} = \sigma \mathbf{r}_f \tag{36}$$

The optimal landing site $\mathbf{r}_f^*$ can be found as

$$\mathbf{r}_f^* = R_\otimes \frac{t_{\mathrm{go}}(\mathbf{v} + \mathbf{v}_f) + 2\mathbf{r}}{\|t_{\mathrm{go}}(\mathbf{v} + \mathbf{v}_f) + 2\mathbf{r}\|} = R_\otimes \frac{t_{\mathrm{go}}\mathbf{v} + 2\mathbf{r}}{\|t_{\mathrm{go}}\mathbf{v} + 2\mathbf{r}\|} \tag{37}$$

Finally, the guidance algorithm for optimal asteroid soft landing is obtained as

$$\mathbf{a} = \frac{6[\mathbf{r}_f^* - (\mathbf{r} + t_{\mathrm{go}}\mathbf{v})]}{t_{\mathrm{go}}^2} - \frac{2(\mathbf{v}_f - \mathbf{v})}{t_{\mathrm{go}}} \tag{38}$$

Similar to the intercept problem, an optimal time-to-go exists for certain initial conditions [7]. The optimal time-to-go is found by solving the final equation:

$$t_{\mathrm{go}}^2 \mathbf{v}^T \mathbf{v} - t_{\mathrm{go}} 6\mathbf{v}^T(\mathbf{r}_f - \mathbf{r}) + 9(\mathbf{r}_f - \mathbf{r})^T(\mathbf{r}_f - \mathbf{r}) = 0 \tag{39}$$

where the condition

$$[6\mathbf{v}^T(\mathbf{r}_f - \mathbf{r})]^2 - (4\mathbf{v}^T\mathbf{v})9(\mathbf{r}_f - \mathbf{r})^T(\mathbf{r}_f - \mathbf{r}) \le 0 \tag{40}$$

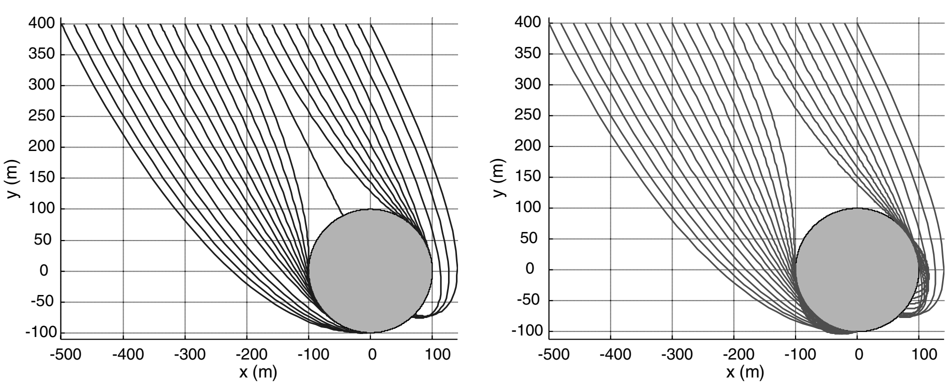
is required for a local minimum of t_{go}. When this minimum does not exist, t_{go} needs to be as large as possible. However, there is no guarantee that any particular t_{go} will not intercept the surface of the asteroid, so a numerical simulation is required to evaluate feasibility using the chosen $t_{\max}$ as the initial guess.

C. Numerical Simulation Example

An asteroid landing problem using the guidance law expressed by Eq. (38) was numerically simulated for a variety of initial conditions. The target asteroid has a radius of 100 m and is assumed to be centered at the origin of the coordinate system. The lander has an initial velocity of $(20, -40, 0)$ m/s. Various initial positions are tested to illustrate the performance of the ZEM/ZEV algorithm with adaptive t_f and landing site. For initial conditions without a local minimum for the performance index, the ultimate upper bound of the flight time is 40 s.

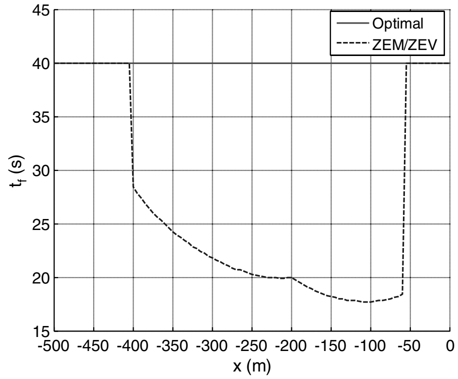
Solving the optimal asteroid landing problem with GPOPS requires only a couple of constraints. There is a terminal event constraint requiring that the final position lies on a circle or radius 100 m. The final time is free, with the designer-chosen upper bound of 40 s. In this case, the optimal solution always takes the maximum amount of time. As in the previous example, generous bounds on the states and controls are provided to make the search space finite.

Figure 4 shows the trajectories using the ZEM/ZEV approach and the open-loop optimal trajectories obtained from GPOPS. Figure 5 compares the calculated flight time and performance index for the ZEM/ZEV law and the open-loop optimal solution.

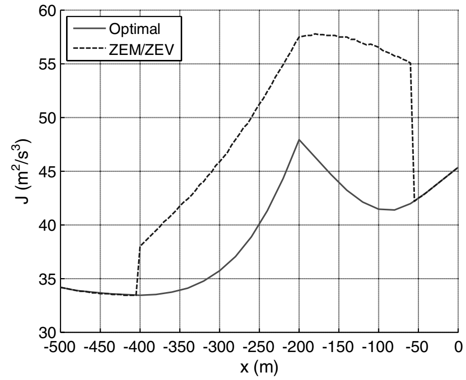


a) ZEM/ZEV with adaptive t_f and landing site b) Open-loop optimal

Fig. 4 Vehicle trajectories for asteroid landing ($t_{\max} = 40$ s).



a) Flight time comparison



b) Performance comparison

Fig. 5 Performance comparison between ZEM/ZEV and open-loop optimal ($t_{\mathbf{max}} = 40$ s).

The landing mission can be successfully completed for all of the initial conditions considered. The initial conditions roughly fall into two different types. The first type includes initial conditions that will collide with, or come close to colliding with, the asteroid if the lander continues on a straight-line trajectory. The second type has initial conditions that will travel well outside the asteroid's footprint. For the second type of initial conditions, the ZEM/ZEV algorithm performs identically to the open-loop optimal solution, which verifies the optimal feature of autonomous landing site calculation approach. For initial conditions on a collision course (or nearly so), a collision hazard is detected and the flight time is adjusted downward until the mission is safe. The reduced mission time leads to more acceleration commanded.

One trajectory of particular interest is the case when the spacecraft starts on a collision course through the center of the asteroid. The ZEM/ZEV algorithm will simply reduce the mission time and arrest the spacecraft's forward velocity, while the optimal solution is to turn the spacecraft to fly around the asteroid while taking the full 40 s.

VI. Orbit Transfer Example with Continuous Thrust

The objective of an orbital transfer/raising problem is to optimally transfer a spacecraft from a lower orbit to a higher orbit, with a specified injection point and velocity, at a given time. The spacecraft can also be brought from a higher orbit to a lower one. For high-thrust engines, the well-known impulsive Hohmann transfer is the minimum-energy transfer, however, for continuous low-thrust engines, other methods must be used.

For simplicity, consider the following spacecraft dynamic equations:

$$\dot{x} = v_x \quad \dot{y} = v_y \tag{41}$$

$$\dot{v}_x = -\mu \frac{x}{(x^2 + y^2)^{1.5}} + a_x \quad \dot{v}_y = -\mu \frac{y}{(x^2 + y^2)^{1.5}} + a_y \tag{42}$$

where (x, y) and (v_x, v_y) denote the position components and velocity components in heliocentric inertial orbit plane, μ is the gravitational parameter of the sun, and (a_x, a_y) are control accelerations along the (x, y) axes.

The only requirement for the control system is to ensure that the spacecraft satisfies the following terminal conditions at the final time t_f:

$$\begin{aligned} x(t_f) &= x_c, & y(t_f) &= y_c \\ v_x(t_f) &= v_{xc}, & v_y(t_f) &= v_{yc} \end{aligned} \tag{43}$$

A. Application of Zero-Effort-Miss/Zero-Effort-Velocity Feedback Guidance Algorithm

The equations of motion are strongly coupled, and an analytic optimal control algorithm does not exist. The ZEM/ZEV algorithm [Eq. (11)] can control the terminal position and velocity at a specified final time. These encompass all of the requirements of the orbit transfer mission, making it a good candidate for this problem. Expressed in the x and y coordinates, the proposed ZEM/ZEV law becomes

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \frac{6}{t_{go}^2} \begin{bmatrix} \mathrm{ZEM}_x \\ \mathrm{ZEM}_y \end{bmatrix} - \frac{2}{t_{go}} \begin{bmatrix} \mathrm{ZEV}_x \\ \mathrm{ZEV}_y \end{bmatrix} \tag{44}$$

where the ZEM and ZEV are obtained by subtracting the predicted terminal states (with no further control accelerations) from the required terminal states, as follows:

$$\begin{bmatrix} \mathrm{ZEM}_x \\ \mathrm{ZEM}_y \end{bmatrix} = \begin{bmatrix} x_c - \tilde{x}_F \\ y_c - \tilde{y}_F \end{bmatrix} \quad \begin{bmatrix} \mathrm{ZEV}_x \\ \mathrm{ZEV}_y \end{bmatrix} = \begin{bmatrix} v_{xc} - \tilde{v}_{xF} \\ v_{yc} - \tilde{v}_{yF} \end{bmatrix} \tag{45}$$

B. Numerical Simulation Example

An orbit transfer example from Earth to Mars is considered here to evaluate the performance of the generalized ZEM/ZEV algorithm. Feedback guidance control is not generally needed for such an orbital transfer mission, because an open-loop optimal trajectory can easily be generated during the long mission time. We examine this case here only as an illustrative example to demonstrate the applicability of the ZEM/ZEV feedback guidance concept.

For ease of analysis, canonical (or normalized) units will be used. For Earth's orbit, the mean distance to the sun is 1 AU (astronomical unit), 1.4959965×10^{11} m. Defining 1 TU (time unit) as 58.132821 days gives the Earth a circular orbital velocity of 1 AU/TU. Mars orbit is at a radius of 1.54 AU, with a velocity of 0.8059 AU/TU. In these units, the gravitational parameter μ is 1 $\mathrm{AU}^3/\mathrm{TU}^2$.

For this mission, the spacecraft starts at (1, 0) AU with velocity of (0, 1) AU/TU. The terminal position is (−0.3986, 1.4875) AU, the terminal velocity is (−0.7784, −0.2086) AU/TU. The flight time is 144 days, or 2.4771 TU.

Finding the optimal control with GPOPS is straightforward in this case, as the initial and final position and velocity are fully specified, as is the mission time. Again, generous limits on the states and controls are given to maintain a finite search space.

Figure 6 shows the position, velocity, and acceleration histories for both the ZEM/ZEV algorithm and the optimal open-loop solution determined by GPOPS. Figure 7 shows the transfer orbits for both cases. The direction and normalized magnitude of the acceleration commands are also shown every 1/10 of the mission.

The plots of position and velocity from the ZEM/ZEV algorithm nearly overlap the optimal plots. Overall, then, the ZEM/ZEV algorithm drives the spacecraft to result in the near-optimal trajectory. Recall that the plots use canonical units, and the differences may seem more significant in more familiar units. The acceleration history plots show more of a difference between the two guidance schemes. The acceleration histories vary the most at the beginning and the end of the mission. At the beginning of the mission the calculated ZEM and ZEV are large, while at the end the time-to-go is small. Even with

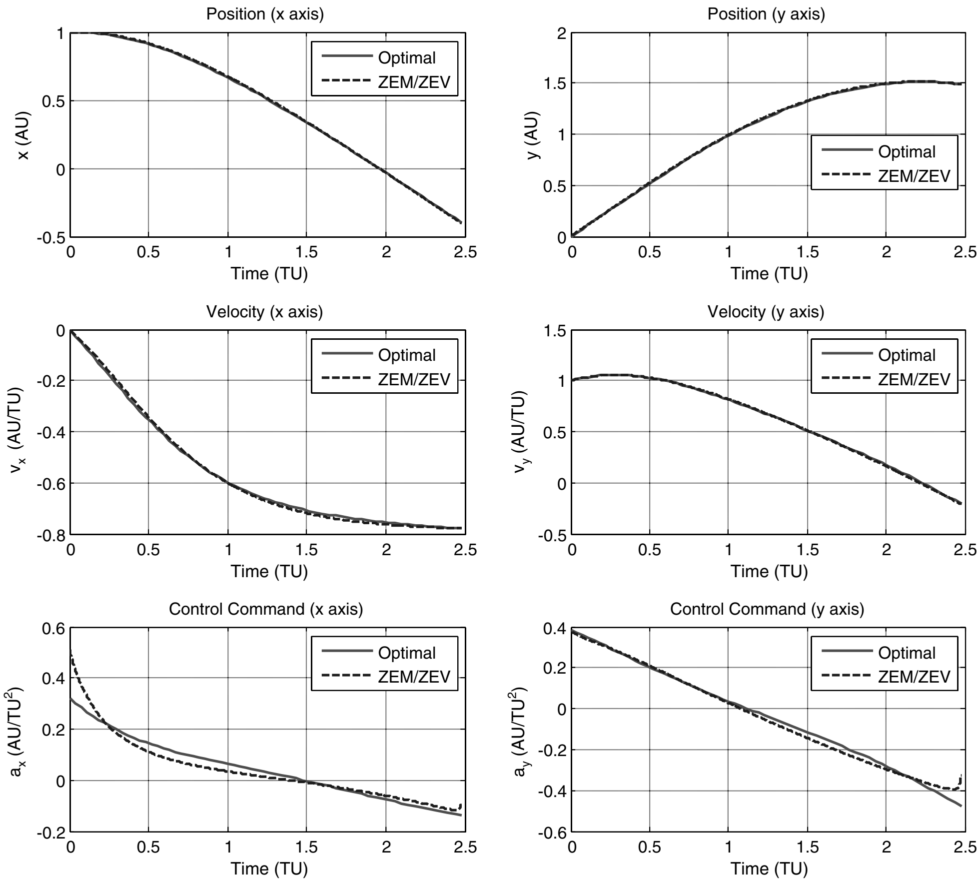


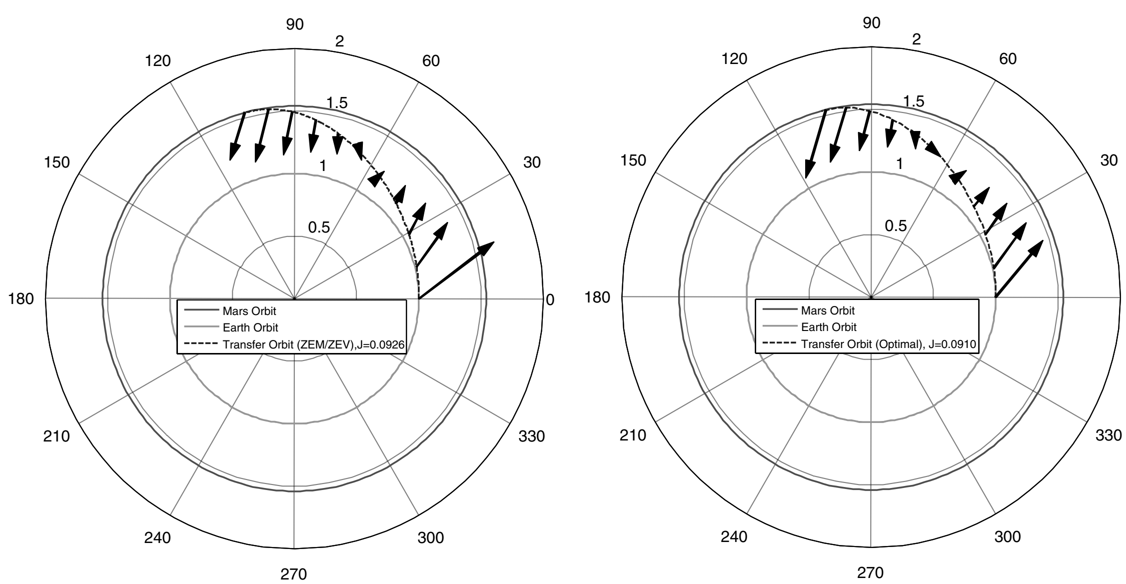
Fig. 6 144-day orbit transfer from Earth to Mars.

these differences, the performance index J is 0.0926, only 1.76% larger than the open-loop optimal J of 0.0910.

In the scenario examined, the ZEM/ZEV algorithm achieves near-optimal performance. The performance is better for shorter missions, and gets worse for longer missions. For longer missions, the prediction error increases, and nonlinearities build up. This technique is not applicable for very low-thrust transfer missions where the spacecraft makes more than one-half of a revolution around the sun. Most of the approaches presented in the literature for such missions are open-loop techniques.

VII. Orbit-Raising Problem

The objective of an orbit-raising problem is to transfer a spacecraft from one orbit to another orbit. For an orbital transfer to a circular orbit, the terminal constraints are that the spacecraft should be placed at a specified distance from the sun with circular orbital velocity. The final radial velocity is zero, while the true anomaly (or any equivalent angular position) is free. Due to the nature of the constraints, polar coordinates are used. The standard dynamical models for this type of orbit-raising problem are described by



a) b)

Fig. 7 144-day orbit transfer from Earth to Mars.

$$\dot{r} = u \quad \dot{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + a_r \quad \dot{v} = -\frac{uv}{r} + a_t \tag{46}$$

where r, u, and v represent the distance of the spacecraft from the sun, the radial velocity, and transverse velocity, respectively, and a_r and a_t are control accelerations in the radial and transverse directions, respectively. The just described required terminal states are

$$r(t_f) = r_f, \qquad u(t_f) = 0, \qquad v(t_f) = \sqrt{\frac{\mu}{r_f}} \tag{47}$$

A. Application of Zero-Effort-Miss/Zero-Effort-Velocity Feedback Guidance

The orbit-raising problem is somewhat unusual in that the control requirements are different along the radial and tangential axes. In the radial direction, there are position and velocity requirements as usual. In the tangential direction, we have the rare case where only the velocity is specified. The feedback algorithm is a combination of Eqs. (11) and (13), as follows:

$$a_r = \frac{6}{t_{go}^2}\mathrm{ZEM}_r - \frac{2}{t_{go}}\mathrm{ZEV}_r \quad a_t = \frac{1}{t_{go}}\mathrm{ZEV}_t \tag{48}$$

where the ZEM and ZEV are, as before, the difference between the required and predicted terminal states defined as

$$\mathrm{ZEM}_r = r_f - \tilde{r}_F \quad \mathrm{ZEV}_r = -\tilde{u}_F \quad \mathrm{ZEV}_t = \sqrt{\frac{\mu}{r_f}} - \tilde{v}_F \tag{49}$$

Due to nonlinearities in the system, the ZEM/ZEV algorithm with direct gravity compensation, Eq. (14), can be used as

$$a_r = \frac{6}{t_{go}^2}(r_f - (r + t_{go}u)) - \frac{2}{t_{go}}(u_f - u) - \left(\frac{v^2}{r} - \frac{\mu}{r^2}\right)$$
$$a_t = \frac{1}{t_{go}}(v_f - v) - \left(-\frac{uv}{r}\right) \tag{50}$$

The first ZEM/ZEV algorithm, Eq. (48), works by numerically predicting the final states, so it is called the ZEM/ZEV predicting algorithm, or ZEM/ZEV-p. The second algorithm, Eq. (50), works by directly compensating for the gravitational acceleration terms, so it is called the ZEM/ZEV compensating algorithm, or ZEM/ZEV-c.

B. Numerical Simulation Example

The same initial conditions as the Mars orbit transfer problem are considered, with the same flight time. In the polar coordinate system, we have $r(t_0) = 1$ AU, $u(t_0) = 0$ AU/TU, $v(t_0) = 1$ AU/TU, and $t_f = 2.4771$ TU. Both proposed ZEM/ZEV algorithms, described by Eqs. (48) and (50), are used, called ZEM/ZEV-p and ZEM/ZEV-c, respectively. The optimal open-loop solution generated by GPOPS is also shown.

Implementing GPOPS for the basic orbit-raising problem is relatively simple. The flight time is fixed. The final radial and tangential velocities are specified, as is the final radius. The final rotation angle is free. Later, a more interesting implementation of a multiphase optimal control problem with GPOPS will be described.

Figure 8 shows the position, velocity, and acceleration histories in the radial and tangential directions for all three cases. The different algorithms perform similarly in the radial direction, but are noticeably different in the tangential direction.

Sharma et al. describes a waypoint method for the same orbit-raising problem [9]. The problem was broken up into several segments, where the terminal state for each segment is a waypoint in the overall problem. A series solution method (SSM) was then used to connect the waypoints. The ZEM/ZEV algorithm can be used as part of a waypoint method to improve performance. For this problem, the

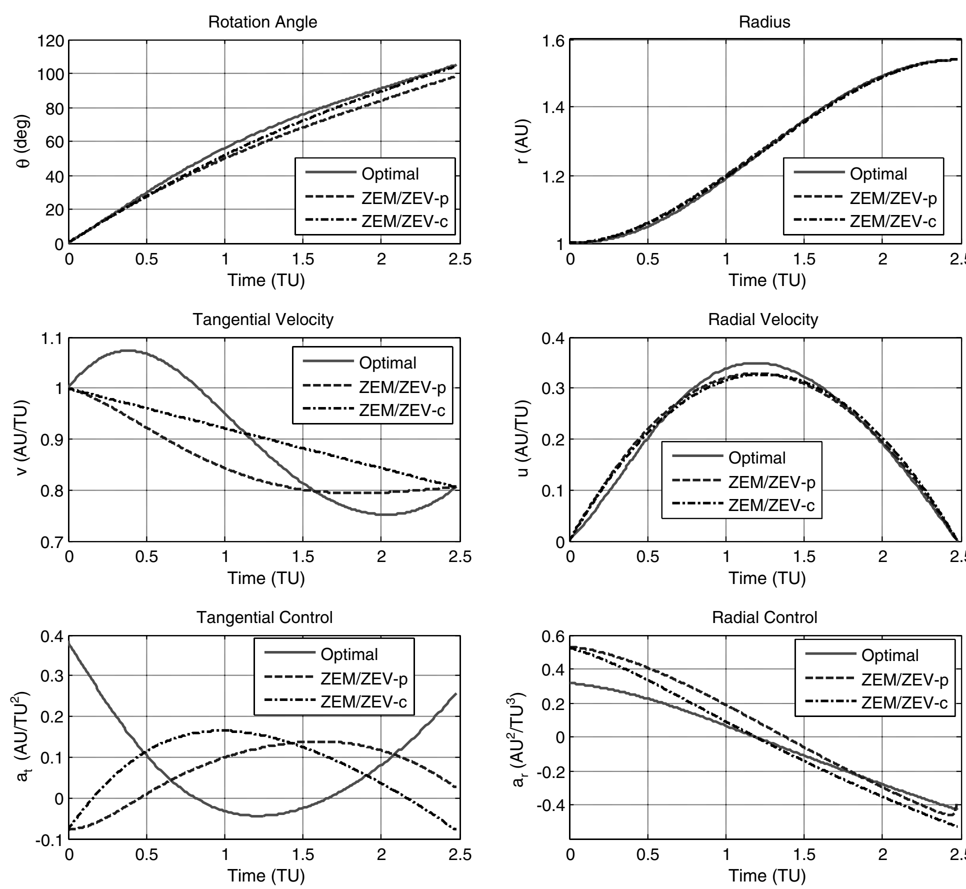


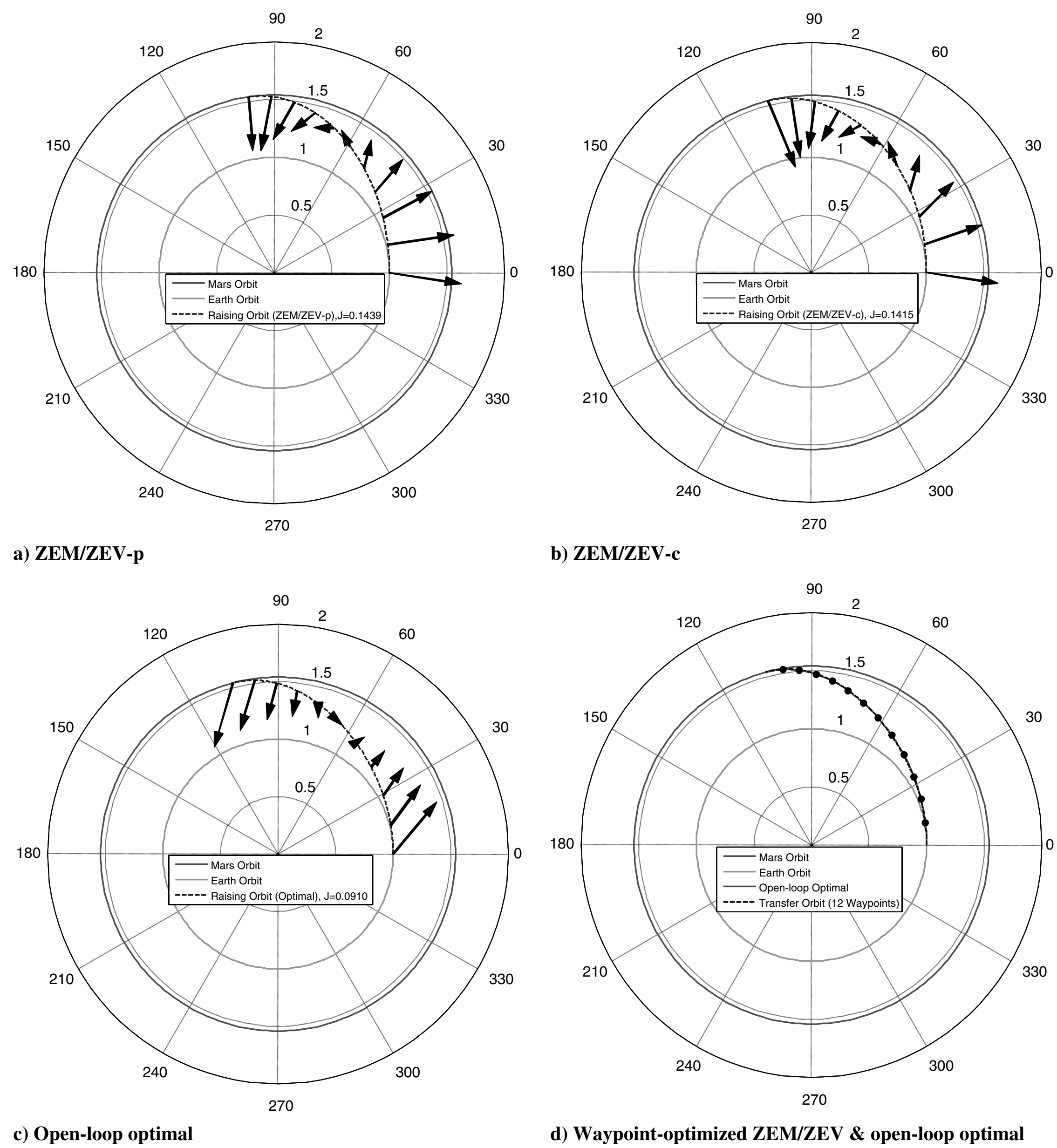
Fig. 8 144-day orbit raising from Earth orbit to Mars orbit.

total flight time is divided into equal-length segments, and GPOPS is used to generate waypoints, enforcing the control law during the flight. These waypoints are then used as intermediate points for control law simulation. The gravity compensation form ZEM/ZEV-c is used for the waypoint method.

Optimal waypoints for the orbit-raising problem can be found with GPOPS. The principle of optimality ensures that the optimal solution in multiple phases is equivalent to the optimal solution as one phase. To find waypoints, then, the problem is broken up into any number of segments of equal duration. The six-dimensional waypoint (three position components and three velocity components) is left as a set of six parameters to be found by GPOPS. Parameter variables are constant during each phase, but not known ahead of time. When GPOPS solves the optimal control problem, the optimal parameters (the optimal waypoints) are found. ZEM/ZEV control can then be used, with each waypoint used as the final position and final velocity for a given phase. A much more in-depth discussion of optimal waypoint determination, applied to a Mars landing problem, is given in [20].

Figure 9 shows the orbit-raising trajectories for all four methods discussed, ZEM/ZEV-p, ZEM/ZEV-c, optimal open-loop, and the waypoint method with 12 waypoints. The first three of these plots show the normalized accelerations every one-tenth of the mission time, while the last shows the locations of the waypoints. Table 2 compares the performance of the ZEM/ZEV waypoint method and the SSM waypoint method. Four cases are considered, using 1, 2, 4, and 12 total waypoints. The case with no waypoints, as well as the open-loop optimal solution, are also shown for comparison.

The two ZEM/ZEV algorithms perform similarly, with a performance index 50–60% larger than the open-loop optimal. The compensating algorithm performs better than the predicting algorithm due to the nonlinear coupled terms. The predicting algorithm does not command as large of an angular change during the mission, as can be seen in the rotation angle plot in Fig. 8. This plot shows that control effort is wasted trying to overcome misleading terms in the dynamic equations. The acceleration vectors in Fig. 9 show that the ZEM/ZEV algorithms spend too much effort in the radial direction, which must be made up for later. The ZEM/ZEV algorithms also show the opposite trends in tangential control from the optimal. Because the control directions are separated, the tangential control channel cannot account for the effects of radial acceleration on tangential velocity. One way to overcome these problems is to simply specify a terminal position and velocity, changing the orbit-raising problem to an orbital transfer problem. For the case of the orbital transfer problem



a) ZEM/ZEV-p

b) ZEM/ZEV-c

c) Open-loop optimal

d) Waypoint-optimized ZEM/ZEV & open-loop optimal

Fig. 9 Orbit-raising trajectories using ZEM/ZEV and open-loop optimal methods.

Table 2 Performance comparisons between ZEM/ZEV algorithms and SSM

	No waypoint	2 waypoints	4 waypoints	12 waypoints	Open-loop optimal J
SSM ([9])	0.1314	0.1051	0.0982	0.0942	0.0910
ZEM/ZEV-c	0.1415	0.1152	0.0996	0.0922	

formulation, the ZEM/ZEV comes within 2% increase of the open-loop optimal performance index value.

The ZEM/ZEV algorithm can also be improved by implementing a waypoint scheme. The adverse effects from nonlinear terms and coupled dynamics are reduced for shorter mission times. By breaking the mission up into many shorter segments, the feedback properties of the ZEM/ZEV algorithm can be preserved while approaching optimal performance. The ZEM/ZEV algorithm compares favorably with the SSM method as the number of waypoints is increased, as can be seen in Table 2.

VIII. Conclusions

Four different applications of the generalized zero-effort-miss/zero-effort-velocity (ZEM/ZEV) feedback guidance algorithm have been investigated in this paper. The application examples were the ballistic missile intercept problem, the asteroid intercept and landing problem, and the orbital transfer/raising problems. For cases when the gravitational acceleration can be assumed to be independent of the vehicle's state, three different feedback-optimal ZEM/ZEV algorithms are considered. For many practical missions, the gravitational acceleration is a function of the vehicle's state. By numerically propagating the system state, corresponding generalized ZEM/ZEV algorithms can be obtained.

Numerical simulations demonstrated that the generalized ZEM/ZEV guidance algorithm can achieve intercept at a specified time. When the mission time is not specified, performance can be improved with a flight-time adaptive approach. ZEM/ZEV feedback guidance is conceptually simple, and is easy to implement. It works for many different cases, as the gains are predefined and do not need to be adjusted based on experience or on the specific problem.

The ZEM/ZEV algorithm can be used for a case, such as asteroid landing, where the only the magnitude of the terminal position is specified. Results of numerical simulations show the feasibility of the approach, including autonomous landing site selection.

For highly nonlinear systems with coupled dynamics, such as the orbital transfer/raising problem, numerical simulations have confirmed the effectiveness of the generalized ZEM/ZEV algorithm. For some missions, the performance of the ZEM/ZEV algorithm is significantly worse than the open-loop optimal solution. In these cases, a series of waypoints can be found using commercially available optimization software. The ZEM/ZEV algorithm is then used between each waypoint. The ZEM/ZEV waypoint algorithm approaches the performance of the open-loop optimal solution, while maintaining the robustness of a closed-loop feedback algorithm.

Acknowledgments

This research work was supported by a research grant from the NASA Iowa Space Grant Consortium awarded to the Asteroid Deflection Research Center at Iowa State University. The authors would like to thank Ramanathan Sugumaran (Director of the NASA Iowa Space Grant Consortium) for his support of this research project.

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