ISSN 1750-9823 (print)
International Journal of Sports Science and Engineering
Vol. 06 (2012) No. 01, pp. 041-053

Optimizing the Pole Properties in Pole Vaulting by Using Genetic Algorithm Based on Frequency Analysis

Ali Fellah Jahromi [1, +], Ahmed Atia [1], Rama B. Bhat [1], Wen-Fang Xie [1]

[1] Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC, H3G 2W1, Canada

(*Received August 30, 2011, accepted September 9, 2011*)

Abstract. The pole vaulting is a sport with a long history involving complex dynamic motion. The aim of pole vaulting for the athlete is to achieve maximum height. The dynamic behavior of the pole-athlete system is identical to an inverted pendulum system with the buckling pole acting as a spring. The purpose of this paper is to study the influence of factors on the pole vaulting performance such as the initial velocity of the athlete, pole stiffness and length. The best results are obtained when all the stored potential energy in the pole is released with the pole reaching the 90 degrees condition for success. The natural frequency of the pole-athlete system is obtained by Fast Fourier Transform (FFT). The storing and releasing time of potential energy should be equal to half the natural period. Based on this condition the pole length and stiffness are determined by using Genetic Algorithm (GA) so as to achieve maximum height. This paper presents an optimization procedure to design a flexible pole based on the frequency analysis. The result of this research can be utilized in sport industries to design optimum poles.

Keywords: Pole Vault, Genetic Algorithm, Fast Fourier Transform, Lagrangian method.

1. Introduction

Pole vaulting has been a competitive sport since 1984 A.D. in ancient Ireland [1]. The Pole Vault was born in Germany as a type of sport in the 18th century. The Pole Vault rules from 18thcentury till now have been approximately the same. But, the material of the pole always kept changing, from solid wood, to bamboo, to glass fiber and to carbon fiber.

Research on Pole Vault sport was focused on two main topics; dynamic motion and material properties of the pole. This paper is focused on dynamic motion analysis of the pole. Ekevad and Lundberg [2] studied the pole vault motion by the finite element method and obtained the ratio between the maximum potential energy and the initial kinetic energy of the athlete as an indicator of the efficiency of the maneuver. In these studies a function of the dimensionless parameters for the pole vault motion was developed. Frère*et al* [3] studied the kinetic and potential energies of the athlete and the strain energy stored in the pole. They reviewed the influence of run-up and take-off velocities and the torque produced by the athlete during pole vaulting. McGinnis and Bergman [4] carried out an experimental study on the internal forces and moments of the athlete during vaulting. Hubbard [5] studied a three segments vault considering internal muscle forces of the athlete and also studied the effect of initial velocities (run-up and take-off) of the vaulter. Ohshima and Ohtsuki [6] optimized the joint torque during pole vaulting by genetic algorithm, simulated the dynamic motion of pole vaulting and calculated the optimum value of the required torque by genetic algorithm.

Dillman and Nelson [7] determined experimentally the kinetic and potential energies of the athlete during the vault and studied the material properties of the pole. Walker and Kirmser [8] studied the effect of pole stiffness by using a one segment pendulum model of the vaulter. McGinnis [9] used the finite element method with different pole stiffnesses with a fixed scheme of motion. Linthorne [10] studied the effect of the take-off velocity and angle considering a point mass for the vaulter and a mass-less perfectly rigid pole. Also he studied the optimum take-off technique and pole characteristics for a typical world-class pole vaulter [11].This study presented two advantages for a flexible fiberglass pole in comparison with rigid wooden and bamboo pole. It showed that the fiberglass pole reduces the amount of energy dissipated and needs lower

+ Ali Fellah Jahromi Tel.: +1 514 848 2424 Ext.: 7268
E-mail address: al_fel@encs.concordia.ca.

take-off angle and lower kinetic energy.

The frequency analysis of the dynamic motion of the pole is also one of the important areas that received attention. Cooley and Turkey [12] identified a method for programming FFT, which was used in many frequency analysis studies. Low [13] studied the frequency response of a vibratory system by FFT, and solved the equation of motion by Runge-Kutta method. This study demonstrated how the FFT method can be used to find the natural frequency of the system from displacement response.

In the recent studies the pole vault motion is modeled by two dynamic methods and the pole is considered as a mass-less spring. The aim is to study the effect of the run up speed of the athlete, pole length and stiffness. The results on the responses of the pole vault are presented in 5 steps. In the first step, responses of the pole vault motion are presented for given values and the motion is simulated until the pole reached 90 degrees (successful vault). In the second step the effect of the athlete's run up speed is studied to find out the minimum and maximum speed to make the vaulter reach 90 degrees. The third and fourth steps study the effect of pole length and stiffness. The last part is to optimize the length and stiffness of the bar by conducting frequency analysis. By using the results of previous parts, the limitation for the pole length and stiffness are identified for the optimization procedure. Then, by using GA, the pole length and stiffness are optimized in order to achieve the maximum height, and equality constraint for frequency of bar oscillation is considered. Finally, the responses of the system with optimized values are plotted.

The rest of the paper is organized in five sections. Section II presents the mathematical model of the pole vault motion. Lagrangian method is used to derive the equations of motion. Using the Fast Fourier Transform to find the natural frequency of the system is presented in section III. The Genetic Algorithm optimization method is given in the section IV. Finally, the results obtained from optimizing the main parameters of the system are presented and discussed in section V. The conclusions are given in section VI.

2. Mathematical Model and Equations of Motion Using Lagrangian Method

The pole-athlete system can be considered as an inverted pendulum with the pendulum rod acting as a spring. In fact, the bar buckles during the pole vault motion although for the present analysis it acts as a radial spring. The mathematical model is shown in Fig.1.The dynamic equations of the pole vault motion are derived by using Lagrangian method.

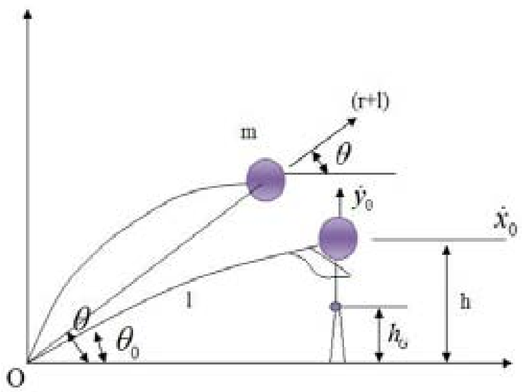

Fig.1: Description of pole vault motion

Denote the following:

$$L' = l + r \tag{1}$$

$$\dot{L}' = \dot{r} \tag{2}$$

$$I_o = m(l + r)^2 = mL'^2 \tag{3}$$

where "l" is the length of the pole, "r" is the radial deflection, "L " is the distance between two end of the pole which is function of "r"; and I_0 is the mass moment of inertia of the athlete about the "box" which the pole rotates. The kinetic (T) and potential (V) energies, the Lagrangian and the Lagrange's equations (L) are written respectively, as:

$$L = T - V \tag{4}$$

$$T = \frac{1}{2} I_o \dot{\theta}^2 + \frac{1}{2} m\dot{r}^2 \tag{5}$$

$$V = mgL' \sin(\theta) + \frac{1}{2} Kr^2 \tag{6}$$

$$L = \frac{1}{2} I_o \dot{\theta}^2 + \frac{1}{2} m\dot{r} - mgL' \sin(\theta) - \frac{1}{2} Kr^2 \tag{7}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i \tag{8}$$

The equations of motion are obtained by using the Lagrangian equations:

$$m\ddot{r} - mL'^2 \dot{\theta}^2 + mg \sin(\theta) + Kr = 0 \tag{9}$$

$$L'\ddot{\theta} + 2\dot{r}\dot{\theta} + g \cos(\theta) = 0 \tag{10}$$

where "*m*" is the mass of the athlete, "*g*" is the acceleration due to gravity, and "*K*" is the radial stiffness of the pole, and also "*θ*" is the pole angle between the pole and earth. The numerical values for these parameters are given in Table I. These values are used in solving the dynamic equations by a numerical method (Runge-Kutta [14]). The real range of this parameter will be presented based on sensitivity analysis of these parameters.

Tab. 1Numerical Values of pole vault motion parameters

Parameter	Value
$\dot{x}_0$	10 (m/s)
$\dot{y}_0$	3 (m/s)
θ_0	30 (Degree)
h_G	0.9 (m)
l	4 (m)
K	300 N/m
h	1.8 (m)

The height achieved during the pole vault motion strongly depends on the initial kinetic energy which is generated by virtue of the initial approach velocity of the athlete. The initial conditions for pole vault motion are shown in Fig. 2

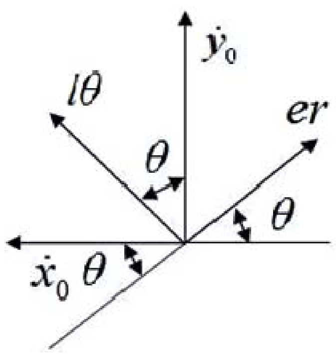

Fig. 2: Vector analysis of the initial point

Based on this figure the equations of the run-up and take-off velocities of the athlete are defined in equations (11) – (13) as:

$$\dot{x}_0 + i\dot{y}_0 = \dot{r}_0\overline{e}_r + (r_0 + l)\dot{\theta}_0\overline{e}_\theta = \dot{r}_0\overline{e}_r + l\dot{\theta}_0\overline{e}_\theta \qquad (11)$$

$$\dot{r}_0 = \dot{y}_0\sin(\theta_0) - \dot{x}_0\cos(\theta_0) \qquad (12)$$

$$l\dot{\theta}_0 = \dot{x}_0\sin(\theta_0) + \dot{y}_0\sin(\theta_0) \qquad (13)$$

where "e_r" and "e_θ" are the unit vectors in the coordinates "r" and "θ", which are defined in the Fig. 2.

3. Fast Fourier Transform

The natural frequency of the pole is calculated from the FFT of the pole deflection. The FFT equation, (14), is used to solve the real and imaginary parts. The square roots of the square summation of these parts are magnitudes; and the frequency related to the first peak of magnitude is the first natural frequency [13].

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N} \qquad K=0,\dots,N-1 \qquad (14)$$

where "N" is the number of points which the FFT is defined based on that. The FFT of the pole deflection shows the natural frequency of the pole, and if half the corresponding natural period becomes equal to ascending time to 90 degrees the stiffness and length becomes optimum.

4. Genetic Algorithm

The Genetic Algorithm is an optimization method for nonlinear systems. This method was introduced by Holland [15] for applications in the Biology, Control, and Artificial Intelligence. The accuracy of the GA method depends on three main objects; fitness function, constraints and population. The fitness function is the objective function which should be minimized. This function should be the function of designed variables of the optimization problem, which are the length and stiffness of the bar. The designed variables also affect the natural frequency of the pole. The fitness function in the pole vault motion, "Equation (15)", is the maximum ascended height regarding the minimum 4 meters as desired variables.

$$H(l,k) = 4 - (r + l)\sin(\theta) \qquad (15)$$

The main constraint is the equality of the half period of the pole oscillation with time to reach 90 degrees, "equation (16)". It means that all spring potential energy of the pole is converted to kinetic energy and this kinetic energy produces gravitational potential energy. Therefore the athlete ascends to the maximum height if the length and stiffness of the pole will result in the mentioned natural frequency. The other constraints are the lower bound and upper bound of the length and stiffness of the pole. The lower bound and upper bound should be selected in feasible region related to allowable length and stiffness "Equation (17)". The population is an effective factor on the number calculation in the GA loop, "The set of design points at the current iteration is called a population" [16]. Increasing the population will increase the accuracy of GA.

$$\frac{\pi}{\omega_n(l,k)} - t_{\frac{\pi}{2}}(l,k) = 0 \qquad (16)$$

$$k_l \le k \le k_u \qquad (17)$$

$$l_l \le l \le l_u \qquad (18)$$

where "l_l" and "l_u" are lower bound, "k_l" and "k_u" are upper bound and "ω_n" is the natural frequency of the pole. The amount of "ω_n" should be calculated by solving differential equation in each loop and getting FFT from pole deflection. Also the time to reach 90 degrees can be calculated by Runge-Kutta and monitoring pole angle during the motion.

5. Results

A parametric study was carried out in order to understand the effect of the parameters on the performance of the pole motion and to optimize the structural parameters (length and stiffness) to get the most efficient pole. The results are presented in two parts, sensitivity analysis of the pole motion and optimization of the pole. In the sensitivity analysis the effect of changing speed of the athlete, pole length and stiffness are studied and the results are presented. The second part of results is assigned to optimize the pole length and stiffness, which are the structural effective parameters on the pole efficiency.

5.1. Sensitivity analysis

In order to study the effect of the initial velocity along the horizontal direction, the run-up speed of the athlete is considered as an unknown parameter, and it is varied to find the threshold velocity that will reach the 90 degrees for a successful attempt. The speed of the athlete ranges from 3.6 (m/s) to 4 (m/s), Fig. 3 and 4 show the effect of this speed range on both pole angle and achieved height.

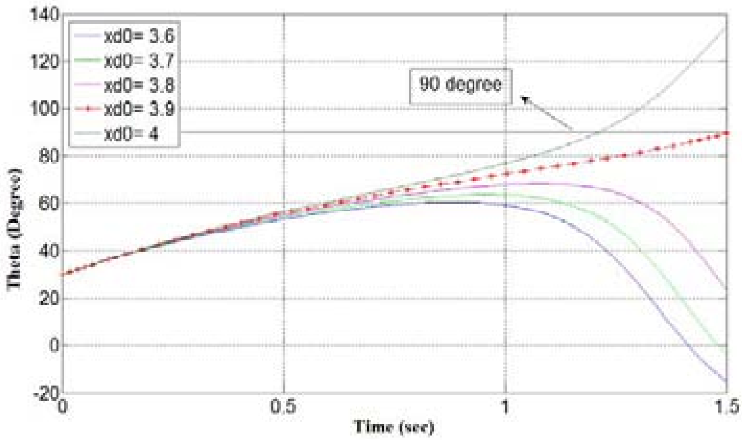

Fig.3: Pole angle for different values of initial speeds

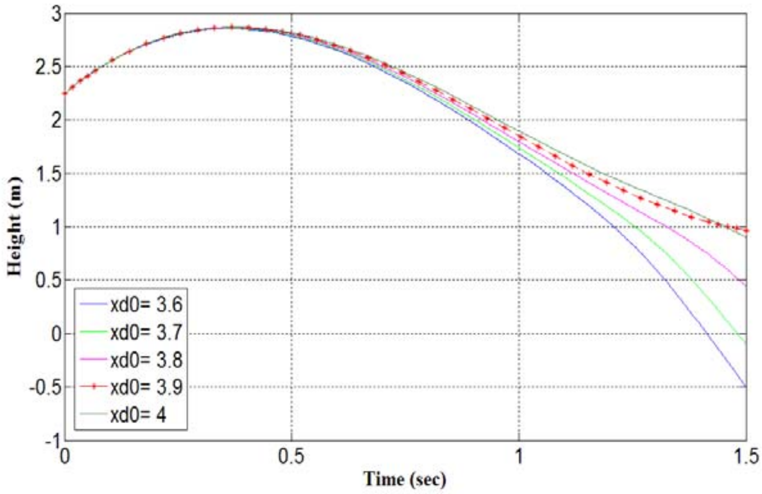

Fig.4: Pole height for different values of initial speeds

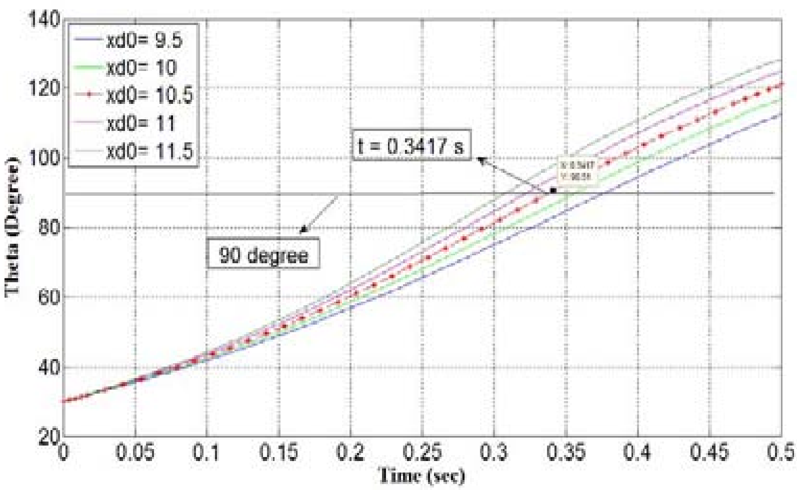

Fig.5: Pole angle f**o**r different values of initial speeds

Fig. 3 and 4 show that if the running speed is less than 3.9 (m/s) the athlete cannot reach 90 degrees. They also show that increasing the velocity has an effect on increasing the height. However, increasing speed in the first moment of the motion produces large deflection in the pole. Consequently after a certain value, the speed of the athlete will have an inverse effect in increasing height. As a result the effect on the increased height in higher speeds for 90 degrees pole angle is shown in Figs. 5 and 6. These figures represent the achieved height that corresponds to changing the velocity from 9.5 (m/s) to 12 (m/s).

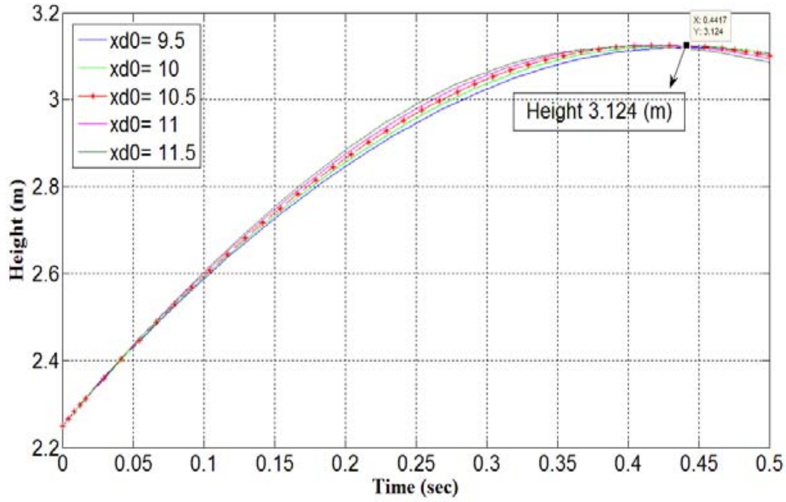

Fig.6: Pole vault ascended for different values of initial speeds

The performance of the pole vault motion strongly depends on converting the kinetic energy to the potential energy. In order to attain the maximum height, all the kinetic energy generated by the athlete through the initial approach velocity must be stored in the spring which in turn must be converted to gravitational potential energy of the athlete. To achieve the best results, the spring potential energy must be converted totally to the kinetic energy and in turn to gravitational potential energy through the 90 degrees angle of the pole. In order to study the performance of the motion, the deflections of the pole during the vaulting for different velocities are shown in Fig. 7.

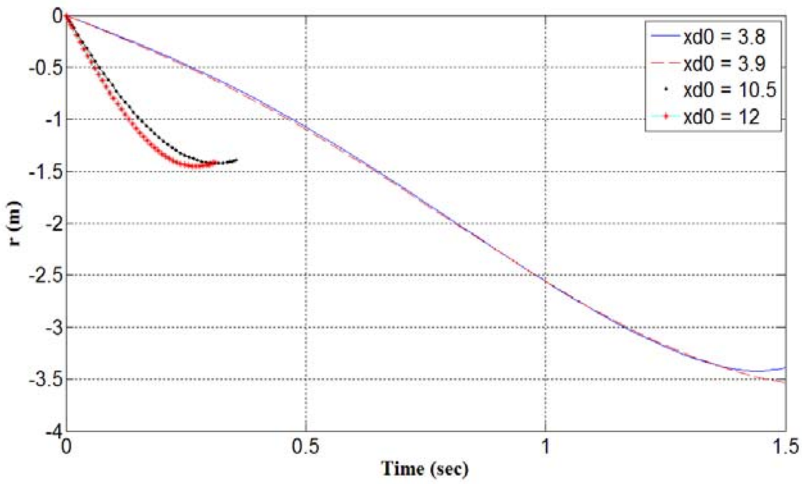

Fig.7: Pole deflection for different values of initial speeds

Fig. 7 shows that when the approach velocity is about 4 (m/s), the pole produces a deflection of about 3.5 (m). But, the pole with 4 (m) length cannot deflect up to 3.5 (m) realistically. Further, the approach velocity of 10 (m/s), is too high for an athlete carrying a pole. Therefore the response of the system for speed around 10 (m/s) is not of practical importance.

The length of the pole influences the natural frequency of the system and the maximum height reached. The effect of the pole length on the pole deflection is studied and the results are presented in Fig. 8. The pole length is varied from 4 (m) to 7.2 (m), and the motion is simulated to show the response of the system up to 90 degrees. The pole angle, the height reached and the pole deflection are shown in Figs. 8-10.

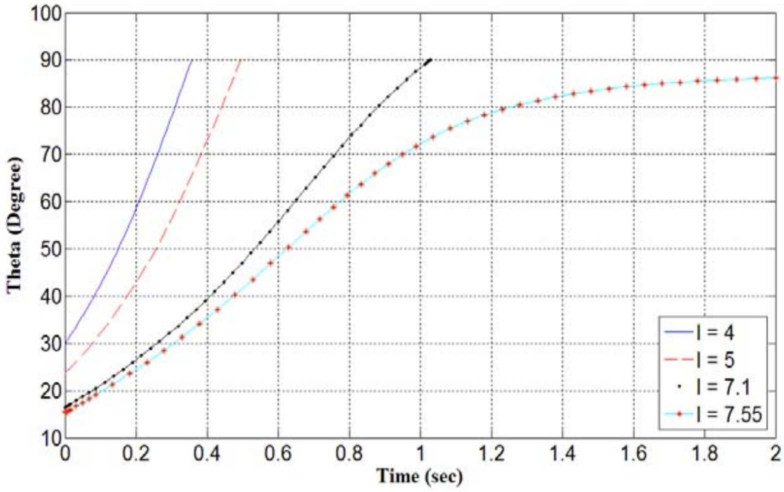

Fig.8: Pole angle for different values of pole lengths

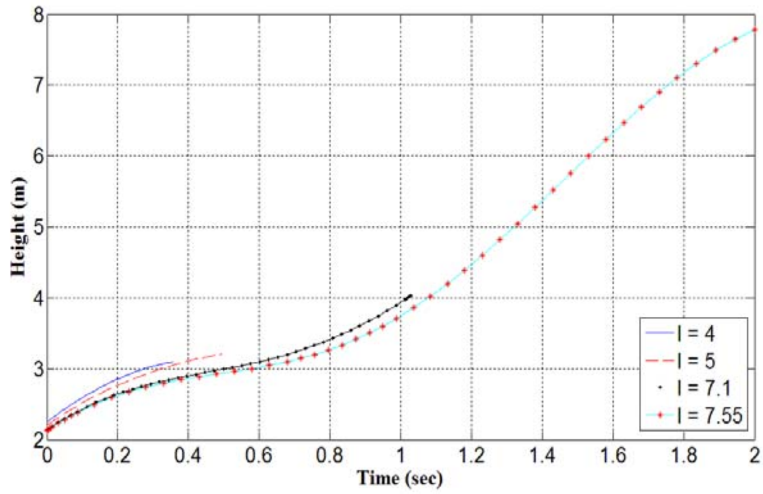

Fig.9: Pole vault ascended for different values of pole lengths

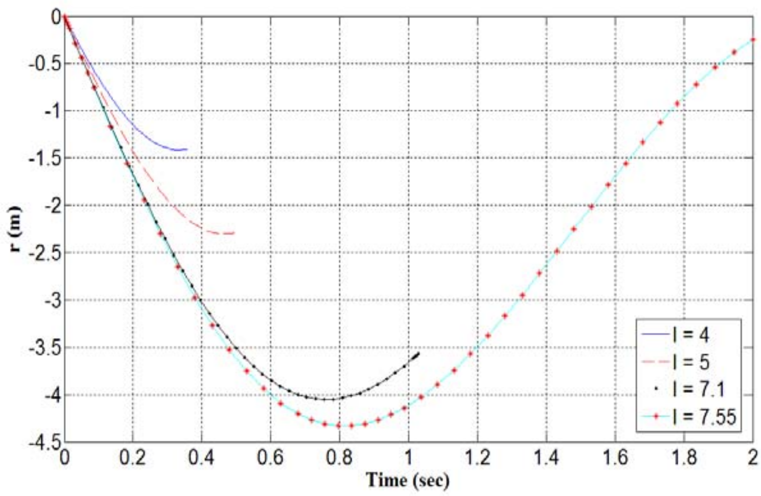

Fig.10: Pole deflection for different values of pole lengths

It is seen from Fig. 8 that the pole length should be less than 7.55 (m) to make the vaulter reach 90 degrees with pole stiffness of 300 (N/m) and the initial velocity in the horizontal direction is10 (m/s). Fig.9 shows that vaulter with pole of length 7.55 (m) can reach heights more than 4 meters and they can achieve up to 7.8 meters. But Fig. 10 shows the pole deflection that corresponds 7.55 (m) length, is about 4.3 (m) (more than 50 % of the length). Therefore the buckling in pole is increased dramatically and the attempt is not successful.

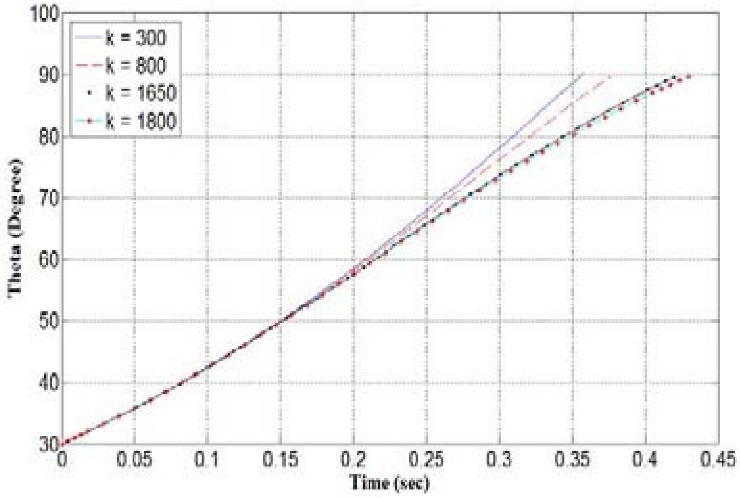

Fig.11: Pole angle for different values of pole stiffness

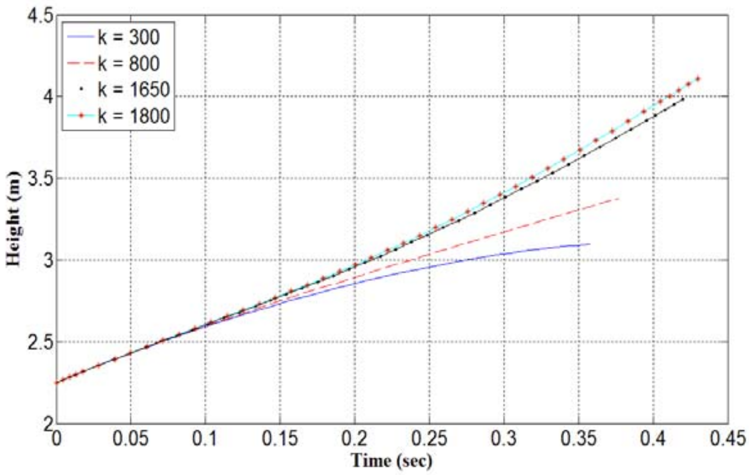

Fig.12: Pole vault ascended height for different values of pole stiffness

When the pole stiffness is increased, the height reached is also increased. In order to study the effect of pole stiffness to reach to 4 (m) height, the stiffness of the pole is varied in the range from 300 (N/m) to 1800 (N/m). Figs. 11-13 show the corresponding pole angle, height and pole deflection.

Figs.11-13 show that increasing the pole stiffness enables the athlete to reach 4 (m) height and pass 90 degrees. Fig. 13 is a proof of the effect of the stiffness in storing energy. It shows that increasing stiffness has direct effect in increasing the stored energy and converting spring potential energy into the kinetic energy of the athlete. The summary of the effect of the initial velocity, length and stiffness in pole vaulting are presented in Table II. This table summarizes the effect of the various parameters on the achieved height and time to reach 90 degrees.

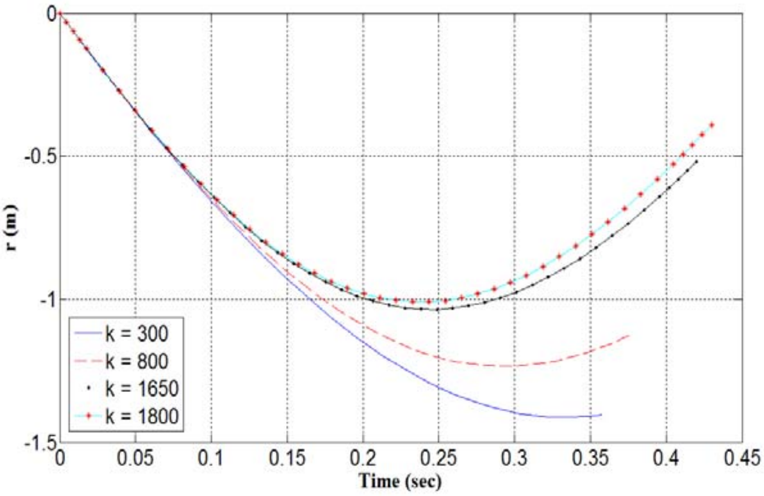

Fig.13: Pole deflection for different values of pole stiffness

Tab.2 Effect of speed, length and stiffness on the pole vaulting

Parameters		Maximum to success	Minimum to Success	Relation to Height	Relation to Time for 90 degrees
Velocity	Variable	10.5 (m/s)	3.9 (m/s)	Direct	Direct
Length	4 (m)				
Stiffness	300 (N/m)				
Velocity	10 (m/s)	7.54 (m)	_	Direct	Inverse
Length	Variable				
Stiffness	300 (N/m)				
Velocity	10 (m/s)	_	1650 (N/m)	Direct	Inverse
Length	4 (m)				
Stiffness	Variable				

5.2. Optimization Based on Frequency Analysis

Tab.3 Parameters and result of GA

Velocity (constant)	8.5 (m/s)		
Length (variable)	4.2 (m/s)	Lower Bound	3.5 (m)
		Upper Bound	4.6 (m)
Stiffness (variable)	1476.7(N/m)	Lower Bound	900 (N/m)
		Upper Bound	1500 (N/m)
Fitness Function Value	4.661 (m)		
Constraint Value	0.0771 (sec)		
Population	40		

Regarding equations (9) and (10) the length and stiffness are effective variables influencing the natural frequency of the system as shown in Fig. 14. But the ratio between these variables has effect on the natural frequency. Consequently infinite number of pairs of the length and stiffness can produce desired natural frequency of the system. In order to find the length and stiffness for pole, the search region should be bounded by lower and upper bounds. Limitations for these variables are selected based on common values for these parameters, which are given in Table III. Another effective parameter is the approach speed of the vaulter, which has a maximum value of 11 (m/s) [17] without carrying the pole. Based on this value and with mass of pole of around 2.25 Kg [18], speed of athlete is selected as 8.5 (m/s). Result of optimization parameters of the pole by identified values and constraint are shown in Table III. The values presented in

Table III are calculated after 20 generations by using Genetic Algorithm and Direct Search Matlab Toolbox.

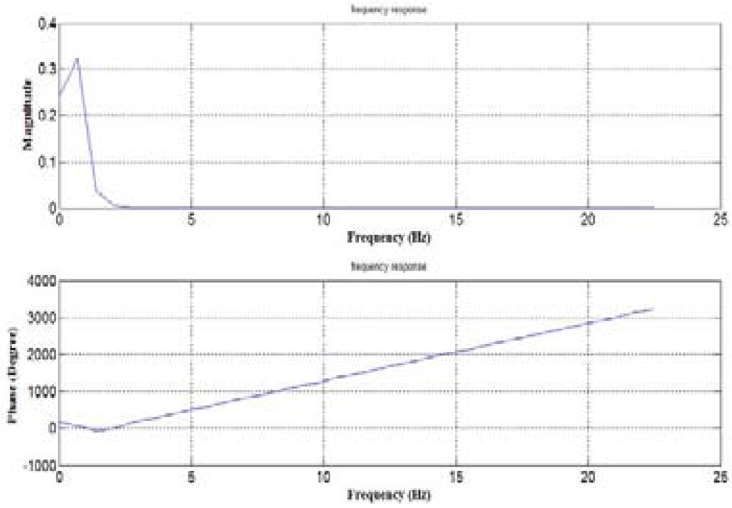

Fig.14: Magnitude and phase of the first natural frequency of system

Figs. 15-19 show the response of the system optimized for the pole stiffness and length. They show that the pole is relapsed totally in the 90 degrees; implying that all the kinetic energy is converted to the gravitational potential energy and equality constraint of GA is satisfied with very low error (11.8 %).Fig. 17 also shows that the pole angular velocity has the minimum value at 90 degrees; also it means that the kinetic energy of the pole caused from the rotation will have the minimum value or gravitational potential energy will have the maximum value. The velocity of the pole deflection (Fig. 18) has maximum value at 90 degrees, which implies that the pole has totally relapsed and all spring potential energy is converted to kinetic energy enabling to increase the height (gravitational potential energy) at this angle. Fig. 19 shows the height of the pole during the motion, and it is approximately equal to the length of the pole adding athlete’s arms. Therefore the fitness function is maximized.

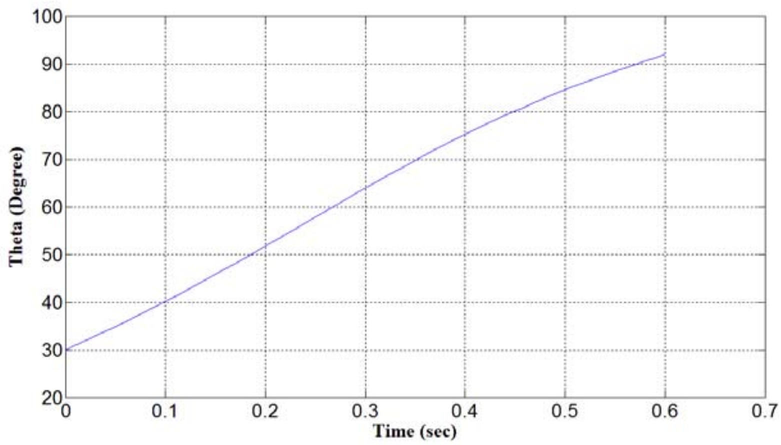

Fig.15: The trajectory of pole optimized pole angle

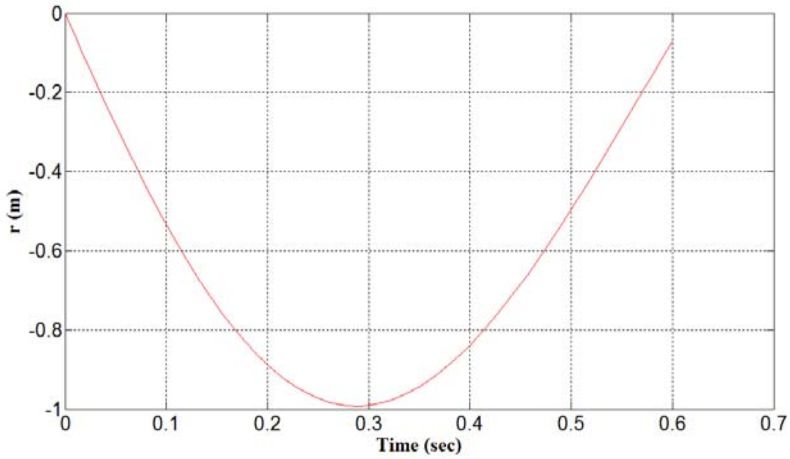

Fig.16: The deflection of the optimized pole

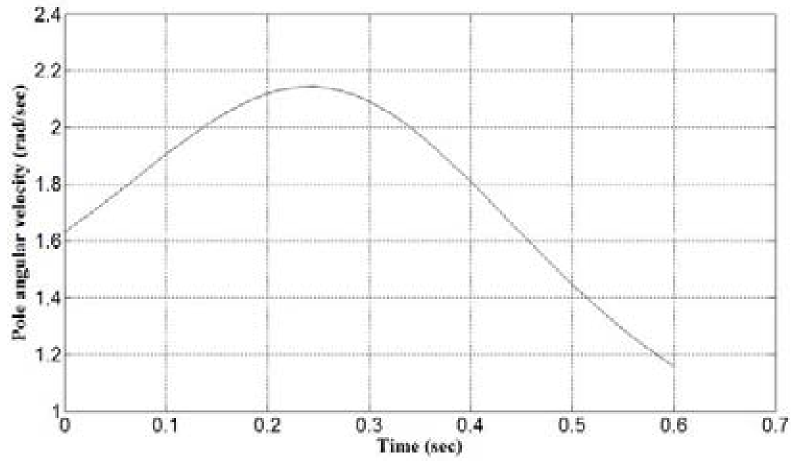

Fig.17: The angular velocity of optimized pole

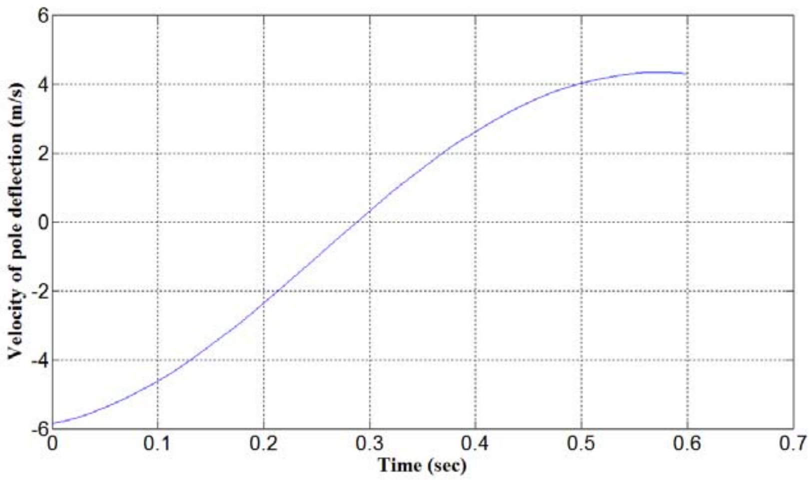

Fig.18: Deflection rate of the optimized pole

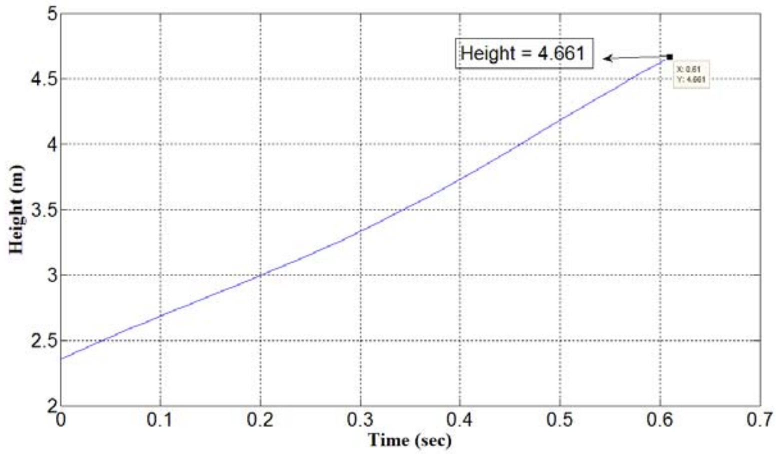

Fig.19: Height achieved based on optimized pole length and stiffness

Figures 6-17 and Table II show that increasing the athlete’s speed, pole length and stiffness enhance the height reached. But increasing length and stiffness decreases the chance to reach 90 degrees and result in a successful attempt. Increasing the speed has a direct effect on reaching 90 degrees and having a successful pole vaulting. The most efficient pole vaulting should convert the kinetic energy of the athlete “by virtue of the initial velocity” to potential energy stored in the spring which is then converted to the kinetic energy of the athlete before finally converting into the gravitational potential energy of the athlete (pole totally relapsed). Based on this study the most important factors to make an efficient pole vaulting are pole length and stiffness. Fig. 17 shows the effect of increasing stiffness in storing energy.

6. Conclusion

The effects of the athlete’s approach speed, pole length and stiffness on the pole vault motion were studied, and margins for these parameters are identified. In this paper the pole length and stiffness are optimized by GA. The objective function of the GA is used to achieve the maximum height over 4 meters. The frequency of the pole deflection is selected as constraint of the optimization algorithm. The natural frequency of the pole is calculated by getting FFT of pole deflection based on nonlinear dynamic equations. The results of optimization show that the GA can optimize the length and stiffness with high efficiency (11.8 % error). By using this method the pole can be optimized for high efficiency in pole vaulting.

7. References

[1] J. Johnson, R. V. Steeg, R. Kring. The History of the Pole Vault. *American Track and Field*. 2008, **10**: 22 - 23.

[2] M. Ekevad, B. Lundberg. Influence of Pole Length and Stiffness on the Energy Conversion in the Pole Vaulting. *Journal of Biomechanics Elsevier Science*. 1997, **33**: 59 - 64.

[3] J. Frere, M. Hermrtte, J. Slawinski and C. T. Chollet. Mechanics of Pole Vaulting: a Review. *Journal of Sports Biomechanics*. 2010, **9**: 123 - 138.

[4] P. M. McGinnis. Dynamic Finite Element Analysis of a Human Implement System in Sport: The Pole Vault. Ph.D. Thesis. University Urbana- Champaign. United States of America. 1984.

[5] M. Hubbard. Dynamics of the Pole Vault. *Journal of Biomechanics.* 1980, **13**: 965 - 976.

[6] S. Ohshima, A. Ohtsuki. Optimization of Torque in Pole Vaulting Using Genetic algorithm (P248). *The Engineering of Sport 7.* M. Estivalet and P. Brisson,Springer Paris. 2008, **2**: 525 - 533.

[7] C. J.Dillman, R. C. Nelson. The Mechanical Energy Transformations of Pole Vaulting with a Fiberglass Pole. *Journal of Biomechanics*. 1968, **1**:175-183.

[8] H. S. Walker, P. G. Kirmser. Biomechanical Parametric Analysis of Pole Vaulting and Optimization of Performance. *Human Body Dynamics: Impact, Occupational, and Athletic Aspects*. Oxford University Press. 1988.

[9] P. M. McGinnis, L. A. Bergman. An Inverse Dynamic Analysis of the Pole Vault. *Journal of Applied Sport Biomechanics*. 1986, **2**:186 - 201.

[10] N. P. Linthorne. Original Investigations Mathematical Model of the Take off Phase in the Pole Vault. *Journal of Applied Biomechanics.* 1994, **10**: 323 - 334.

[11] N. P. Linthorne. Energy Loss in the Pole Vault Take-off and the Advantage of the Flexible Pole. *Journal of Sport*

Engineering. 2000, **3**: 205 - 218.

[12] J. W. Cooley, J. W. Tukey. An Algorithm for the Machine Computation of the Complex Fourier Series. *Mathematics of Computation*. 1965, **19**: 297–301.

[13] K. H. Low. Displacement and Frequency Analyses of Vibratory Systems. *Journal of Computers and Structures*. 1995, **54**: 743–755.

[14] J.C. Butcher. *Numerical Methods for Ordinary Differential Equations.* John Wiley and Sons. 2003.

[15] J. H. Holland. *Adaption in Natural and Artificial Systems*. MIT Press/Bradford Books Edition. 1992.

[16] J. S. Arora. *Introduction to Optimum Design*. 2nd edition. Elsevier Academic Press. 2004.

[17] G. Elert, “Speed of the Fastest Human, Running,” Internet:http://hypertextbook.com/facts/2000/KatarzynaJanuszkiewicz.shtml, [July 11, 2011].

[18] “UCS Pole Vault Crossbar”. Internet: http://www.ucsspirit.com/-vaulting-poles/product detail.cfm/category/Pole-Bags/product/Pole-Vault-Crossbar, [July 11, 2011].