Adaptive Modulation with Moments based Signal-to-Noise Ratio Estimator

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Abstract — **Adaptive modulation techniques in wireless communications are reactive ways designed in communication systems to thrive in unpredictable channel environments. The attractive use of adaptive communications will prove to bring more robustness and flexibility compared to fixed modulation schemes. In order for adaptive modulation to work correctly, it requires an accurate estimation of the channel condition at the receivers' end to make decisions and take action. Channel state information (CSI) has several of other uses in wireless communication systems. Accordingly, a communication link which adapts the degree of modulation scheme according to the estimated signal-to-noise ratio (SNR) values is proposed. The system estimates the current channel condition in the form of CSI and feedback to the transmitter. Hence, the objective of the adaptive system is to stay opportunistic in favourable circumstances while achieving acceptable quality margin in a time-varying communication link. In this paper, the overall system is measured using metrics of spectral efficiency and average bit error rate. Monte Carlo simulations of different signals and channel conditions corroborate our analysis and discussion.**

Keywords — Adaptive modulation, SNR estimation, BER, Communication systems

I. INTRODUCTION

The demands and services for reliable and high speed wireless communications is growing as the number of wireless devices are also growing synonymously. The ability to stay connected wherever we go has become essential if not standard. In that sense, given the variable channel conditions in reality, there will be a requirement to maximize the spectral efficiency (SE) or data rate while minimizing the bit error ratio (BER). The wireless propagating channel places limitations to wireless communication systems due to variation in distance, mobility and the physical environment of the path. For instance, wireless networks demand efficient routing protocols to increase the throughput for better network efficiency [1]. There are various methods to maintain efficient spectrum usage as it is considered limited resource in this age of information. An example is adaptive channel assignment techniques using evolutionary algorithms where the problem of channel assignment efficiency is maximized [2]. However, this paper focuses on maximizing a singular channel. Adaptive modulation and coding (AMC) allows the communication system to adjust the data rate, transmit power, instantaneous BER, and channel code rate to overcome the noise and fading effects [3, 4]. Different modulation schemes cater to different channel gains. Also another challenging problem is to find the balance between which parameters to adapt for the overall best performance. These adaptive techniques are advantageous for next generation wireless systems because they provide optimum solutions to efficiently use the resources.

In wireless communications, channel state information (CSI), crucial to adaptive modulation and various wireless applications, is derived from the receiver by estimating the instantaneous signal-to-noise ratio (SNR). The transmitter then can use the CSI to decide what modulation parameters to employ, unlike fixed traditional systems designed to handle worst case conditions [5]. Most literature does not elaborate how the transmission scheme will utilize the feedback link. Most assumes a special case of non-ideal or ideal channel estimation information plus a feedback path with or without delay. The effects of non-ideal estimation and feedback delay are analyzed in [6, 7, 8]. An overall impact of SNR estimation error on adaptive modulation is written in [8]. The justification of using effective coding and uninterrupted reverse channel allows most literature refers to a perfect case of feedback without error or delay. SNR estimation provides valuable assistance in modern wireless communication systems. Other applications of SNR information include: information war, threat analysis, and electronic surveillance system. The degree of how frequent fading in a channel can vary will greatly affect the CSI integrity. It is hard if the channel is changing too fast until the CSI acquisition information becomes outdated.

Fundamentally, two SNR estimators, Data-Aided (DA) and Non-data Aided (NDA) form the divide of estimator types. DA estimators obtain knowledge of the transmitter sequence by receiving pilot symbols periodically. A known symbol will be inserted with the signal, once at the receiver it will gauge how far the extent of distortion on the signal [9]. Comparatively, NDA estimators do not require such knowledge. Estimators are separated by I/Q and envelope based estimators, I/Q estimators processes in-phase and quadrature components of the received signal, thus synchronization required. In comparison, envelope-based estimators only focus on magnitude of the received signal, thus can be applied even if the communication link is not coherent. The maximum likelihood (ML) SNR estimator provides good statistical performance but it comes with higher computational complexity compared to envelop-based estimators [7]. Reference [10] provides insight to various estimators under normalized simulation environments for fair comparisons. In summary, the authors concluded that the best estimator actually depends on what context the application is in. The implementation of an envelope-based estimator is less complex. In this paper, a moments based envelop SNR estimator is paired with adaptive modulation to form an adaptive communication link.

The consequent parts of the paper are planned as follows. Section II provides a literature study of the topic in focus. Section III describes the system model and SNR estimation method. Section IV covers the adaptive modulation scheme with SNR estimator and section V discusses results of the adaptive modulation scheme. Finally, section VI makes up the conclusion.

II. ADAPTIVE MODULATION WITH PERFECT CHANNEL STATE INFORMATION

The extensive literature of adaptive communication systems extend from as far back in the late 1960's [11]. After that, due to lack of technical depth in estimation methods and practical constraints in hardware advancements, the new found interest started to gain momentum approximately two decades thereafter. Goldsmith discussed some results on AMC with fading channels, utilizing trellis and turbo-coded MQAM modulation for flat-fading [12]. Goldsmith and Varaiya, showed that Shannon capacity of the communication link can be achieved by adapting the rate and power of transmitter in their work [13]. This was further developed to [5], where a practical variable-rate variable-power adaptive modulation communication is derived and analysed. The adaptive system's improvement in spectral efficiency is fulfilled while satisfying BER and power boundaries. It is also shown that the technique has up to 10 dB gain over constant rate variable power modulation whereas an even bigger 20 dB gain over non-adaptive modulation.

Following the discoveries of adaptive modulation scheme's performance gain, a proposal to implement AMC in cellular wireless standards was given in [14], specifically in the CDMA2000 standard. Furthermore, AMC is also imparted in WCDMA high speed downlink packet access (HSDPA), IEEE 802.11x family standards and also WiMax standards for wireless internet access.

III. MOMENTS-BASED SNR ESTIMATION

This section introduces a novel envelope-based NDA estimator from [15]. This estimator applies to constant modulus (CM) modulations and non-constant modulus cases. In this paper, only CM modulation schemes are considered. The authors in [13] showed that by adapting the rate and power of the transmitter, the transmission link is able to achieve close to the Shannon capacity.

A. System Model

The system model and related representations are described henceforth. In this literature, complex signals are dealt with discrete sampling. Practical limitations for AMC system prior designing requires a feasible feedback path and a channel with slow varying conditions because the feedback must be able to keep up. This feedback frequency can be adjusted so that the link will always have updated information. However, with too frequent signaling between receiver and the transmitter can also mean inefficient use of bandwidth. The system model is illustrated in Fig. 1.

Assume a discrete channel model, so that the sampled symbol at the receiver's matched filter output given by

$$r_k = \sqrt{g_k}\, x_k + n_k\,,\ k = 1, \ldots, N \tag{1}$$

Where x_k represents the transmitted complex symbols, $\sqrt{g_k}$ is stationary and ergodic channel gain, and n_k are independent and identically distributed Gaussian noise with unknown variance $2\sigma^2$. With samples $r_0, r_1, \cdots, r_N - 1$ the system is predicted to estimate the SNR value. This paper's envelop-based SNR estimator relies on $|r_k|$.

B. Second- and Fourth-Order Moments (M_2M_4) NDA Estimator

Adaptive modulation requires CSI which can be derived at the receiver by evaluating pilot symbols embedded in the transmitted signal. These pilot symbols are known at both ends. While pilot symbols provide a straight forward way to conduct channel acquisition, they consume transmit power and bandwidth, which in turn reduces spectral efficiency of the overall system. Hence, we propose a NDA estimator which is responsible to inform transmitter of the channel state.

The symbols are drawn from a finite constellation which is known to the receiver and has *I* different amplitudes, having i^{th} amplitude A_i and probability P_i ($i = 1, ..., I$). The constellation p^{th} moment is denoted by

$$M_p = E\{|x_k|^p\} = \sum_{i=1}^{I} P_i A_i^p \tag{2}$$

It is mentioned in [15], sample moment of the envelope are measured by their respective time averages as the following function with K_{sym} as the total number of received symbols:

$$M_p \approx \frac{1}{K_{sym}} \sum_{k=1}^{K_{sym}} |r_k|^p \tag{3}$$

from which is an estimate if the SNR, $\rho = S/N$. The equation of true moment of the envelope is given by [14]

$$M_p \approx (2\sigma^2)^{\frac{p}{2}} \sum_{i=1}^{I} P_i \Gamma\left(\frac{p}{2}+1\right) e^{-\rho A_i^2} {}_1F_1\left(\frac{p}{2}+1;1;\rho A_i^2\right) \quad (4)$$

Where ${}_1F_1(\bullet\,;\bullet\,;\bullet)$ is the confluent hypergeometric function, and $\Gamma(\bullet)$ is the gamma function. From (4), it can be observed that ρ and σ are related to the moment. Since, the moment-based estimator uses at least two different moments. Suppose $k \neq l$, let functions of ρ be:

$$f_{k,l}(\rho) := \frac{M_k^l(\sigma^2,\rho)}{M_l^k(\sigma^2,\rho)} \quad (5)$$

Note that (5) depends on A_i, ρ, and p_i but not on standard deviation, σ. Then the moments-based SNR, ρ estimator is expressed as

$$\hat{\rho}_{k,l}(\rho) := f_{k,l}^{-1} \frac{M_k^l}{M_l^k} \quad (6)$$

In [15], to overcome the tractability of the inverse function f, a look-up table was proposed to easily compute the estimation formula. However, in this paper, values of $l = 2$ and $l = 4$ are used as we are only dealing with second and fourth moments. Thus, its inverse is possible and resulting in closed form solution given by:

$$\hat{\rho}_{2,4} := \frac{1-2\frac{M_2^2}{M_4} - \sqrt{(2-a)\left(\frac{2M_2^4}{M_4^2}-\frac{M_2^2}{M_4}\right)}}{a\frac{M_2^2}{M_4}-1} \quad (7)$$

where, $a = \sum_{i=1}^{I} p_i A_i^4$. For PSK constellations, (7) is equivalent to the M_2M_4 SNR estimator for complex signals in [10], whereby,

$$\hat{\rho}_{2,4} := \frac{\sqrt{2M_2^2 - M_4}}{M_2 - \sqrt{2M_2^2 - M_4}} \quad (8)$$

I. ADAPTIVE MODULATION WITH SNR ESTIMATION

A. *Adaptive M-Psk Modulation*

In this proposed model shown in Fig. 1, there is a delay- and error-free feedback path to the transmitter. This feedback is responsible for returning CSI of SNR estimates. Let γ_κ denote the instantaneous SNR received at time k. The received signal is also assumed to have ideal coherent phase. Since g_k is stationary, then it is also independent on time k, we denote this distribution $p(\gamma)$.

The receiver adapts the modulation constellation according to SNR received from the feedback. Given a finite set of constellations available, $\{\gamma_i\}_{i=0}^{N-1}$ defines the range of γ where the constellations are associated. One constellation is assigned to an instantaneous SNR region of $[\gamma_i, \gamma_{i+1})(0 \le i \le N-1)$, but when an SNR value drops below γ_0, the communication link stops transmitting. γ_0 is called the cut-off SNR. This means that if the channel reaches an extensive degrade in quality, the channel should not be used. In this paper, – 6 dB is the cut-off value as it is decided by the performance if the SNR estimator. This paper considers only $0 \le i \le 2$, where $M_i = \{4,8,16\}$. .

B. *Channel Capacity*

Firstly, the channel capacity of a wireless transmission over AWGN channel is described by the equivocation theory,

$$C = \max\, f_R(r)\{H(R)\} - H(N) \quad (9)$$

where H(N) is the entropy of additive white Gaussian noise and entropy of the output H(R) is:

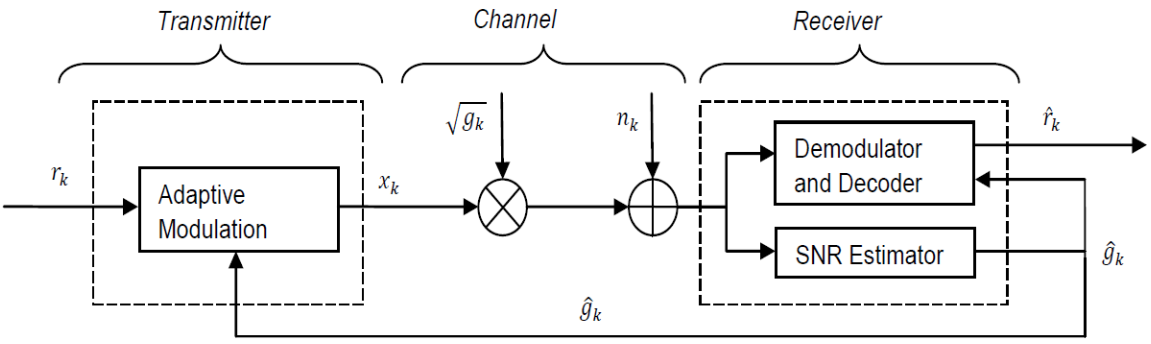

Figure 1. System model with feedback.

$$H(R) = -\int_{-\infty}^{\infty} f_R(r)\log_2 f_R(r)dy \qquad (10)$$

and the entropy if the white Gaussian noise if given by:

$$H(N) = \frac{1}{2}\log_2(\pi e N_0) \qquad (11)$$

Depending on the channel, we describe our channel capacity in the case of discrete time AWGN with finite modulation constellations by combining equations (9, 10, 11):

$$C = -\int_{-\infty}^{\infty} f_R(r)\log_2 f_R(r)dy - \frac{1}{2}\log_2(\pi e N_0) \qquad (12)$$

The probability of the output function is given by:

$$f_R(r) = \frac{1}{|M|}\sum_x \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-\left(\sqrt{g}x+n\right)^2}{2\sigma^2}} \qquad (13)$$

During transmission, besides factors like transmit power and coding, for a finite alphabet input *M* there is a limit to how many bits can be transferred at given time. It can be concluded immediately that:

$$C = \log_2 M \text{ for } \frac{S}{N} \to \infty \qquad (14)$$

C. Assumptions and Performance Measure

In this paper, Spectral efficiency (SE) and average BER is evaluated. SE, denoted as R/B, equals the average data rate over unit bandwidth. At point in time, $k(\gamma) = \log_2[M]$ bits/symbol is sent across the channel where M-ary PSK is used, this also equals the spectral efficiency for a fixed *M*. In typical cases, spectral efficiency is influenced by BER and SNR. The Shannon-Hartley equation (15), describes just that and BER is also a factor because it relies on the theories of mutual information and entropy, which are not discussed in this general formula. *C* is the channel capacity in unit bps.

$$\frac{C}{B} = \log_2(1 + \frac{S}{N}) \qquad (15)$$

For a discrete number of constellations the spectral efficiency is given by:

$$\frac{R}{B} = \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_i+1} p(\gamma)d\gamma \text{ bps/Hz} \qquad (16)$$

For BER it is assumed as:

$$\overline{\text{BER}} = \frac{E[\text{number of error bits per transmission}]}{E[\text{number of bits per transmission}]} \qquad (17)$$

The Symbol Error Rate (SER) expression for M-psk in AWGN channel with grey mapping is described in [16]:

$$\text{SER}_{\text{M-psk}}(\gamma) = 2Q(\sqrt{2\gamma}\sin(\frac{\pi}{M})) \qquad (18)$$

Due to grey mapping, two adjacent symbols differ only in a single bit. Therefore, the most probable case of selecting the neighbouring symbol in the event of noise distortion k-bit symbol can only have 1-bit error. So the approximation of $\text{BER}_{\text{M-Psk}}$ is:

$$\text{BER}_{\text{M-psk}} \approx \frac{1}{k} erfc(\sqrt{\gamma}\sin(\frac{\pi}{M})) \qquad (19)$$

However, this expression is not easy to work upon with its arguments. In that case, we resort to approximation of another simpler BER expression using curve fitting while keeping it in tight bounds. In [3], an approximation of BER for M-Psk constellations that is valid for k(γ) ≥ 2 within 1.5dB of error for BER ≤ 10^{-3} is adapted as:

$$\text{BER}_{\text{M-psk}} \le 0.05 e^{\left(\frac{-6\gamma}{2^{2M}-1}\right)} \qquad (20)$$

Rearranging (20), we obtain an expression to maintain the BER while producing the possible maximum constellation size:

$$\text{M}(\gamma) = \sqrt{\frac{-6\gamma}{\ln(20\text{BER})} + 1} \qquad (21)$$

The spectral efficiency that is bounded by (14) is hence obtained by substituting (21) into the expression $R/B = \log_2 M$, resulting in the expression:

$$\frac{R}{B} = \log_2\left(\sqrt{\frac{-6\gamma}{\ln(20\text{BER})} + 1}\right) \qquad (22)$$

II. RESULTS AND ANALYSIS

The M_2M_4 moments NDA SNR estimator performs very well from 5 dB onwards for BPSK, 8-PSK, 16-PSK and 32-PSK. The estimator's performance is mainly acceptable. However, estimation values below 5 dB exhibit a minimal level of standard deviation from the exact valuein Fig. 2. In Fig. 3, at –10 dB the estimators show an error of approximately 4 dB from the exact value. From that point onwards, its performance gradually improves with better SNR values. It is also noted that the M_2M_4 does not discriminate between all types of M-psk modulation. The performance is about the same for all where they perform

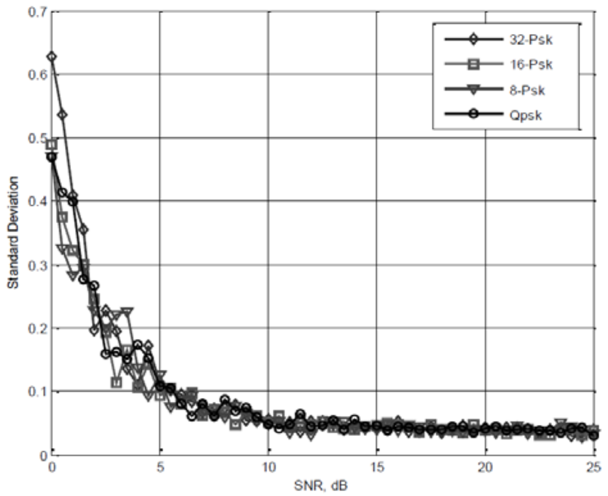

Figure 2. Standard deviation of SNR estimator for various modulations.

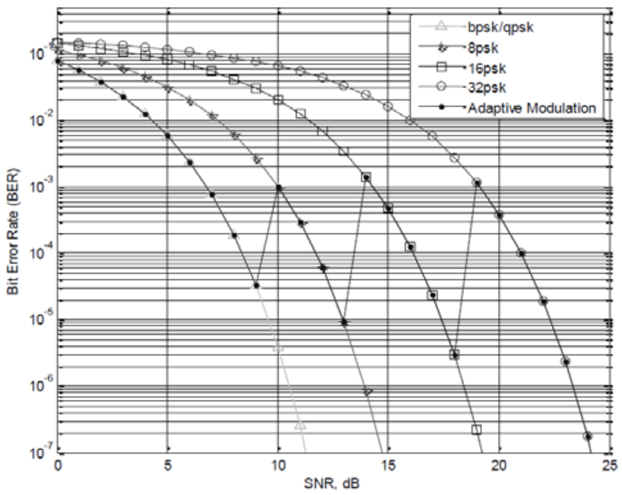

Figure 4. BER performance of adaptive M-PSK modulation.

poorly in low SNR conditions and then from –5 dB SNR onwards shows good accuracy. In real world implementation, the error of the estimator in low SNR will affect adaptive M-psk modulation which in reality the channel conditions are bad, but the estimator gives a better SNR feedback value to the transmitter. In the end, that can lead to a wrong decision hence contribute to a higher BER.

The simulation on adaptive M-PSK modulation is conducted based on a quality of service (QoS) constraint, which is the BER. In the simulations, a BER of 10^{-3} is set to demonstrate how the link changes according to feedback. Comparing the BER performance between the fixed M-PSK modulation methods with adaptive modulation in Fig. 4, we see that in low SNR values (6 – 9 dB), adaptive modulation has a better BER performance compared to the other higher order of modulations. For adaptive modulation, there is advantage in having low error rate at low SNR conditions and also higher bit rate performance at favorable channel conditions. This ensures that reliable communication is maximized no matter in good or bad channel conditions.

Besides having the best performance compared to other types of estimators as shown in [10], the complexity of the M_2M_4 estimator is a factor to consider. Advantageously, simulation implementations for this estimator are not complex, therefore it computes rapidly even over long transmission duration of symbols.

In Fig. 5, the SE at (22) is evaluated with BER constraints of 10^{-3} and 10^{-6} in comparison with Shannon capacity. Both BER limited curves of adaptive M-PSK modulation are still below the Shannon limit curve which means it is still inside the practical region. As the curves strive to reach closer to the Shannon limit, it requires effective coding techniques and spectrum efficient efforts.

The channel capacity between adaptive modulation and M-PSK modulation methods are compared. It can be observed in Fig. 6 that adaptive modulation reaches its maximum capacity (14) as SNR increases. Besides, its

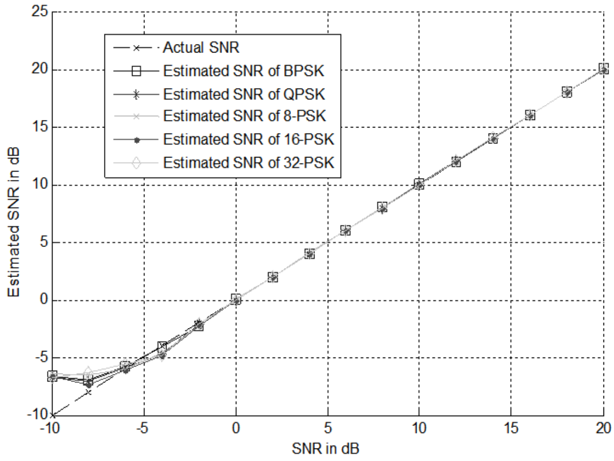

Figure 3. Plot of actual SNR vs estimated SNR

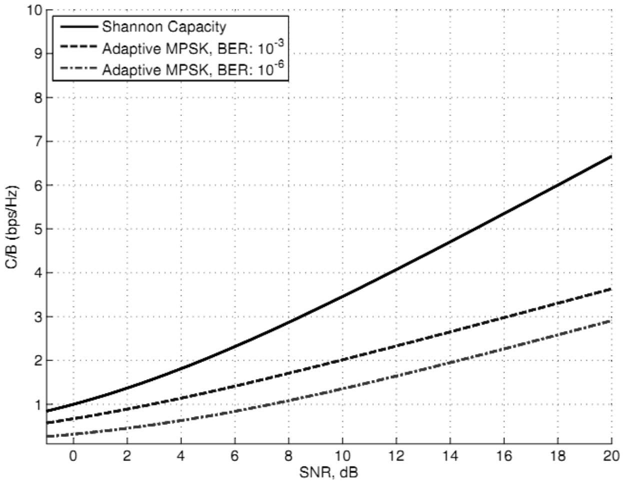

Figure 5. Spectral efficiency of adaptive M-PSK modulation in AWGN.

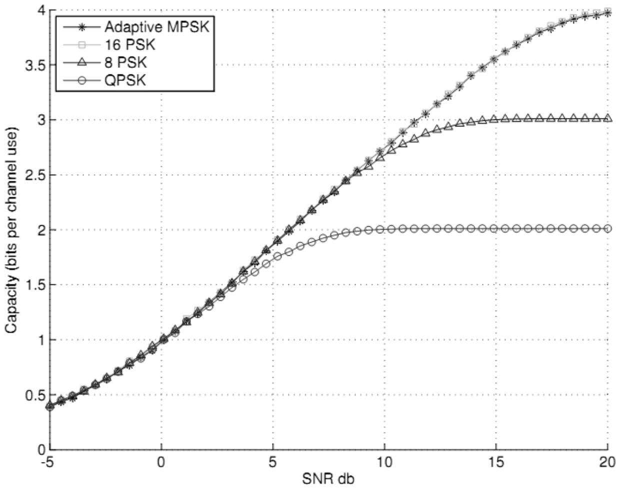

Figure 6. Channel capacity comparison with fixed bandwidth.

adaptive M-psk modulation used in this paper with finite alphabet only goes to a maximum cardinality of 16 at favorable conditions. However, do note that the BER performance of adaptive modulation is better at lower SNR values due to its adaptive nature. Gradually as SNR improves, the rate will tune higher. So, at higher SNR situations it performs close to a fixed 16 PSK modulation scheme.

To examine the effect of adaptive modulation on various parameters, further analysis is conducted on top of the findings in [17]. The average throughput, average spectral efficiency and overall BER were simulated using parameters of 65 MHz System Bandwidth, BER Threshold of 10^{-3} for adaptive modulation and common flat fading channel with average velocity of 3m/s. The results obtained are tabulated in Table I. The fixed modulation schemes performed as they should. With higher modulation levels, the BER suffered highest being 32-PSK. However, note that the average throughput of 32-PSK is highest among all four PSK schemes. As for Adaptive modulation with QoS constraint of 10^{-3}, the average throughput is the second highest. On top of that, the overall BER performance is better compared to Fixed 32-PSK.

TABLE I. COMPARISON RESULTS BETWEEN FIXED AND ADAPTIVE M-PSK MODULATION SCHEME

	Performance Metrics		
	Average Spectral Efficiency(b/s/Hz)	***Average Throughput (Mbps)***	***Overall BER***
32-PSK	5.000	325.0	15571.0e-7
16-PSK	4.000	260.0	166.58e-7
8-PSK	3.000	195.0	27.019e-7
QPSK	2.000	130.0	0.70591e-7
Adaptive M-PSK	4.995	324.7	289.53e-7

III. CONCLUSION

In this paper, second- and fourth-order moments based SNR estimator proves to be reliable, simple and accurate. Envelope-based SNR estimators are less complex yet produce excellent performance for M-psk constellations. This qualifies the estimator to complement the adaptive system described. We have shown that for a certain range of low SNR environment, the BER performance is much better in adaptive modulation compared to fixed modulation schemes. We also compared the spectral efficiency of said system with the theoretical bound.

However, there are many constraints to take account before considering it as a practical implementation. This paper provides analytical approach towards adaptive modulation's capabilities in the context of measuring overall performance with valid assumptions.

For future work, the adaptation parameters could include coding for better resilience against fading channels as forward error correction (FEC) coding has proven effective in noisy channels. Using different coding rates can introduce more types of transmission strategies, hence more suitable approach to different CSI. Furthermore, the balance between using FEC which consumes higher forward bandwidth against the spectral advantage of AMC is an interesting topic to venture.

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