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Comparisons of Simulation Methods for Motions of a Moored Body in Waves

Based on the linear water wave theory, numerical simulations are carried out for motions in waves of a body moored by a nonlinear-type mooring system. Numerical results obtained by using the equation of motion described in the time domain with a convolution integral (C.I. method) are compared with those of the second-order linear differential equation with constant coefficients (C. C. method). These results are also compared with experimental values measured from the initial stage when the action of exciting forces starts and the validity of C.I. method is discussed.

Introduction

Since the introduction of the principle of superposition into the naval architecture by St. Denis and Pierson [1], the responses of a ship and/or offshore structure to the sea are usually described in terms of the frequency response functions. In the linear dynamic system, the responses of a ship to irregular waves can be represented by a linear summation of its responses to the components of the irregular waves.

However, in the nonlinear dynamic system, such as a ship moored by a nonlinear mooring system, the responses cannot be expressed in the frequency domain since the linear superposition principle is not applicable in this case. The solutions are obtained by the numerical integration of the equations of motion with respect to time and the equations of motion to be used are usually classified into the following two types. The one is the equation of motion with a convolution integral (C.I. method) and the other is the second-order linear differential equation with constant coefficients (C.C. method). The C.C. method is not exact since the frequency of a motion cannot be determined, a priori, when the nonlinear terms are involved in the equation. However it has an advantage with respect to the computing time compared with C.I. method. From the practical point of view, Shuku, et al. [2] studied the motions of a moored floating storage barge in shallow water by using C.C. method. The constant coefficients were assumed to be the hydrodynamic coefficients at some representative frequency associated with the incident waves.

On the moored ship problems Oortmerssen [3] derived numerical results based on C.I. method and compared them with experimental values. Following his work Hotta [4] and Wichers [5] have dealt with C.I. method and compared the numerical results with the ones of C.C. method and/or experiments. However, most of these works have been concerned with the responses at stationary state and in the work of Oortmerssen a factor (1.0-exp(0.01t)) was introduced to avoid the shock force due to the wave-exciting force in a transient stage.

The purpose of this paper therefore is to compare the two calculation methods for the motions of a moored body in waves from the initial stage when the action of exciting forces starts. For comparisons with measured values, the viscous damping force on a body is estimated from the free oscillation test and introduced in both equations of motions. In this paper, for simplicity, the sway motion of a two-dimensional body moored by a nonlinear-type mooring system is assumed and a significant difference between two calculation methods is found in the following cases.

- When the sway motions close to the natural frequency of a moored body occur.
- When the floating body which is initially at rest with a loosened rope and/or chain starts its motion (i.e., in a transient stage under the action of wave forces).
- When the subharmonic sway motions appear.

Integro-Differential Equation of Motion

Impulse Response and Memory Effect Function. In order to formulate the problem for an impulse response, consider a two-dimensional body floating at rest in still water and the coordinate system illustrated in Fig. 1. Let us then consider the general formulation of hydrodynamic forces acting on the body due to an impulsive motion velocity in the kth mode. Here $k=1$, 2 and 3 are referred to sway x_1, heave x_2, and roll motion x_3, respectively. Under these assumptions, we obtain the following governing equation and boundary conditions for the velocity potential $\Phi_k(x,y;t)$:

$$[L]\,\nabla^2\Phi_k=0 \quad \text{for } y>0 \tag{1}$$

$$[F]\,\Phi_{ktt}-g\Phi_{ky}=0 \quad \text{on } y=0 \tag{2}$$

$$[H]\,\frac{\partial\Phi_k}{\partial n}=n_k\delta(t) \quad \text{on the body} \tag{3}$$

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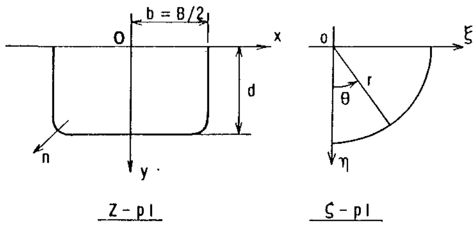

Fig. 1 Coordinate system for Lewis form transformation

$$[I]\Phi_k = \Phi_{kn} = 0 \quad \text{for } t<0 \tag{4}$$

where

$$n_1 = \frac{\partial x}{\partial n},\ n_2 = \frac{\partial y}{\partial n},\ n_3 = x\frac{\partial y}{\partial n} - y\frac{\partial x}{\partial n}$$

and $\delta(t)$ is the usual Dirac delta function.

Cummins [6] firstly introduced ship motion description by a succession of small impulsive displacements. Adachi [7], Yeung [8] and Ikebuchi [9] solved the time-dependent boundary value problem by a direct approach in which the time-varying Green's functions were involved. On the contrary, the time-dependent boundary value problem can be converted to the problem in the frequency domain and the problem will be derived in a more simplified manner than the one of a direct approach. Ogilvie [10] reviewed the method of Cummins by introducing the memory effect functions and discussed the relation between time and frequency domain description of motions. Oortmerssen applied the time domain equation of motion to a moored ship in waves in which the time-dependent boundary value problem was converted into the frequency-dependent boundary value problem as was shown by Ogilvie.

Following Oortmerssen, we also describe the problem in the frequency domain and for this purpose we define the Fourier transforms as follows:

$$F^*(\omega) = \int_{-\infty}^{\infty} dt\ \exp(-i\omega t)F(t)$$

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \exp(i\omega t)F^*(\omega) \tag{5}$$

By taking the Fourier transforms of equations (1)–(4), we have

$$[L]\nabla^2\Phi_k^* = 0 \quad \text{for } y>0 \tag{6}$$

$$[F]K\Phi_k^* + \Phi_{ky}^* = 0 \quad \text{on } y=0 \tag{7}$$

where

$K = \omega^2/g$

$$[H]\ \frac{\partial \Phi_k^*}{\partial n} = n_k \quad \text{on the body} \tag{8}$$

This is the well-known boundary value problem described in the frequency domain and the problem is solved by taking into account the radiation condition in addition to the foregoing governing equations. Let us denote the force component of jth direction due to the motion of kth mode by $F_{kj}(t)$; then we have

$$F_{kj}(t) = -\int_c p_k n_j dc = \rho\int_c \frac{\partial \Phi_k}{\partial t} n_j dc \tag{9}$$

And by using the hydrodynamic coefficients in the frequency domain, we have

$$F_{kj}^*(\omega) = \rho\int_c n_j dc i\omega\Phi_k^* = \{-b_{kj}(\omega) - i\omega\ a_{kj}(\omega)\} \tag{10}$$

where

$a_{kj}(\omega)$ = added-mass coefficient

$b_{kj}(\omega)$ = damping coefficient

Hence, by use of Fourier inversion formula and equation (10), equation (9) can be rewritten as

$$F_{kj}(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \exp(i\omega t)\{b_{kj}(\omega) + i\omega\ a_{kj}(\omega)\} \tag{11}$$

Since $a_{kj}(\infty)$ has generally a finite constant value, the integral of equation (11) does not converge. In order to have a convergent integral we rewrite equation (11) as

$$F_{kj}(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \exp(i\omega t)[b_{kj}(\omega) + i\omega\{a_{kj}(\omega) - a_{kj}(\infty)\}] - \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \exp(i\omega t)a_{kj}(\infty)i\omega \tag{12}$$

Applying the following relation [11]:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \exp(i\omega t) = \delta(t) \tag{13}$$

Nomenclature

- a_1, a_3 = Lewis form parameters
- a_{kj} = added-mass coefficient of jth mode due to motion in kth mode
- $\bar{A}_1$ = ratio of amplitudes of generated waves to those of swaying motions
- b = $B/2$ half-breadth of cylinder
- b_{kj} = damping coefficient in jth mode due to motion in kth mode
- c = body contour
- $C_{D_1}^*$ = viscous damping coefficient
- $C_{M_1}^*$ = mechanical damping coefficient due to pulley
- f_{11} = restoring force resulting from mooring system
- F_{kj} = force component of jth mode due to motion in the kth mode
- F^* = Fourier transform of F
- $F_j^{(k)}$ = force component of jth mode when motion velocity $\dot{x}_k(t)$ is given
- F_{1w} = $F_{1w1} + F_{1w2}$ wave-induced external forces which consist of wave-exciting force F_{1w1} and drifting force F_{1w2}
- g = acceleration of gravity
- G = Green's function
- K = ω^2/g wave number
- K_{kj} = memory effect function in jth mode due to motion in kth mode
- K_{kj} = nondimensional K_{kj}
- K_{kj}^* = Fourier transform of K_{kj}
- M_1 = mass of body
- n = normal vector pointing outside body
- n_j = generalized direction cosine
- p_k = hydrodynamic pressure in kth mode
- R_{ij} = steady-state drifting force coefficients
- t = time
- t' = $t\sqrt{g/b}$ nondimensional time
- x_k = motion in kth mode
- δ = Dirac delta function
- ϵ_1 = phase difference between generated waves and swaying motions
- ζ = surface elevation of incident waves
- ρ = density of water
- Φ_k = velocity potential in kth mode
- Φ_k^* = Fourier transform of Φ_k
- ω = circular frequency of motion
- ω_o = representative frequency of motion

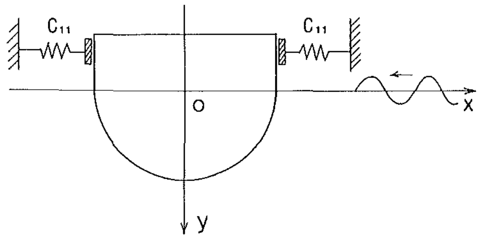

Fig. 2 Two-dimensional moored body

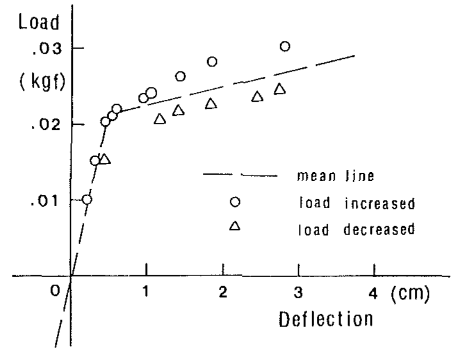

Fig. 3 Load deflection curve for fender-type mooring system

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ i\omega\ \exp(i\omega t) = \delta'(t) \tag{14}$$

we obtain

$$F_{kj}(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \exp(i\omega t)[b_{kj}(\omega) + i\omega\{a_{kj}(\omega) - a_{kj}(\infty)\}] - a_{kj}(\infty)\delta'(t) \tag{15}$$

By taking into account the causality of potential, it follows that

$$F_{kj}(t) = -\frac{2}{\pi}\int_{0}^{\infty} d\omega\ \cos(\omega t) b_{kj}(\omega) - a_{kj}(\infty)\delta'(t) \tag{16}$$

Therefore, the memory effect function $K_{kj}(t)$ is given by the following expression:

$$K_{kj}(t) \equiv -F_{kj}(t) - a_{kj}(\infty)\delta'(t) = \frac{2}{\pi}\int_{0}^{\infty} d\omega\ \cos(\omega t) b_{kj}(\omega) \tag{17}$$

For the calculation of memory effect functions by using equation (17), the damping coefficients must be known for all frequencies and this is done by connecting the exact values in a medium frequency range with the asymptotic values in high frequencies. The damping coefficients in a medium frequency range are obtained by using Ursell-Tasai's method [12] for a Lewis form section. Oortmerssen obtained iteratively the asymptotic values of the damping coefficients in high frequencies. In this paper they are derived analytically by solving the boundary value problem formulated on the high-frequency assumption.

Integro-Differential Equation of Motion. Substituting equation (17) into equation (16) the impulsive response is expressed with the memory effect function as follows:

$$F_{kj}(t) = -K_{kj}(t) - a_{kj}(\infty)\delta'(t) \tag{18}$$

Let us assume that $F_j^{(k)}$ is the force component of jth direction when the motion velocity $\dot{x}_k(t)$ is given from $t = 0$. Then it is given by the convolution integral of impulse response and motion velocity.

$$F_j^{(k)}(t) = -\int_0^t K_{kj}(t-\tau)\dot{x}_k(\tau)d\tau - a_{kj}(\infty)\int_0^{\infty}\dot{x}_k(\tau)\delta'(t-\tau)d\tau \tag{19}$$

By considering the initial condition, we have

$$\int_0^{\infty}\dot{x}_k(\tau)\delta'(t-\tau)d\tau = \ddot{x}_k(t) \tag{20}$$

and equation (19) reduces to

$$F_j^{(k)}(t) = -\int_0^t K_{kj}(t-\tau)\dot{x}_k(\tau)d\tau - a_{kj}(\infty)\ddot{x}_k(t) \tag{21}$$

In addition to the hydrodynamic force given by equation (21), a floating body in waves is subjected to the inertial reaction force, hydrostatic restoring force and the external force due to waves. Applying Newton's second law of motion the differential equations which describe the body motions in waves are given. In this paper, as shown in Fig. 2, only sway motion of a moored body is considered, and the equation of motion results in the following expression:

$$\{M_1 + a_{11}(\infty)\}\ddot{x}_1(t) + \int_0^t K_{11}(t-\tau)\dot{x}_1(\tau)d\tau + f_{11}\{x_1(t)\} = F_{1w}(t) \tag{22}$$

where

M_1 = mass of body, $x_1(t)$ = swaying motion
$a_{11}(\infty)$ = added mass at infinite frequency
$K_{11}(t)$ = memory effect function for swaying motion
$f_{11}\{x_1(t)\}$ = restoring force resulting from the mooring system and it is the function of $x_1(t)$
$F_{1w}(t) = F_{1w1}(t) + F_{1w2}(t)$ = wave-induced external forces which consist of wave-exciting force and drifting force

Since equation (22) involves the convolution integral term, we will refer it as "C.I. equation of motion."

Second-Order Linear Differential Equation With Constant Coefficients

If a body is forced in a simple harmonic motion and the stationary conditions are attained, the hydrodynamic force acting on a body is expressed by the following equation:

$$F_j^{(k)}(t) = -b_{kj}(\omega_0)\dot{x}_k(t) - a_{kj}(\omega_0)\ddot{x}_k(t) \tag{23}$$

where

ω_0 = circular frequency of motion
$\dot{x}_k(t)$ = motion velocity in kth mode

Let us assume that hydrodynamic forces acting on a body can be given in equation (23) by using a representative frequency even when irregular waves and/or nonlinear mooring system are concerned. Then the equation of motion of the body at any instant can be written in the form:

$$\{M_1 + a_{11}(\omega_0)\}\ddot{x}_1(t) + b_{11}(\omega_0)x_1(t) + f_{11}\{x_1(t)\} = F_{1w}(t) \tag{24}$$

A suitable representative frequency has to chosen for ω_0 and thus the coefficients of the equation of motion become constant. Hence we will refer equation (24) as "C.C. equation of motion."

Model Experiments

In order to verify the numerical results obtained from the two different equations of motion for a moored body in waves, model experiments were conducted by using a small tank ($L \times B \times D = 14\text{m} \times 0.3\text{m} \times 0.75\text{m}$) in the Naval Architectural Department of Osaka University. The model was a half-immersed circular cylinder with a 0.1075-m radius.

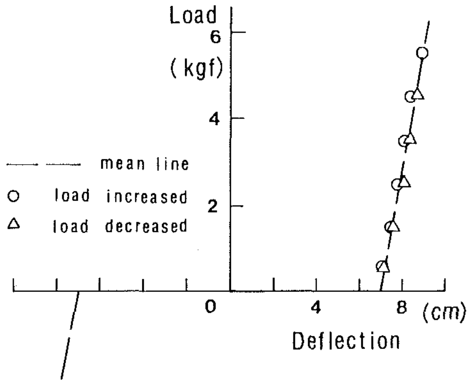

Fig. 4 Load deflection curve for rope and/or chain-type mooring system

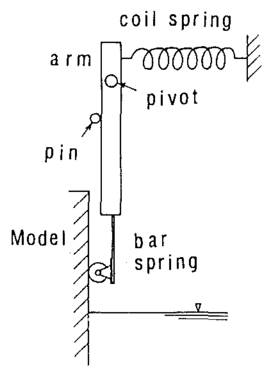

Fig. 5 Mooring system used for model tests

The width of cylinder was 0.297 m and the clearance between the one side wall of model and the tank wall was 1.5 mm. The model experiments were divided into two series according to the type of mooring system, although both are nonlinear types. The first series of experiments was concerned with a mooring system as shown in Fig. 3 and the second one was with a mooring system as shown in Fig. 4. Hereafter, for convenience, we will refer to the former system as a fender-type and the latter one as a rope-type mooring system. For a fender type, the nonlinear characteristic for the restoring force of the mooring system was simulated by means of a composite of a bar spring and a coil spring as shown in Fig. 5; its characteristic curve is given in Fig. 3. For a rope-type mooring system, bar springs are set at a short distance from both sides of a floating body and the nonlinear characteristic curve is given in Fig. 4. In these experiments the roll motion has been restricted by strings and only the sway motion has been measured by means of a potentiometer. The restoring forces were measured by strain gages on bar springs. The irregular waves were generated from the up and down motions of the plunger of wave maker controlled by the electrical signals. These signals were obtained as follows. Firstly, the time histories of the digital signals were calculated from the given wave spectrum by dividing into a finite number of discrete rectangulars. Secondly, these signals were converted into analogue signals through the D-A converter and recorded on a magnetic tape of data recorder [13]. All signals were simultaneously recorded both on magnetic tape recorder and paper chart recorder. The sketch of the test setup is shown in Fig. 6. The experimental results are shown in the following chapter along with the numerical results.

Comparisons Between Calculations and Experiments

Calculation of Memory Effect Function. As was mentioned before, the memory effect function results in the following expression:

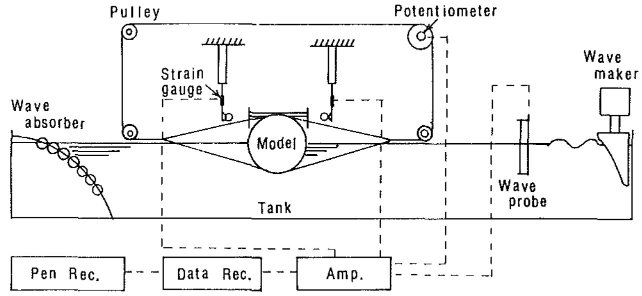

Fig. 6 Experimental setup for the second series tests with a rope-type mooring system

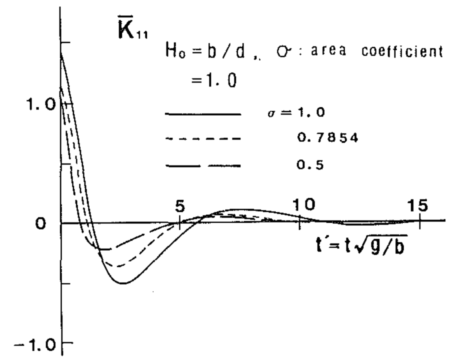

Fig. 7 Memory effect functions for three Lewis form cylinders with $H_0 = 1.0$ in swaying motion

$$K_{kj}(t) = \frac{2}{\pi}\int_0^\infty d\omega \cos(\omega t) b_{kj}(\omega) \tag{25}$$

The damping force coefficient for a two-dimensional body oscillating with the circular frequency ω is given by

$$b_{11}(\omega) = \rho g^2/\omega^3 \cdot \bar{A}_1^2(\omega) \tag{26}$$

where $\bar{A}_1(\omega)$ = ratio of amplitudes of the generated waves to those of the swaying motions.

Putting equation (26) into equation (25), the memory effect function for sway motion can be expressed as

$$K_{11}(t) = \frac{2\rho g^2}{\pi}\int_0^\infty \frac{\bar{A}_1{}^2(\omega)}{\omega^3}\cos\omega t d\omega \tag{27}$$

In order to perform the integration of equation (27), the asymptotic values of the wave amplitude ratio in the high-frequency range are derived as follows (for details see Appendix of reference [14, pp. 219–220]):

$$\bar{A}_1 = 2\left|1 + \frac{F}{K}\right| \quad \text{for } \omega \rightarrow \infty \tag{28}$$

where

$$F = \frac{(1 + a_1 + a_3)}{b(1 - a_1 - 3a_3)^2} \times \sum_{m=1}^{\infty} \frac{\alpha_m}{U}(-1)^m 2m \cdot \left\{2m + \frac{1 + a_1 + 9a_3}{1 - a_1 - 3a_3}\right\}$$

The memory effect function is nondimensionalized as follows:

$$\bar{K}_{11}(t') = K_{11}(t)/\rho g(B/2) \tag{29}$$

and the memory functions for Lewis form sections are shown in Fig. 7 as a function of nondimensionalized time $t' = t\sqrt{g/b}$.

Incident Waves and External Forces on the Body due to Waves. The incident waves at the location of the body were assumed to be obtained from the waves measured by removing the body out of the experimental tank and the

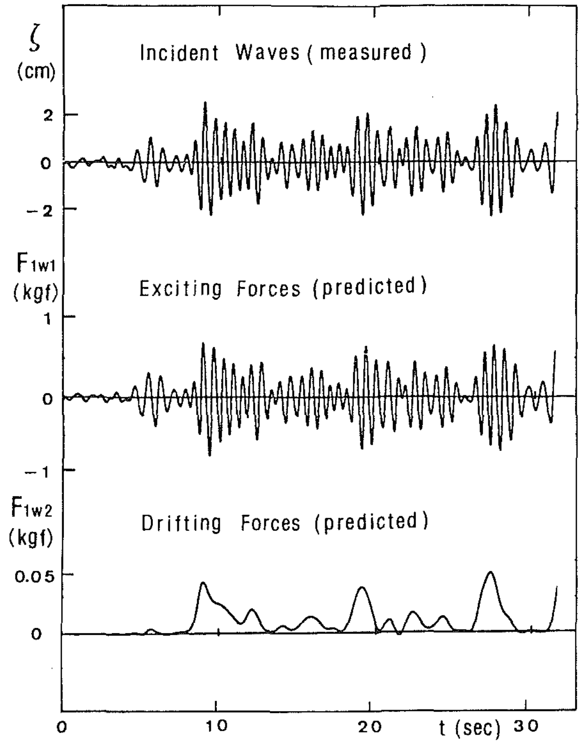

Fig. 8 Incident irregular waves and wave forces for fender-type mooring system

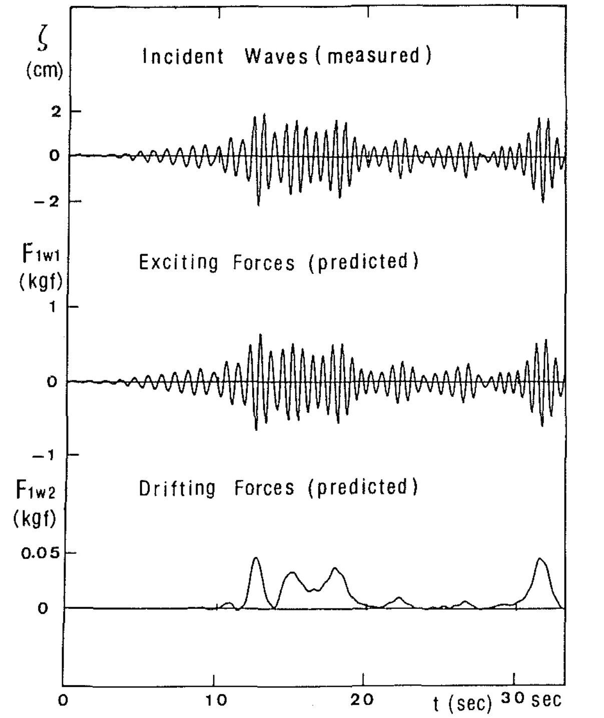

Fig. 9 Incident irregular waves and wave forces for rope-type mooring system

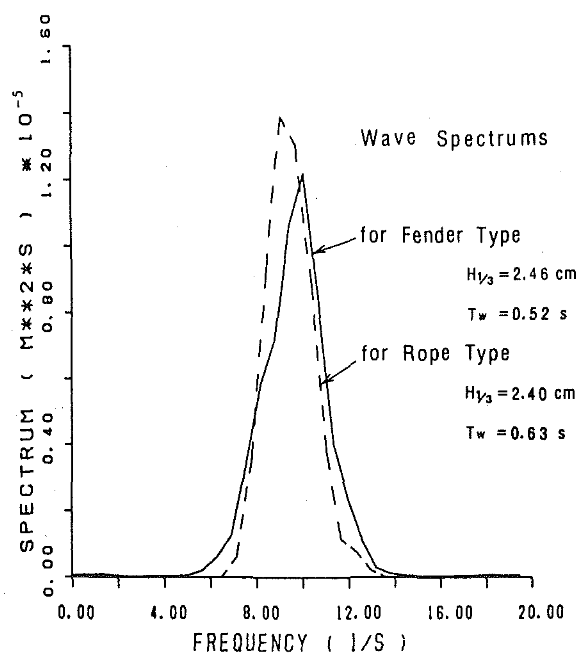

Fig. 10 Wave spectrums of incident irregular waves

reproducibility of waves was checked by a wavemeter installed at the location between the model and the wave maker.

In order to compare the numerical results with the experimental ones, the external forces acting on the body were estimated by the following procedure. Firstly, the measured incident waves were decomposed by the method of Fourier analysis and expressed as follows:

$$\zeta = \sum_{i=1}^{120} a_i \cos(\omega_i t + \psi_i) \tag{30}$$

Secondly, the wave-exciting forces were predicted by linear summation of the wave forces to the components of the incident waves as follows [12]:

$$F_{1w1}(t) = \rho g^2 \sum_{i=1}^{120} \frac{\bar{A}_1(\omega_i)}{\omega_i^2} \cos[\omega_i t + \psi_i + \epsilon_1(\omega_i)] \tag{31}$$

where

$\bar{A}_1(\omega_i)$ = ratio of amplitudes of the generated waves to those of the swaying motions at the frequency of ω_i

$\epsilon_i(\omega_i)$ = phase difference between the generated waves and the swaying motions

In addition to the aforementioned linear wave forces, higher order wave forces can be considered. For the usual ship motion prediction, higher order wave forces are neglected since the magnitudes of forces are small compared with the ones of linear wave forces. However, when the moored body is considered, long period motions close to natural frequency of the mooring system may occur and the damping forces in this frequency region are small. Therefore, in this case, higher order wave forces with near the natural frequency will play an important roll for the occurrence of large motions even if the magnitudes of the forces are small. These forces correspond to the second-order slow drift forces and they should be taken into account in this study. The prediction methods for the forces are already mentioned by several researchers, such as in [15–19] and here only the final expression for the forces will be shown. Based on the Hsu's assumption [15], the wave-drifting forces are expressed as [16]

$$F_{1w2}(t) = \frac{1}{2}\rho g \sum_i \sum_j R_{ij} a_i a_j \cos[(\omega_i - \omega_j)t + (\psi_i - \psi_j)] \tag{32}$$

This is the same expression as the one of Pinkster [17]. R_{ij} is the steady-state drifting force coefficient [18] and can be obtained when the reflecting coefficients of the body are known [19].

Figure 8 shows the measured incident waves in the first series experiments and the predicted external forces due to waves. Figure 9 shows the same results for the second series experiments. Figure 10 shows the incident wave spectrums corresponding to Figs. 8 and 9.

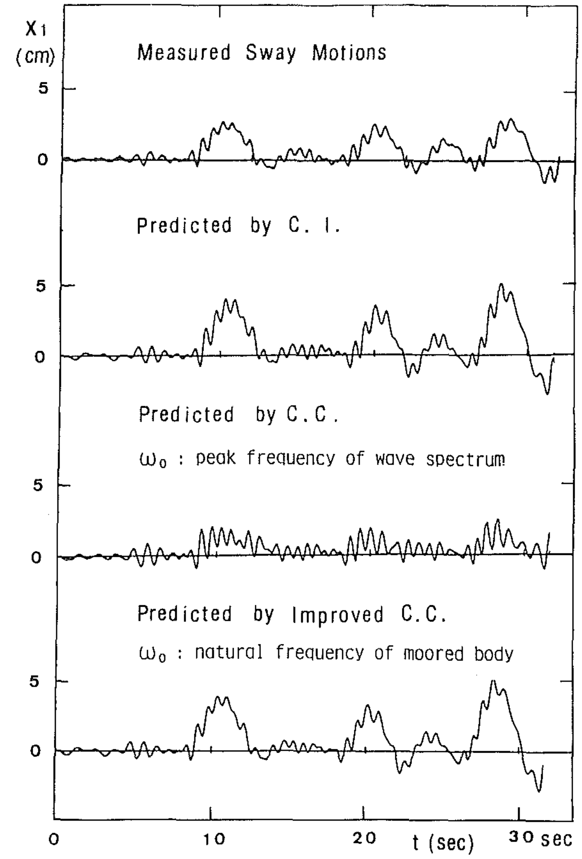

Fig. 11 Measured and predicted sway motions of a body with fender-type mooring system

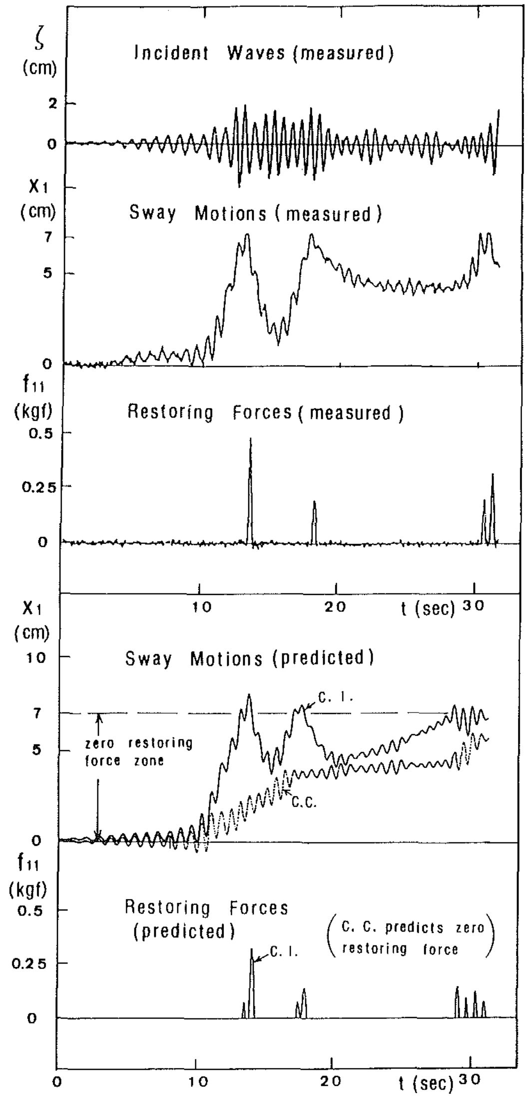

Fig. 12 Measured and predicted sway motions of a body with rope-type mooring system

Comparisons Between Calculations and Experiments. In order to assess the validity of the calculation methods of C.C. and C.I., the simulation results are compared with the values of experiments mentioned in the former chapter. The numerical calculations were performed by Euler's method with a time step $\Delta t'(=\Delta t \times \sqrt{g/b}) = 0.1$. For the comparisons with experiments, the effects of viscous damping and mechanical damping due to a pulley on the results were considered by introducing the terms $C^*_{D_1}\ \dot{x}_1 |\dot{x}_1| + C^*_{M_1} \dot{x}_1$ on the L.H.S. of equation of motion. The coefficients $C^*_{D_1}$ and $C^*_{M_1}$ were estimated from free sway oscillation experiments which were conducted separately from the experiments in waves.

Fender-Type Nonlinear Mooring System. In Fig. 11, the measured and calculated time histories of the sway motions of the moored body in a train of irregular waves are compared. It is found that there exists a fairly good agreement between the results of C.I. method and experiments with respect to the low frequency motion of the moored body. However the results of C.C. method (the third curve in the figure), in which the hydrodynamic coefficients are assumed to be the values at the peak frequency of the irregular waves, does not give such a low frequency motion of the moored body.

The following reason may be considered for the difference between the results of C.I. and C.C. methods. In a transient stage, free oscillation components of the motion are more dominant compared with the wave-forced oscillation components particularly when the restoring forces are small. The former components, in this stage, will cause large motions different from the wave-excited motions and the large motions will continue due to small wave damping. In C.I. method, the hydrodynamic forces can be selected automatically depending on the instantaneous motions. On the contrary, in C.C. method, it is difficult to predict exact body motions in a transient stage when the hydrodynamic coefficients are given at the peak frequency of the wave spectrum.

From the foregoing fact, C.C. method can be corrected by assuming the hydrodynamic coefficients at the natural frequency of the moored body instead of the peak frequency of the wave spectrum and the results of corrected C.C. method (the bottom curve in the figure) show a better agreement with measured ones.

Kobayashi, et al. [20] also showed a technique for an improvement of C.C. method. In their method the numerical calculation is divided into two parts. The one is related to the higher frequency of incident waves. The other is related to the slow drift oscillation corresponding to the natural frequency of the moored body. The numerical results gave a good agreement with the measured ones.

Rope-Type Nonlinear Mooring System. In Fig. 12, the measured and calculated time histories of sway motions of the moored body in irregular waves are shown. In a transient stage when the body starts its motion under the effect of waves, a significant difference between two calculated results is found. According to the results of C.C. method, in which the hydrodynamic coefficients are assumed to be the values at

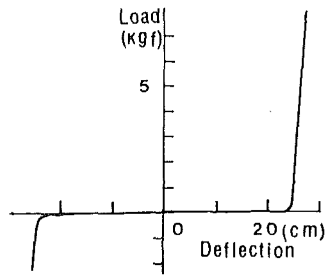

Fig. 13 Load deflection curve for steel wire rope

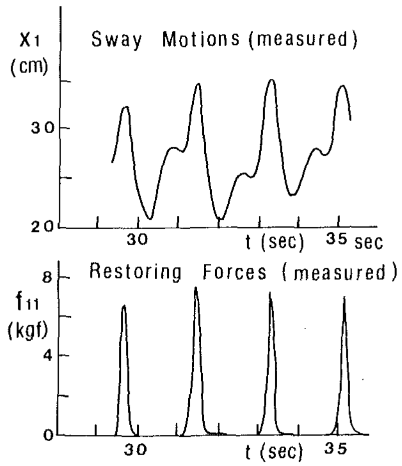

Fig. 14 Measured sway motions and restoring forces

peak frequency of irregular waves, the body did not reach the point where it is subjected to the effect of restoring force. On the contrary, the results of C.I. method showed the effect of restoring force quickly after the action of external force began. The C.I. method gave the results which are close to the experiments than those of C.C. method.

Rope-Type Nonlinear Mooring System With the Effect of Simulated Wind Forces. The present calculations may be compared with corresponding results obtained by Shoji's experiments [21] for a circular cylinder. Figure 13 shows the restoring force characteristic curve for the wire rope mooring system used in the experiments. Figure 14 illustrates the time histories of sway motion and restoring force. In his experiments a wind force was given by suspending a horizontal force on the cylinder. The calculations were performed using a sinusoidal regular wave with the same amplitude corresponding to Shoji's experiments and on the R.H.S. of equation of motion a wind force was added besides wave forces. The calculated results are shown in Fig. 15 and a better agreement between the results of C.I. method and experiments are revealed. Particularly, the C.I. method is superior to the C.C. method, in which the hydrodynamic coefficients are given by the values at the frequency of regular waves, with respect to the subharmonic sway motions. The following reason for the occurrence of subharmonic motions may be considered. As was mentioned before, free oscillation components of the motions will cause large body motions in a transient stage and sometimes subharmonic motions appear due to the action of the strong nonlinear restoring force on the body. Therefore the precise estimation of motions in a transient stage is very important when nonlinear effects of the mooring system are concerned. In C.C. method it is difficult, in general, to predict exact body motions in a transient stage and to show subharmonic motions coming from transient

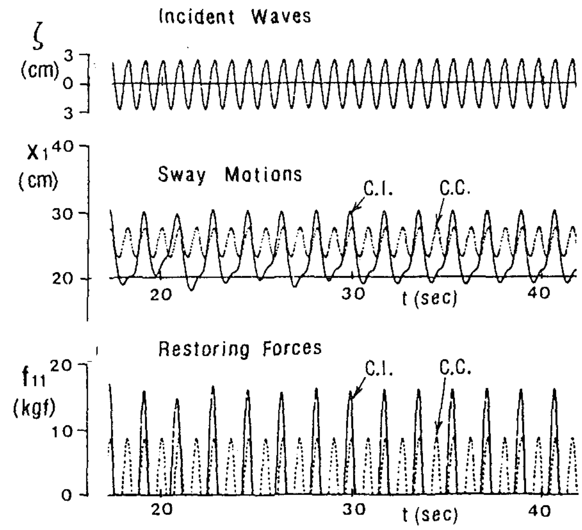

Fig. 15 Predicted sway motions and restoring forces

phenomena. Further investigations are needed to ascertain the foregoing discussions.

From the foregoing three comparisons it may be said that the results of C.I. method agree better with the experiments than the ones of C.C. method. In a transient stage and/or when the resonant phenomena occurred, large sway motions close to the natural frequency of a mooring system are dominant and precise estimation for the damping forces is very important for the prediction of such motions. In C.C. method, the hydrodynamic forces are estimated by assuming the expected frequency of motion, a priori, and it is generally difficult to predict reasonable damping forces for all experienced motions. On the contrary, in C.I. method, the hydrodynamic forces can be selected automatically depending on the instantaneous motions. This fact can explain the reason why C.I. method is able to estimate more precise damping forces at near-resonant frequency than in the case of C.C. method and it will give good results compared with the experimental ones. With respect to C.C. method, some improvements may be expected when the hydrodynamic coefficients are estimated at the natural frequency of the mooring system rather than at the wave frequency for a fender-type nonlinear mooring system.

Conclusions

The numerical simulations using the integro-differential equation of motion are compared with the second-order differential equation of motion and the former results show a good agreement with measurements in the following cases:

1 when the sway motions close to the natural frequency of a moored body occur;
2 when a floating body at rest with a loosened rope and/or chain starts its motion (i.e., in a transient stage under the action of wave forces);
3 when the subharmonic sway motions appear.

One reason for this might be the fact that the former equation of motion can select suitable hydrodynamic coefficients automatically depending on an instantaneous motion while in the latter case the hydrodynamic coefficients are fixed. Similar discussions as 1 and 3 are found in the work of Oortmerssen.

With respect to C.C. method some improvements may be expected by the selection of the suitable representative frequency depending on the motion characteristics.

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