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A New Kind of Derivations in BCI-Algebras

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Abstract

A new kind of derivation in BCI-algebras is introduced, and related properties are investigated. For a self map d_q^f of a BCI-algebra X, conditions for the kernel of d_q^f to be both a subalgebra and an ideal of X are provided.

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1 Introduction

Several authors [1, 2, 7, 10] have studied derivations in rings and near-rings. Jun et al. [4] applied the notion of derivation in ring and near-ring theory to BCI-algebras, and as a result they introduced a new concept, called a (regular) derivation, in BCI-algebras. Using this concept as defined they investigated some of its properties. Using the notion of a regular derivation, they also established characterizations of a p-semisimple BCI-algebra. For a self-map d of a BCI-algebra, they defined a d-invariant ideal, and gave conditions for an ideal to be d-invariant. In [12], Zhan et al. introduced the notion of left-right (resp., right-left) f-derivation of a BCI-algebra, and investigated some related properties. Using the idea of regular f-derivation, they gave characterizations of a p-semisimple BCI-algebra.

In this paper, we introduce a new kind of derivation in BCI-algebras, and investigate related properties. We provide conditions for the kernel of a self d_q^f to be a subalgebra/ideal of a BCI-algebra X.

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2 Preliminaries

A nonempty set X with a constant 0 and a binary operation $*$ is called a *BCI-algebra* if for all $x, y, z \in X$ the following conditions hold:

(I) $((x*y)*(x*z))*(z*y) = 0$, (II) $(x*(x*y))*y = 0$,

(III) $x*x = 0$, (IV) $x*y = 0$ and $y*x = 0$ imply $x = y$.

A BCI-algebra X has the following properties: for all $x, y, z \in X$

(a1) $x*0 = x$. (a2) $(x*y)*z = (x*z)*y$.

(a3) $x \leq y$ implies $x*z \leq y*z$ and $z*y \leq z*x$.

(a4) $(x*z)*(y*z) \leq x*y$. (a5) $x*(x*(x*y)) = x*y$.

(a6) $0*(x*y) = (0*x)*(0*y)$. (a7) $x*0 = 0$ implies $x = 0$.

For a BCI-algebra X, denote by X_+ (resp. $G(X)$) the BCK-part (resp. the BCI-G part) of X, i.e., X_+ is the set of all $x \in X$ such that $0 \leq x$ (resp. $G(X) := \{x \in X \mid 0*x = x\}$). Note that $G(X) \cap X_+ = \{0\}$ (see [6]). If $X_+ = \{0\}$, then X is called a *p-semisimple BCI-algebra.* In a p-semisimple BCI-algebra X, the following hold:

(a8) $(x*z)*(y*z) = x*y$. (a9) $0*(0*x) = x$ for all $x \in X$.

(a10) $x*(0*y) = y*(0*x)$. (a11) $x*y = 0$ implies $x = y$.

(a12) $x*a = x*b$ implies $a = b$. (a13) $a*x = b*x$ implies $a = b$.

(a14) $a*(a*x) = x$.

Let X be a p-semisimple BCI-algebra. We define addition "+" as $x + y = x*(0*y)$ for all $x, y \in X$. Then $(X, +)$ is an abelian group with identity 0 and $x - y = x*y$. Conversely let $(X, +)$ be an abelian group with identity 0 and let $x*y = x - y$. Then X is a p-semisimple BCI-algebra and $x + y = x*(0*y)$ for all $x, y \in X$ (see [8]).

For a BCI-algebra X we denote $x \wedge y = y*(y*x)$, in particular $0*(0*x) = a_x$, and $L_p(X) := \{a \in X \mid x*a = 0 \Rightarrow x = a, \forall x \in X\}$. We call the elements of $L_p(X)$ the *p-atoms* of X. For any $a \in X$, let $V(a) := \{x \in X \mid a*x = 0\}$, which is called the *branch* of X with respect to a. It follows that $x*y \in V(a*b)$ whenever $x \in V(a)$ and $y \in V(b)$ for all $x, y \in X$ and all $a, b \in L_p(X)$. Note that $L_p(X) = \{x \in X \mid a_x = x\}$, which is the p-semisimple part of X, and X is a p-semisimple BCI-algebra if and only if $L_p(X) = X$ (see [5, Proposition 3.2]). Note also that $a_x \in L_p(X)$, i.e., $0*(0*a_x) = a_x$, which implies that $a_x*y \in L_p(X)$ for all $y \in X$. It is clear that $G(X) \subset L_p(X)$, and $x*(x*a) = a$ and $a*x \in L_p(X)$ for all $a \in L_p(X)$ and all $x \in X$. For more details, refer to [1, 8, 10, 13, 14].

3 A new kind of derivations

In what follows, let X denote a BCI-algebra, f an endomorphism of X, and d_q^f a self map of X defined by $d_q^f(x) = f(x) * q$ for all $x \in X$ and $q \in X$ unless otherwise specified.

Definition 3.1. A self map d_q^f of X is called an *inside f_q-derivation* of X if it satisfies:

$$(\forall x, y \in X) \left(d_q^f(x * y) = \left(d_q^f(x) * f(y) \right) \wedge \left(f(x) * d_q^f(y) \right) \right). \tag{3.1}$$

If d_q^f satisfies the following identity:

$$(\forall x, y \in X) \left(d_q^f(x * y) = \left(f(x) * d_q^f(y) \right) \wedge \left(d_q^f(x) * f(y) \right) \right), \tag{3.2}$$

then it is called an *outside f_q-derivation* of X. If d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X, we say that d_q^f is a *f_q-derivation* of X.

In Definition 3.1, if we take $f = i_X$, the identity map, then d_q^f is denoted by d_q and is called an *inside q-derivation* (resp. *outside q-derivation*) of X. Note that if X is commutative, then the notion of inside f_q-derivation and outside f_q-derivation coincide.

Example 3.2. Consider a BCI-algebra $X = \{0, a, b\}$ with the following Cayley table:

$*$	0	a	b
0	0	0	b
a	a	0	b
b	b	b	0

Table 1: Cayley table

Define an endomorphism

$$f : X \to X,\ x \mapsto \begin{cases} 0 & \text{if } x \in \{0, a\}, \\ b & \text{if } x = b, \end{cases}$$

Table 2 shows that d_q^f is an outside f_q-derivation of X with $q = a$.

Similarly, it is verified that d_q^f is an inside f_q-derivation of X with $q = a$.

Example 3.3. Let $X = \{0, 1, 2, 3, 4, 5\}$ be a BCI-algebra with the multiplication table (Table 3) (see [9]).

Table 2: $d_a^f(x*y) = (f(x)*d_a^f(y)) \wedge (d_a^f(x)*f(y))$

x	0	0	0	a	a	a	b	b	b
y	0	a	b	0	a	b	0	a	b
$x*y$	0	0	b	a	0	b	b	b	0
$f(x*y)$	0	0	b	0	0	b	b	b	0
$d_a^f(x*y)$	0	0	b	0	0	b	b	b	0
$f(x)$	0	0	0	0	0	0	b	b	b
$d_a^f(x)$	0	0	0	0	0	0	b	b	b
$f(y)$	0	0	b	0	0	b	0	0	b
$d_a^f(y)$	0	0	b	0	0	b	0	0	b
$d_a^f(x)*f(y)$	0	0	b	0	0	b	b	b	0
$f(x)*d_a^f(y)$	0	0	b	0	0	b	b	b	0
$(f(x)*d_a^f(y)) \wedge (d_a^f(x)*f(y))$	0	0	b	0	0	b	b	b	0

*	0	1	2	3	4	5
0	0	0	3	2	3	2
1	1	0	5	4	3	2
2	2	2	0	3	0	3
3	3	3	2	0	2	0
4	4	2	1	5	0	3
5	5	3	4	1	2	0

Table 3: Cayley table

Define an endomorphism

$$f : X \to X,\ x \mapsto \begin{cases} 0 & \text{if } x \in \{0,1\}, \\ 2 & \text{if } x \in \{2,4\}, \\ 3 & \text{if } x \in \{3,5\}. \end{cases}$$

Table 4 shows that d_q^f is an inside f_q-derivation of X with $q = 2$. If we take $q = 2$, then d_q^f is not an outside f_q-derivation of X since $d_2^f(4*5) = 2 \neq 0 = \Big(f(4)*d_2^f(5)\Big) \wedge \Big(d_2^f(4)*f(5)\Big)$.

Proposition 3.4. *Every self map d_q^f of X preserves order, that is, if $x \leq y$ in X then $d_q^f(x) \leq d_q^f(y)$ in X.*

Proof. Let $x, y \in X$ be such that $x \leq y$. Then $x*y = 0$, and so

$$d_q^f(x)*d_q^f(y) = (f(x)*q)*(f(y)*q) \leq f(x)*f(y) = f(x*y) = f(0) = 0.$$

It follows from (a7) that $d_q^f(x)*d_q^f(y) = 0$, that is, $d_q^f(x) \leq d_q^f(y)$. □

Table 4: $d_2^f(x*y) = \Big(d_2^f(x) * f(y)\Big) \wedge \Big(f(x) * d_2^f(y)\Big)$

x	y	$x*y$	$f(x*y)$	$d_2^f(x*y)$	$f(x)$	$d_2^f(x)$	$f(y)$	$d_2^f(y)$	A	B	$A \wedge B$
0	0	0	0	3	0	3	0	3	3	2	3
0	1	0	0	3	0	3	0	3	3	2	3
0	2	3	3	2	0	3	2	0	2	0	2
0	3	2	2	0	0	3	3	2	0	3	0
0	4	3	3	2	0	3	2	0	2	0	2
0	5	2	2	0	0	3	3	2	0	3	0
1	0	1	0	3	0	3	0	3	3	2	3
1	1	0	0	3	0	3	0	3	3	2	3
1	2	5	3	2	0	3	2	0	2	0	2
1	3	4	2	0	0	3	3	2	0	3	0
1	4	3	3	2	0	3	2	0	2	0	2
1	5	2	2	0	0	3	3	2	0	3	0
2	0	2	2	0	2	0	0	3	0	3	0
2	1	2	2	0	2	0	0	3	0	3	0
2	2	0	0	3	2	0	2	0	3	2	3
2	3	3	3	2	2	0	3	2	2	0	2
2	4	0	0	3	2	0	2	0	3	2	3
2	5	3	3	2	2	0	3	2	2	0	2
3	0	3	3	2	3	2	0	3	2	0	2
3	1	3	3	2	3	2	0	3	2	0	2
3	2	2	2	0	3	2	2	0	0	3	0
3	3	0	0	3	3	2	3	2	3	2	3
3	4	2	2	0	3	2	2	0	0	3	0
3	5	0	0	3	3	2	3	2	3	2	3
4	0	4	2	0	2	0	0	3	0	3	0
4	1	2	2	0	2	0	0	3	0	3	0
4	2	1	0	3	2	0	2	0	3	2	3
4	3	5	3	2	2	0	3	2	2	0	2
4	4	0	0	3	2	0	2	0	3	2	3
4	5	3	3	2	2	0	3	2	2	0	2
5	0	5	3	2	3	2	0	3	2	0	2
5	1	3	3	2	3	2	0	3	2	0	2
5	2	4	2	0	3	2	2	0	0	3	0
5	3	1	0	3	3	2	3	2	3	2	3
5	4	2	2	0	3	2	2	0	0	3	0
5	5	0	0	3	3	2	3	2	3	2	3

where $A = d_2^f(x) * f(y)$ and $B = f(x) * d_2^f(y)$.

If we take $f = i_X$, the identity map in Proposition 3.4, then we have the following corollary.

Corollary 3.5. *Every self map d_q of X preserves order, that is, if $x \leq y$ in X then $d_q(x) \leq d_q(y)$ in X.*

A BCI-algebra X is said to be *quasi-associative* ([11]) if it satisfies:

$$(\forall x, y \in X)\,((x * y) * z \leq x * (y * z))\,. \tag{3.3}$$

Note that a BCI-algebra X is associative if and only if it is both p-semisimple and quasi-associative (see [3]).

Theorem 3.6. *Let X be a quasi-associative BCI-algebra. If X is p-semisimple, then d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$.*

Proof. Let $q, x, y \in X$. Then

$$\begin{aligned} & d_q^f(x * y) = f(x * y) * q = (f(x * y) * q) * 0 \\ & = (f(x * y) * q) * ((f(x * y) * q) * (f(x * y) * q)) \\ & = ((f(x) * f(y)) * q) * (((f(x) * f(y)) * q) * ((f(x) * f(y)) * q)) \\ & = (f(x) * (f(y) * q)) * ((f(x) * (f(y) * q)) * ((f(x) * q) * f(y))) \\ & = \left(f(x) * d_q^f(y)\right) * \left((f(x) * d_q^f(y)) * (d_q^f(x) * f(y))\right) \\ & = \left(d_q^f(x) * f(y)\right) \wedge \left(f(x) * d_q^f(y)\right). \end{aligned}$$

Hence d_q^f is an inside f_q-derivation of X for any $q \in X$. Also we have

$$\begin{aligned} & d_q^f(x * y) = f(x * y) * q = ((f(x) * f(y)) * q) * 0 = ((f(x) * q) * f(y)) * 0 \\ & = ((f(x) * q) * f(y)) * (((f(x) * q) * f(y)) * ((f(x) * q) * f(y))) \\ & = \left(d_q^f(x) * f(y)\right) * \left((d_q^f(x) * f(y)) * ((f(x) * f(y)) * q)\right) \\ & = \left(d_q^f(x) * f(y)\right) * \left((d_q^f(x) * f(y)) * (f(x) * (f(y) * q)\right) \\ & = \left(d_q^f(x) * f(y)\right) * \left((d_q^f(x) * f(y)) * (f(x) * d_q^f(y))\right) \\ & = \left(f(x) * d_q^f(y)\right) \wedge \left(d_q^f(x) * f(y)\right). \end{aligned}$$

Therefore d_q^f is an outside f_q-derivation of X for any $q \in X$. □

Corollary 3.7. *If X satisfies the following conditions:*

(1) $(\forall x \in X)\,(0 * x = 0 * (0 * x))$,

(2) $(\forall x \in X)\,(0 * x = 0 \ \Rightarrow x = 0)$,

then d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$.

Proof. If X satisfies the condition (1), then X is quasi-associative. Also, If X satisfies the condition (2), then X is p-semisimple. Hence d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$ by Theorem 3.6. □

Corollary 3.8. *Let X be a quasi-associative BCI-algebra. If X satisfies one of the following conditions:*

(1) $(\forall x \in X)\,(0 * (0 * x) = x)\,,$

(2) $(\forall x, y \in X)\,(x * (0 * y) = y * (0 * x))\,,$

then d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$.

Proof. If X satisfies one of conditions (1) and (2), then X is p-semisimple. Therefore d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$ by Theorem 3.6. □

Corollary 3.9. *Let X be a p-semisimple BCI-algebra. If X satisfies one of the following conditions:*

(1) $(\forall x \in X)\,(0 * x \leq x)\,,$

(2) $(\forall x, y \in X)\,(0 * (x * y) = 0 * (y * x))\,,$

(3) $(\forall x, y \in X)\,((0 * x) * y = 0 * (x * y))\,,$

(4) $(\forall x, y \in X)\,((x * y) * (y * x) \in \{x \in X \mid 0 \leq x\})\,,$

then d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$.

Proof. If X satisfies one of conditions (1), (2), (3) and (4), then X is quasi-associative. Therefore d_q^f is both an inside f_q-derivation and an outside f_q-derivation of X for any $q \in X$ by Theorem 3.6. □

If we take $f = i_X$, the identity map in Theorem 3.6, then we have the following corollary.

Corollary 3.10. *Let X be a quasi-associative BCI-algebra. If X is p-semisimple, then d_q is both an inside q-derivation and an outside q-derivation of X for any $q \in X$.*

Proposition 3.11. *If d_q^f is an inside f_q-derivation of a p-semisimple BCI-algebra X, then*

(1) $(\forall x \in X)\,\left(d_q^f(0) = d_q^f(x) * f(x)\right).$

(2) *If f is injective, then d_q^f is injective.*

(3) *If $d_q^f(0) = 0$, then $d_q^f(x) = f(x)$ for all $x \in X$.*

Proof. (1) For any $x \in X$, we have

$$\begin{aligned} d_q^f(0) &= d_q^f(x * x) = \big(d_q^f(x) * f(x)\big) \wedge \big(f(x) * d_q^f(x)\big) \\ &= \big(f(x) * d_q^f(x)\big) * \Big(\big(f(x) * d_q^f(x)\big) * \big(d_q^f(x) * f(x)\big)\Big) = d_q^f(x) * f(x) \end{aligned}$$

by (a14).

(2) Assume that f is injective. Let $x, y \in X$ be such that $d_q^f(x) = d_q^f(y)$. Then $f(x) * q = f(y) * q$, which implies from (a13) that $f(x) = f(y)$. Since f is injective, it follows that $x = y$. Therefore d_q^f is injective.

(3) Assume that $d_q^f(0) = 0$ and let $x \in X$. Then $0 = d_q^f(0) = d_q^f(x) * f(x)$ by (1), and so $d_q^f(x) = f(x)$ by (a11). □

If we take $f = i_X$, the identity map in Proposition 3.11, then we have the following corollary.

Corollary 3.12. *If d_q is an inside q-derivation of a p-semisimple BCI-algebra X, then*

(1) $(\forall x \in X)\,(d_q(0) = d_q(x) * x)$.

(2) *d_q is injective.*

(3) *If $d_q(0) = 0$, then $d_q(x) = x$ for all $x \in X$.*

Proposition 3.13. *If d_q^f is an outside f_q-derivation of X with $d_q^f(0) = 0$, then*

(1) $(\forall x \in X)\,\big(d_q^f(x) \le f(x)\big)$.

(2) $(\forall x, y \in X)\,\big(d_q^f(x) * f(y) \le f(x) * d_q^f(y)\big)$.

(3) $(\forall x, y \in X)\,\big(d_q^f(x * y) = d_q^f(x) * f(y) \le d_q^f(x) * d_q^f(y)\big)$.

Proof. Let d_q^f be an outside f_q-derivation of X with $d_q^f(0) = 0$. For any $x \in X$, we have $d_q^f(x) = d_q^f(x * 0) = \big(f(x) * d_q^f(0)\big) \wedge \big(d_q^f(x) * f(0)\big) = (f(x) * 0) \wedge \big(d_q^f(x) * 0\big) = f(x) \wedge d_q^f(x) = d_q^f(x) * \big(d_q^f(x) * f(x)\big) \le f(x)$ by (a1) and (II). This proves (1).

(2) Combining (1) and (a3) induces (2).

(3) By (2), we know that $\big(d_q^f(x) * f(y)\big) * \big(f(x) * d_q^f(y)\big) = 0$ for all $x, y \in X$. Hence

$$\begin{aligned} d_q^f(x * y) &= \big(f(x) * d_q^f(y)\big) \wedge \big(d_q^f(x) * f(y)\big) \\ &= \big(d_q^f(x) * f(y)\big) * \Big(\big(d_q^f(x) * f(y)\big) * \big(f(x) * d_q^f(y)\big)\Big) \\ &= \big(d_q^f(x) * f(y)\big) * 0 = d_q^f(x) * f(y) \le d_q^f(x) * d_q^f(y) \end{aligned}$$

by (1) and (a3). □

If we take $f = i_X$, the identity map in Proposition 3.13, then we have the following corollary.

Corollary 3.14. *If d_q is an outside q-derivation of X with $d_q(0) = 0$, then*

(1) $(\forall x \in X)\,(d_q(x) \leq x)$.

(2) $(\forall x, y \in X)\,(d_q(x) * y \leq x * d_q(y))$.

(3) $(\forall x, y \in X)\,(d_q(x * y) = d_q(x) * y \leq d_q(x) * d_q(y))$.

For a self map d_q^f of X, consider the set $\ker(d_q^f) := \left\{x \in X \mid d_q^f(x) = 0\right\}$, which is called the *kernel* of d_q^f.

Theorem 3.15. *If d_q^f is an outside f_q-derivation of X with $d_q^f(0) = 0$, then the kernel of d_q^f is a subalgebra of X.*

Proof. Let $x, y \in \ker(d_q^f)$. Then $d_q^f(x) = 0$ and $d_q^f(y) = 0$. It follows from Proposition 3.13 that $d_q^f(x*y) \leq d_q^f(x)*d_q^f(y) = 0$ so from (a7) that $d_q^f(x*y) = 0$. Hence $x * y \in \ker(d_q^f)$, and therefore $\ker(d_q^f)$ is a subalgebra of X. □

Corollary 3.16. *If d_q is an outside q-derivation of X with $d_q(0) = 0$, then the kernel of d_q is a subalgebra of X.*

Theorem 3.17. *If X satisfies the right self-distributive law, then the kernel of d_q^f is a subalgebra of X.*

Proof. Let $x, y \in \ker(d_q^f)$. Then $d_q^f(x) = 0$ and $d_q^f(y) = 0$. Since X satisfies the right self-distributive law, it follows that

$$d_q^f(x * y) = f(x * y) * q = (f(x) * f(y)) * q = (f(x) * q) * (f(y) * q) = 0,$$

and so $x * y \in \ker(d_q^f)$. Therefore $\ker(d_q^f)$ is a subalgebra of X. □

Corollary 3.18. *If X satisfies the right self-distributive law, then the kernel of d_q is a subalgebra of X.*

Theorem 3.19. *If X satisfies the right self-distributive law, then the kernel of d_q^f is an ideal of X.*

Proof. By Theorem 3.17, we know that $0 \in \ker(d_q^f)$. Let $x, y \in X$ be such that $x * y \in \ker(d_q^f)$ and $y \in \ker(d_q^f)$. Then $d_q^f(x * y) = 0$ and $d_q^f(y) = 0$. Thus $d_q^f(x) = f(x) * q = (f(x) * q) * 0 = (f(x) * q) * (f(y) * q) = (f(x) * f(y)) * q = f(x * y) * q = d_q^f(x * y) = 0$, and so $x \in \ker(d_q^f)$. Thus the kernel of d_q^f is an ideal of X. □

Corollary 3.20. *If X satisfies the right self-distributive law, then the kernel of d_q is an ideal of X.*

Combining Theorems 3.17 and 3.19, we have the following corollary.

Corollary 3.21. *If X satisfies the right self-distributive law, then the kernel of d_q^f is a closed ideal of X.*

Corollary 3.22. *If X satisfies the right self-distributive law, then the kernel of d_q is a closed ideal of X.*

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