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Nonlinear Marine Structures With Random Excitation

Various important types of offshore structure contain significant nonlinearities or time-varying coefficients in their equations of motion. Well-known examples include tension leg platforms, free-hanging risers, single-buoy moorings, ships moored against fenders and vessels constrained by stiffening moorings. When subject to sinusoidal wave excitation, time domain mathematical models of these structures can display large subharmonic or chaotic motions. This paper shows that such behavior is often an artifact of the regularity of the excitation and is usually unlikely to present a significant problem in a random sea. Narrow-band vessel response can, however, generate near-harmonic motions to create conditions in which these instabilities may become important.

Introduction

Some of the classical nonlinear and time-varying equations of engineering mathematics appear in the modeling of the dynamic behavior of offshore structures. The dynamics of free-hanging risers, tension leg platforms and suspended loads can be cast in the form of Mathieu's equation; wave excitation causes the time variation of the spring parameter (references [1–3]). Articulated loading towers, constrained by a connection to a massive tanker or vessels moored against fenders can be characterized by a second-order differential equation with a bilinear spring rate whose value changes with the sign of the displacement (reference [4]).

Disturbingly large subharmonic resonances or chaotic motions can result if the nonlinear equations (reference [4]) or the spring in the Mathieu equation varies harmonically (reference [1]). This paper presents physical and mathematical arguments which indicate that these large responses are caused by a phase lock between the motion of the structure and the external excitation, something which is generally unlikely to last for long if a structure is subject to a random excitation.

To test these predictions, two typical systems are simulated and randomness is introduced into the previously regular forcing in three different ways; as additive white noise, as frequency wander and as bandwidth spread. The responses are Fourier analyzed and maximum, minimum, mean and rms values are recorded. Random inputs cause the Poincaré points to wander in a "Poincaré region"; these are displayed as a function of the randomness parameter. The size of the subharmonic motions decays quickly with increasing values of the randomness parameter and they are generally small for realistically random wave forcing signals.

Where the motion of a vessel is a significant input to a dynamic system, the filtering action of the vessel's dynamics driven by the wave action can generate a relatively regular motion; Patel [3] reports observations of the Mathieu excitation in the motion of a load carried by a floating crane. Similarly, free-hanging risers could be excited by near-harmonic heaving of the supporting vessel. Alternatively, the filtering action of distance could produce a narrow-banded spectrum at a distance from a storm; the waves in such an extreme narrow-banded swell might be small but more dangerous than those from a larger but broader-banded spectrum.

Complex dynamic behavior of this type is likely to be very important in the generation of noise by rotating machinery which can impart an extremely regular forcing signal.

Much of the study to date of nonlinear systems has focussed on deterministic problems and has ignored the stochastic nature of both typical inputs and the chaotic and subharmonic responses. Equally, many studies of random phenomena have focussed on stochastic analysis of essentially linear systems. This admittedly limited and exploratory paper will hopefully illustrate the value of an integrated approach and demonstrate the importance of each field to workers in the other.

Mathematical Models

The detailed derivation and analysis of the mathematical models analyzed here is presented in the references. The bilinear spring and the Mathieu equation were chosen as representative nonlinear and time varying systems; the conclusions which result are probably valid for a range of important physical systems and nonlinearities.

The tension leg platform (reference [1]) can move horizontally under the action of incident waves, but is restrained by taut tethers which attach it to the seabed; their restoring force can be approximated by a linear spring of strength $T(t)/L$ where L is the tether length and $T(t)$ is the instantaneous tension which varies about a mean value, T, in response to the vertical wave forces acting on the platform. If the platform mass and added fluid inertia is written as M and N is the effective fluid damping due to drag and wavemaking effects, the motion is represented as a second-order differential equation with a time-varying spring.

$$M\ddot{x} + N\dot{x} + xT(t)/L = F(t) \tag{1}$$

Contributed by the OMAE Division and presented at the 6th International Symposium and Exhibit on Offshore Mechanics and Arctic Engineering, Houston, Texas, March 1–5, 1987, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the OMAE Division, 1986; revised manuscript received September 18, 1987.

where $F(t)$ is the exciting force due to the waves. The time-varying tension is caused by the vertical wave forces acting on the platform and changes at the wave frequency. Jefferys and Patel [5] discuss the analysis of this equation and elucidate the worst possible behavior for $T(t)$, which maximizes the work done and hence the steady oscillation amplitude of the platform. The external forcing is at a frequency well removed from the resonant frequency, and hence can do no work on resonant motions; it can be ignored in a stability analysis.

The system can be expressed as a damped Mathieu equation by appropriate normalization and renaming

$$\ddot{x} + 2cw_n\dot{x} + w_n^2[1 + f(t)]x(t) = 0 \tag{2}$$

where w_n is a natural frequency and c is a damping ratio. The spring rate proportional variation $f(t)$ is initially assumed sinusoidal at frequency w; substitution for x removes the damping term and yields the classic Mathieu equation.

Hsu [2] discusses the analysis of a free-hanging marine riser, suspended from a floating drilling vessel, and shows that the Mathieu equation again results; this time the time-varying spring stiffness is caused by the vertical motions of the suspension, which introduces a time-varying spring into the second-order differential equation, which characterizes each mode of the riser.

Patel et al. [3] show a similar effect in the motion of the hook of a crane vessel; the vertical motion of the jib tip changes the tension of the load-carrying cable, which leads to a Mathieu equation for the pendulum motion of the load.

Bilinear springs are fairly common; Thompson et al. [4] analyze the motion of a single-buoy mooring, a columnar buoy, pivoted at its base, which transfers oil from a sea bed pipeline to a moored tanker. When the connection to the tanker is slack, only hydrostatic force restores the buoy to its upright position; if the displacement of the buoy exceeds a certain value, the mooring hawser tightens and its longitudinal stiffness is added to the restoring force. As a first approximation, the tanker may be assumed infinitely massive and the hawser may be assumed to tighten at zero displacement.

The equation of motion is

$$M\ddot{x} + N\dot{x} + K(x)x = F(t) \tag{3}$$

where I is the buoy inertia, N is a damping and $K(x)$ is a position-dependent spring, which changes rate by a factor of α according to the sign of x

$$K(x) = K\, x > 0 \qquad K(x) = \alpha K\, x < 0 \tag{4}$$

The resulting normalized equation is

$$n^2\ddot{x} + 2cn\dot{x} + p(x)x = f(t) \tag{5}$$

$$P(x) = (1 + \sqrt{\alpha})^2/(4\alpha n^2) \qquad x > 0 \tag{6}$$

$$P(x) = (1 + \sqrt{\alpha})^2/(4n^2) \qquad x < 0 \tag{7}$$

The undamped average natural frequency is denoted by n, and c is a damping constant. A forcing function, $f(t)$, initially assumed to be sinusoidal excites the system.

In the limit as α, the spring rate ratio, tends to infinity, this system becomes an "impact oscillator"; the state cannot enter the half-plane where $x < 0$ and "bounces" back into the positive half-plane with the opposite velocity whenever the boundary is encountered. This is a reasonable model of a ship moored against fenders and restrained by soft catenary moorings.

Thompson et al. [4] show that for small values of α typically less than three, subharmonic behavior is difficult to find; this is comforting in that it implies that small departures from nonlinearity will not cause qualitatively nonlinear behavior.

The behavior is linear in each half of the phase space and the response is merely scaled by the forcing so this is a particularly "simple" form of nonlinear system. Nonetheless, when forced by regular sinusoidal excitation it can display extremely complex subharmonic and chaotic behavior which is infinitely initial condition and parameter sensitive (reference [4]). These responses can coexist with stable small solutions at the fundamental frequency; which one results depends purely on the initial conditions, a problem which can cause serious problems in simulations or model tests.

"Tugger" lines attached to a crane load to restrain Mathieu oscillations could also generate a similar equation of motion, leading to an alternative form of instability!

Cause of Subharmonic Motions

The existence, stability and size of subharmonic motions of harmonically forced and autonomous systems can be predicted by describing function techniques; for a comprehensive treatment, see Mees [7]. This method is very powerful so long as the forcing is regular, but is less easy to apply if the excitation is (pseudo) random; the arguments summarized in the forthcoming are intended to provide insight into the physical basis of the problem and indicate the importance of the regular forcing.

An "instability" is a persistently growing motion which occurs if energy is injected into a system faster than it can be dissipated. The amplitude of oscillation continues to grow only while the energy input is greater than the rate of dissipation. Turbo-generators, for instance, possess numerous whirl and torsional resonances at frequencies lying between zero and their operational speed, but this causes no problem. They are run through the resonant frequencies quickly and spend little time with their rotational speed in the critical regime so the oscillation amplitude never reaches its steady (unacceptable) level.

In the Mathieu instability, the time-varying spring force does not work on the system faster than it can be dissipated by a linear damper at any response amplitude. The work done by the spring is

$$E = K\int_o^T (1 + f(t))x(t)\dot{x}(t)\, dt \tag{8}$$

Integrating by parts

$$E = -\left[\frac{K}{2}x^2(t)\right]_o^T + \frac{K}{2}\int_o^T \dot{f}(t)x^2(t)\, dt \tag{9}$$

The first term represents the energy stored in the time-independent spring, while the second is the integrated work done by the time-varying part of the spring. Hence, the power input by the spring is

$$\dot{E} = K\dot{f}(t)x^2(t)/2 \tag{10}$$

Clearly, net positive work will be done if the spring rate increase, $K \cdot \dot{f}(t)$, is large whenever the modulus of $x(t)$ is large. The amplitude of oscillation increases until the average energy dissipation equals the energy input, so long as the phase relationship between the spring change, $f(t)$, and the motion, $x(t)$, is maintained. If the motion is harmonic and at

Nomenclature

a = ratio of spring rates
c = damping ratio
e_i = randomness parameter
$F(t), f(t)$ = forcing of excitation
$K, K(x)$ = spring rate
M = inertia
N = damping
n = normalized frequency
w = frequency
x, X = displacement

the natural frequency of the system, as it must be for dangerously large oscillations to occur in the absence of external forces, the spring must vary at twice this resonant frequency, the result predicted by classical theory.

Linear damping dissipates either more or less energy than the spring puts in at any amplitude of harmonic motion; if an oscillation occurs, its amplitude will finally be limited by nonlinear effects. In marine systems, quadratic damping usually dominates and limits any Mathieu instabilities to finite, albeit potentially unacceptable amplitudes.

The energy dissipation per cycle of linear and quadratic dampers, N_L, N_Q subject to motion of amplitude X is

$$dE_L = N_L w^2 X^2/2 \tag{11}$$

$$dE_Q = (4/3\pi) N_Q w^3 X^3 \tag{12}$$

If the spring variation is sinusoidal with amplitude A, then maximum motion amplitude is

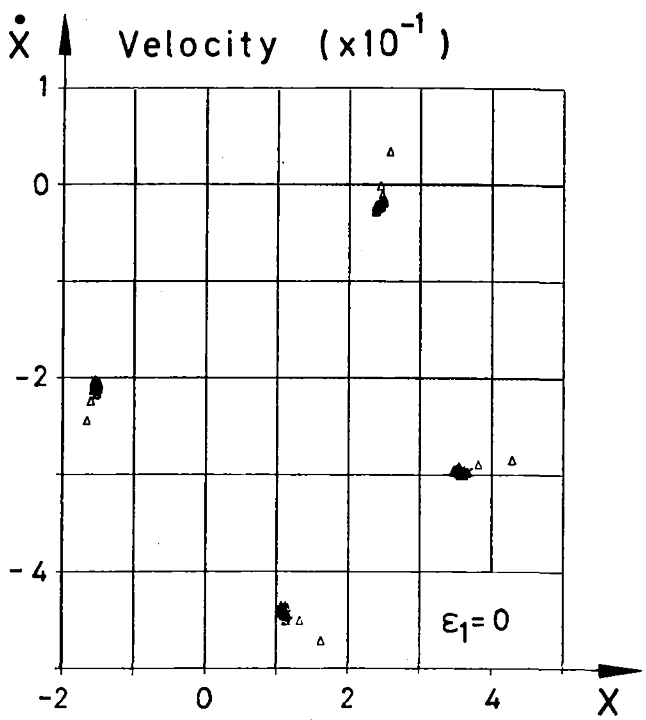
$$X = \frac{3\pi(A - 2N_L w)}{16 N_Q w^2} \tag{13}$$

If the excitation, $f(t)$, is not phase locked to the response, as is likely in many marine applications, the work done by the spring can be negative as often as it is positive; the resulting oscillation will be small unless a great deal of energy is input by a sequence of a very few waves of about the right period and phase. Thus, the spring variation must be regular and greater than a limit defined by the level of damping if significant oscillations are to occur.

The other nonlinear systems are conservative; work is done by the external force acting on the system. If it is harmonic and the displacement is expressed as a Fourier series of sub and superharmonics (reference [7]), the net work done is due only to the product with the velocity term at the forcing frequency.

$$E = \int_o^T \dot{x}(t) f(t)\, dt \tag{14}$$

$$x(t) = \sum_{R=1}^{N} a_k \cos(wt/k + \phi_{k-}) + b_k \cos(kwt + \phi_{k+}) \tag{15}$$



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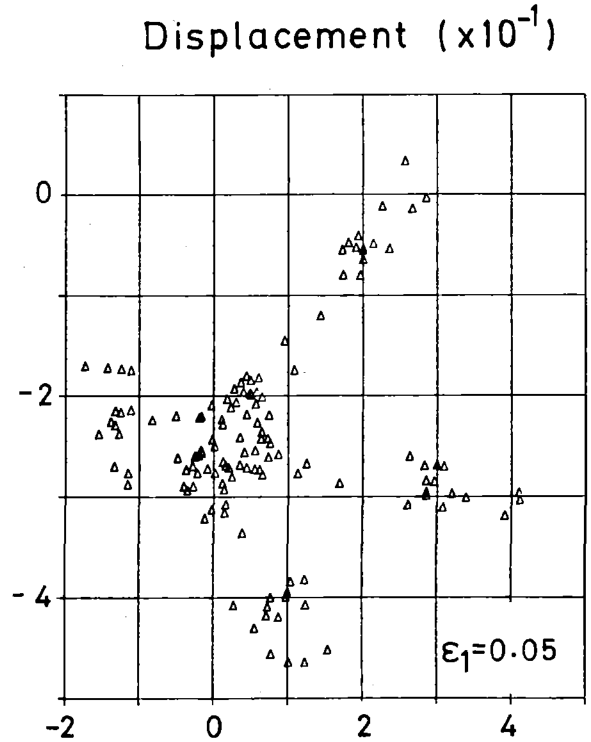

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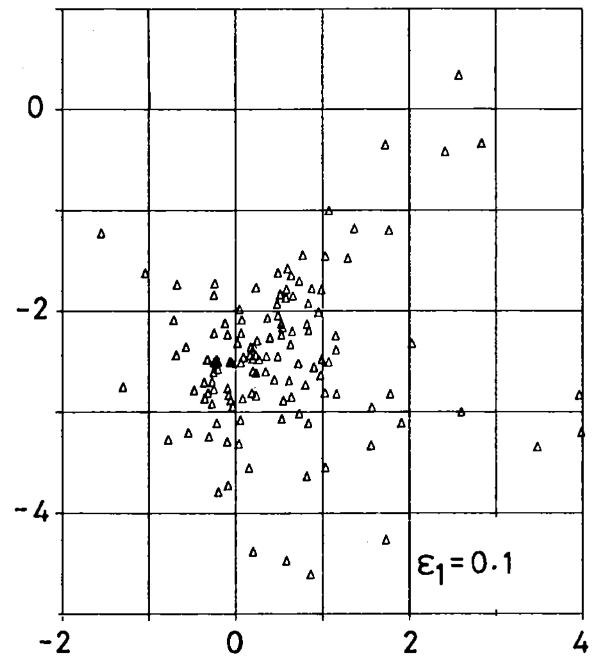


Fig. 1 White noise Poincaré points bilinear spring

(c)

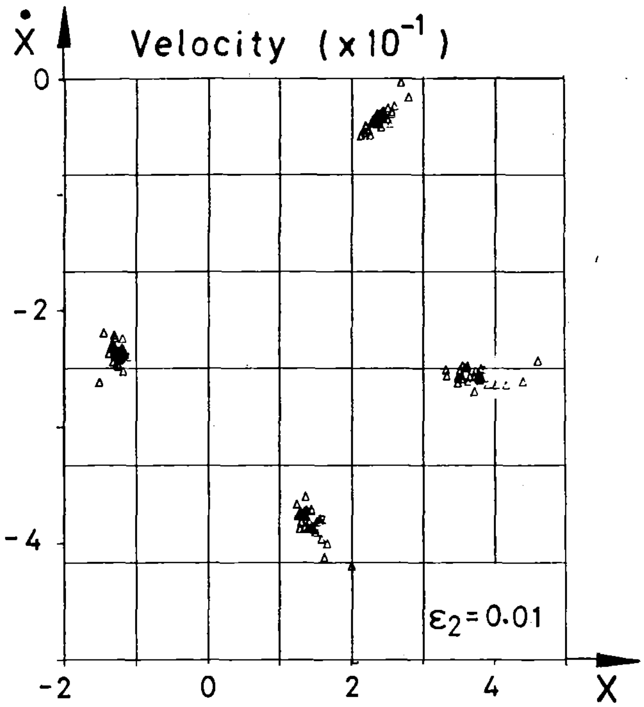
$$\dot{E} = (a_1 \cos \phi_{1-} + b_1 \cos \phi_{1+}) |f| /2 \tag{16}$$

Any signal, $x(t)$, which satisfies the equations of motion and yields a positive value for $\dot{E}$ can grow; the forcing puts no energy into sub or superharmonic motions directly. The nonlinearity couples the fundamental response to low-frequency motions, which can grow to large amplitude if the subharmonic coincides with a resonance. In other words, some of the harmonics generated when the resonant response passes through the nonlinearity are at the forcing frequency; the essence of the describing function technique is to match these harmonics according to the constraints imposed by the differential equation.

Types of Randomness

To investigate the effect of random inputs, the basic sine wave is to be perturbed in some "continuous" fashion which can be characterized by a randomness parameter, e. Three basic approaches are discussed here, only the last of which yields a realistic Gaussian signal.

1 The sine wave can be modified by additive random noise which is some fraction, e_1 of the amplitude of the oscillation. The spectrum of the exciting force is a delta function at the forcing frequency with a steady component at



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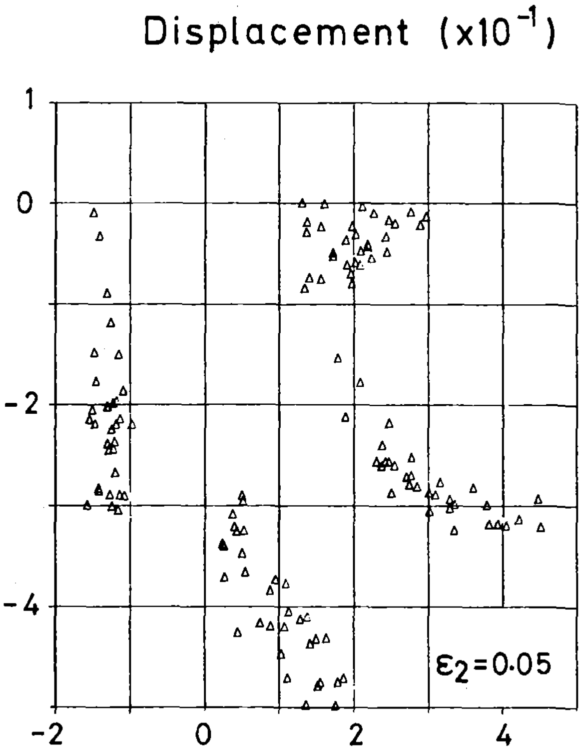

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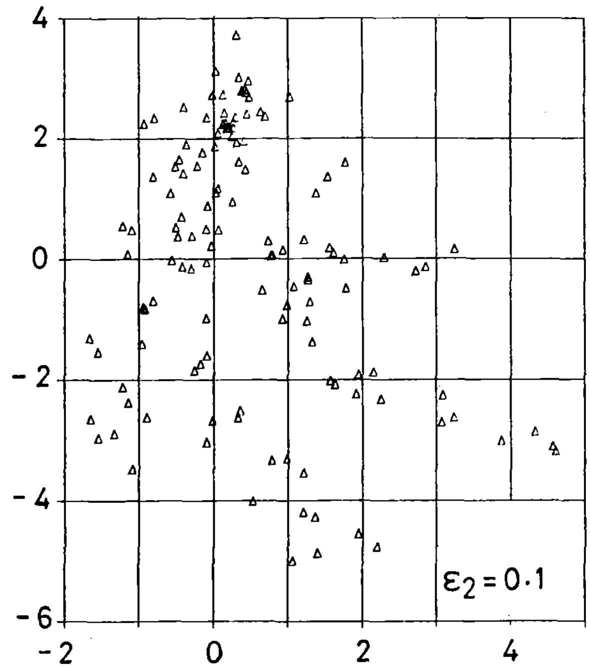


(c)

Fig. 2 Frequency wandering Poincaré points bilinear spring

all frequencies.

$$f(t) = a(\sin w_o t + e_1 w(t)) \tag{17}$$

In the simulations described in the forthcoming, the input signal for the simulation was generated at equal time steps and is perturbed by a random number of standard deviation e_1 each time

2 The frequency of the sine wave may be allowed to "walk" at random throughout a frequency band of half-width, e_2, centered on the base frequency, w_o.

$$f(t) = a \sin wt \tag{18}$$

$$w_o(1 - e_2) < w < w_o(1 + e_2) \tag{19}$$

Clearly, there is some arbitrariness in the rate at which the frequency is allowed to change; in the work reported here, a change was allowed every time the phase reached 2π. Webster and Trudell [8] discusses signal generation by this technique in some detail.

The resulting time history looks very like that of the original sine wave, while the spectrum fills a block of half-width $e_2 w_o$ around the base frequency w_o. The amplitude of the components is chosen so that the total power is independent of bandwidth e_2.

3 Alternatively, the signal can be composed of a summation of equal amplitude sinusoids lying within the bandwidth specified in the foregoing.

$$f(t) = \sum_{i=1}^{N} a_i \sin(w_i t + \phi_i) \tag{20}$$

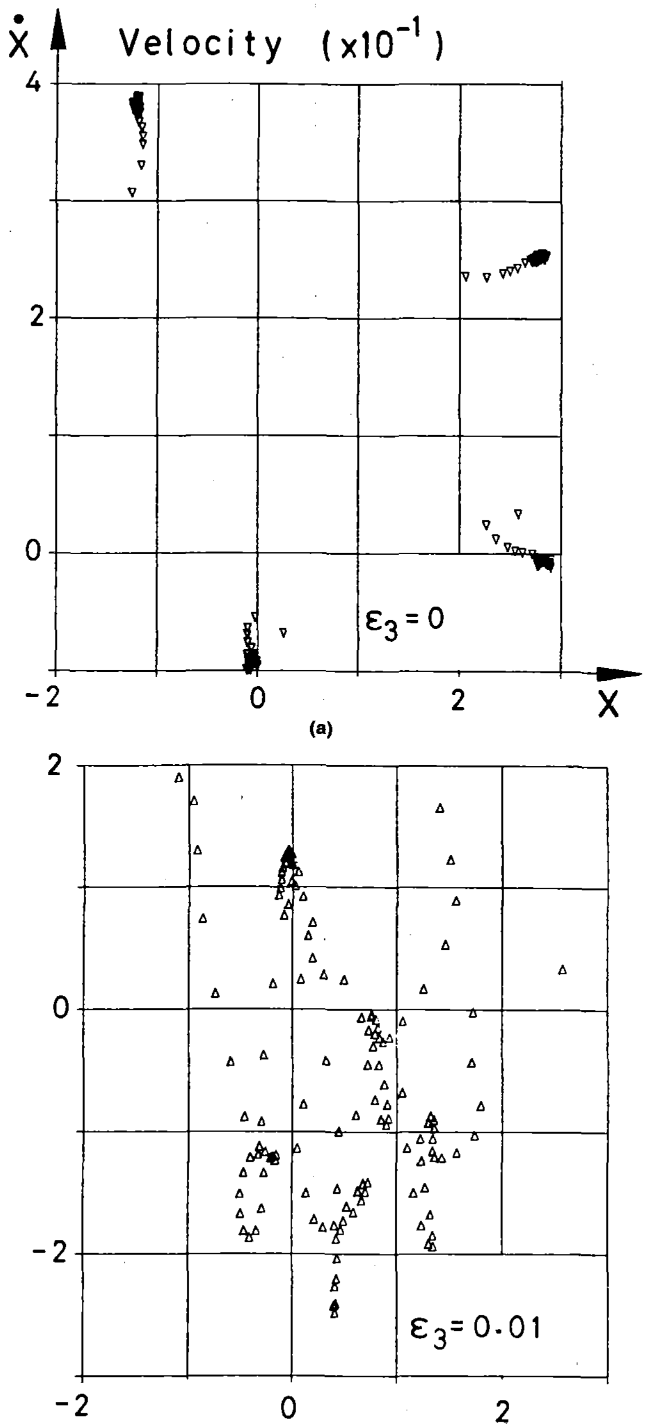
$$(1 - e_3)w_o < w_i = kDW < (1 + e_3)w_o \tag{21}$$

where

$$DW = 2\pi/T \tag{22}$$

In this work, the signal was generated by inverse Fourier transformation of the frequency domain representation; the frequency components were multiples of a base frequency, DW, chosen to yield a time series of adequate length, T. The frequencies, w_i, were chosen to lie on FFT "teeth" so that all components completed an integer number of cycles during the simulation period. The fundamental w_o was at a frequency of 16 DW and the amplitudes were selected so that the total power remained constant. Component phases may be set initially equal or may be randomized.

Although the spectra of this signal and the previous one are very similar, a nonlinear system will respond to them very differently. The response of a linear system to an input is the sum of the responses to its separate components; a nonlinear system responds to the time series "directly" and superposition is invalid. The first two signals display a very non-Gaussian probability distribution, whereas the third approaches the Gaussian distribution as the number of sinusoids in the frequency band approaches infinity. Tucker et al. [9] point out that a signal simulated by deterministically chosen amplitudes, as in this paper, displays inadequate variance of



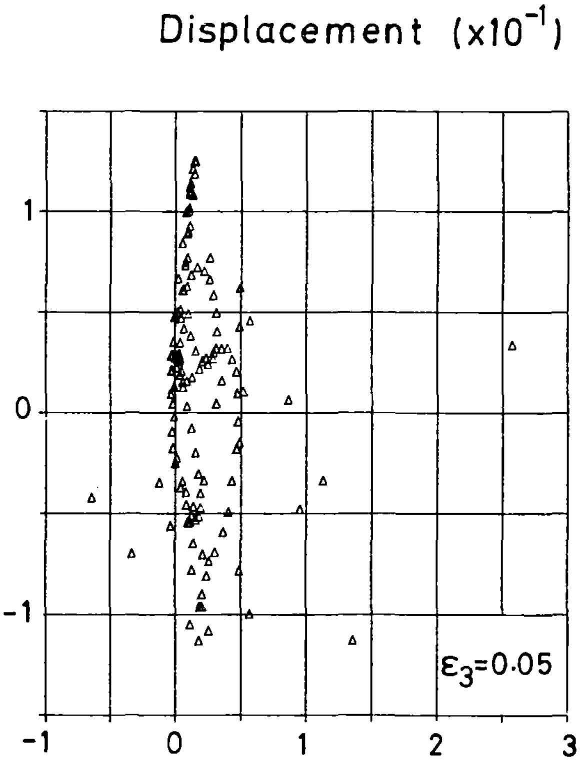


Fig. 3 Bandwidth spread Poincaré points bilinear spring

power; strictly, to obtain "correct" results, we should also randomize the component amplitudes if a properly random seaway was to be simulated by the third technique.

This third type of signal is least like a sine wave and is expected to have the greatest impact on the nonlinear phenomena.

A typical North Sea JONSWAP spectrum has most of its energy in a band of half-width 0.15, but energy outside this limit could have some impact on the behavior of the nonlinear and time-varying systems investigated here. Narrower spectra than JONSWAP (itself narrower than the Pierson-Moskowitz) can be caused by swell from a distant storm; the longest and hence fastest waves arrive first and can be fairly monochromatic. Our results are not intended to represent responses to typical spectra, but merely illustrate the fundamental physics of the problem.

In all cases, the input was generated so that there were 16 data samples per cycle of the fundamental frequency and 2048 samples in total. Thus, the fundamental executes 128 cycles during the simulation.

Digital Simulation

Previous work (reference [4]) on the bilinear and impact oscillators with monochromatic forcing has used the analytic solution to the motion of the system in the two half-planes in a numerical solution scheme; this approach is not possible with the Mathieu system and is difficult with the randomly forced bilinear oscillator. An implementation of a variable step length Adams method with interpolation to yield equally spaced output worked well on the Mathieu problem and the bilinear oscillator with small values (less than 10) of the spring ratio α. Although the spring ratio is discontinuous at $t = 0$, the spring force is itself zero there and the numerical errors introduced by this approach were insignificant; good correlations with the semi analytic approach of reference [4] resulted under harmonic forcing.

The impact oscillator (reference [4]) has also been simulated using a related routine which allows intervention during the integration; when the displacement enters the negative half-plane, the simulation is stopped and the velocity at $x = 0$ is recovered by interpolation. The simulation is then restarted with zero displacement and the opposite velocity; again, this technique correlated well with earlier results when harmonic forcing was employed and allowed stochastic simulation of the impact oscillator.

Analysis of Response

The response was analyzed in essentially the same way, whatever the nature of the forcing input.

A minor generalization of the definition of Poincaré points was necessary when the forcing frequency wandered. Under regular harmonic excitation, these "snapshots" of the state of the system are recorded at the fundamental frequency at an arbitrary point in the cycle; if a subharmonic of order n is present, they fall on n points in the position-velocity plane. To use them under frequency-wandering excitation, they must be recorded at some regular phase with respect to the forcing; here the zero upcrossing of the forcing function was used. If the randomness was introduced by bandwidth spread or additive white noise, the Poincaré point were recorded at the fundamental frequency and an arbitrary phase.

Maximum and minimum values were recorded and the mean and rms were computed.

The response signals were split into 4 equal length blocks, fast Fourier transformed and the resulting power spectra averaged to improve statistical reliability. Within each block, the fundamental executes 16 cycles, the second-order subharmonic 8 and the fourth-order subharmonic 4. Thus, subharmonics of the order 2^n, and any superharmonics fell exactly on FFT teeth, and windowing problems were eliminated.

Examples

This pilot study is limited to the bilinear and Mathieu systems with specified parameters. These nonlinear and time

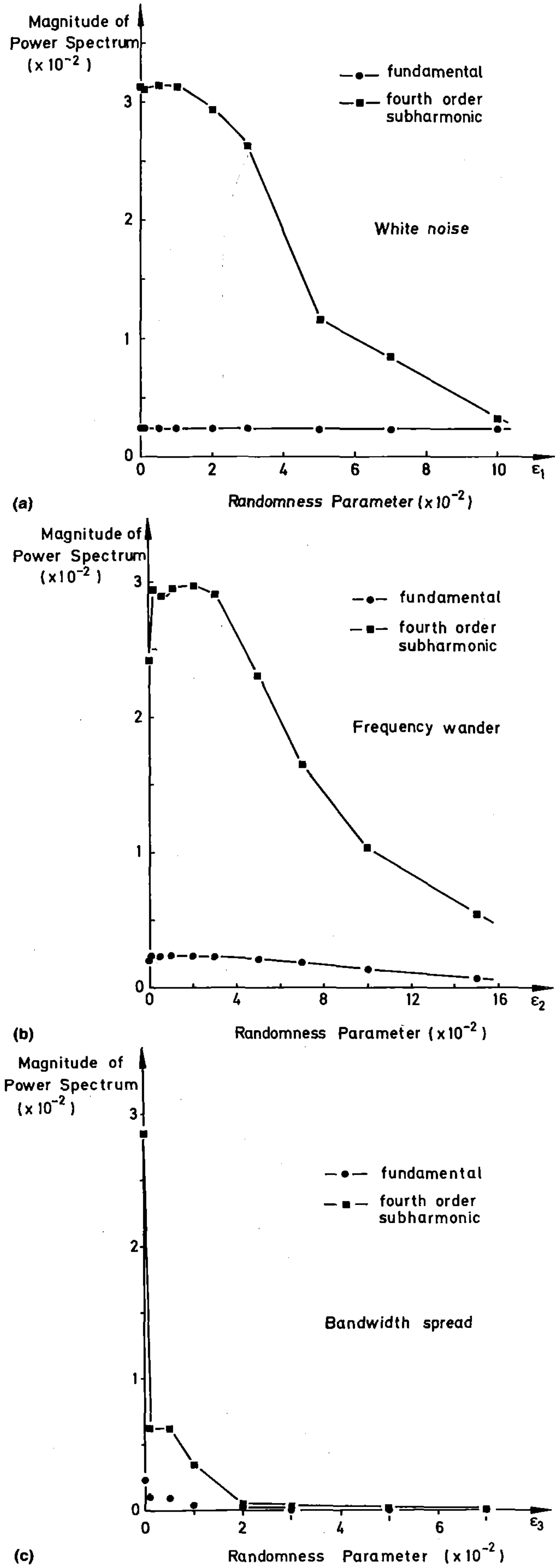


Fig. 4 Effect of white noise, frequency wander and bandwidth spread on subharmonic and fundamental

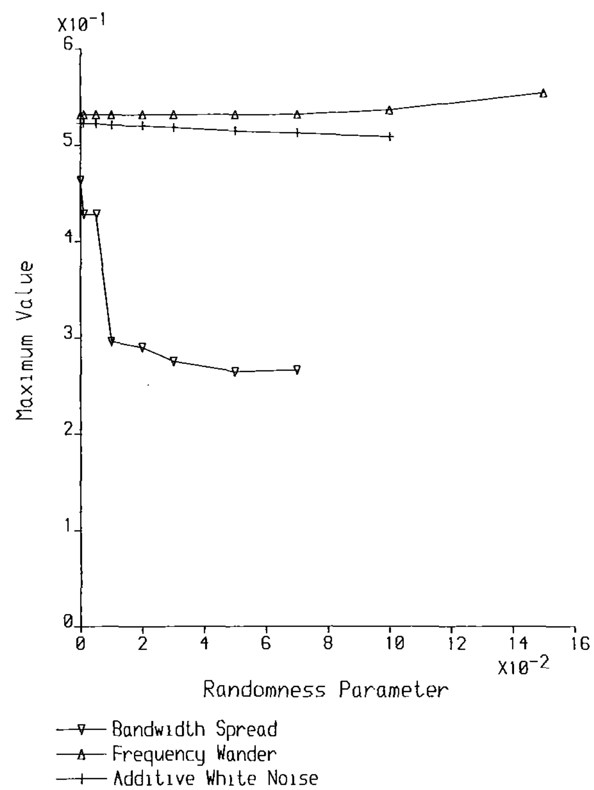


Fig. 5 Maxima for three types of randomnesses—bilinear spring

varying systems provide an illustration of the relation between the regular forcing and subharmonic responses which is valid for other parameter settings investigated but not reported here, and probably other systems.

(*a*) The Bilinear Oscillator. Thompson et al [4] have carried out an extensive analysis of the bilinear oscillator discussed in the foregoing; the response is simply scaled by the size of the forcing, but is a strong function of frequency and initial condition. This study uses parameters which led to either a strong fourth-order subharmonic or a small first-order solution, depending on the initial conditions. The damping, c, is set at 0.1, the forcing frequency, n, is 3.95 the ratio of spring stiffnesses, α is 10; the system was always started on the subharmonic response.

The forcing function $f(t)$ takes one of the three forms discussed in the foregoing; the effect of each on the behavior was investigated in depth.

Figures 1, 2, and 3 show the same information for the three different types of randomness, additive white noise, frequency wander and frequency superposition. Each plot shows the Poincaré points for different values of the randomness parameter e; note the different starting phase for the frequency-wandering case means that the four points are in different regions of the phase space. In each case, as the randomness increases, the Poincaré region expands in a characteristic fashion until they merge; at the highest value of e, there is no obvious grouping and the spectra must be consulted for evidence of fourth-order subharmonic response.

The plots of the fourth and first-order subharmonic response against e (Figs. 4(*a*), (*b*), (*c*)) show that the first harmonic amplitude is not strongly affected by randomness, while the fourth-order response is dramatically decreased by it. As expected, the most nonsinusoidal signal, the frequency superposition causes the most degradation. The rms of the signal is simply the sum of the powers of these two components since no other frequencies are significant.

The maxima are plotted against e in Fig. 5; the maxima fall more slowly than the spectral values, suggesting that while randomness decreases the rms, occasional "appropriate" sequences of inputs can still generate a particularly strong response. The time series are not long enough to make firm predictions about the long-term extreme values, something which is rather difficult for the non-Gaussian time series produced by such a nonlinear system. This is an area of current research.

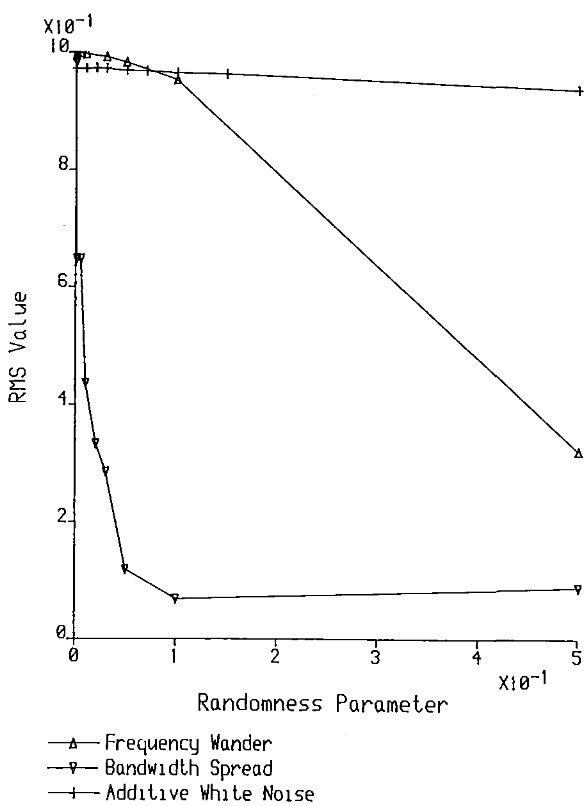


Fig. 6 Mathieu rms values

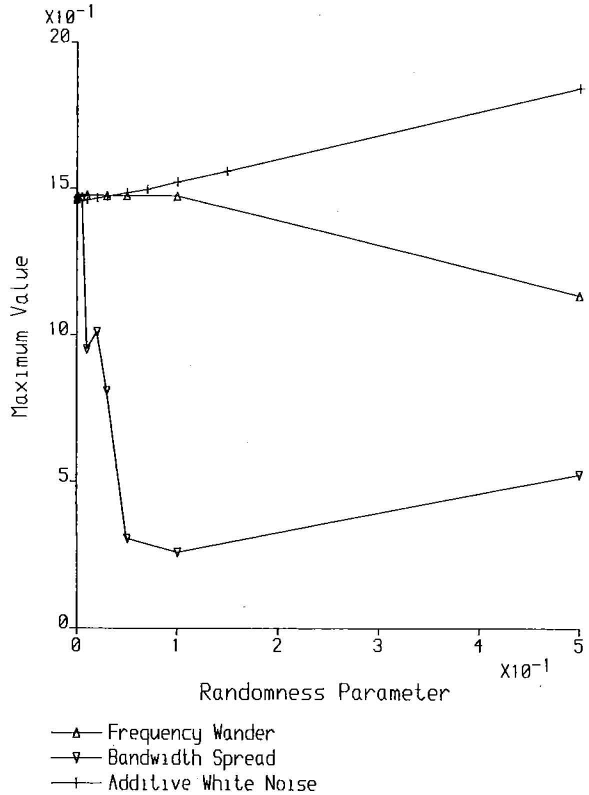


Fig. 7 Mathieu maxima

(*b*) The Mathieu Oscillator. The Mathieu oscillator was forced by a fundamental frequency at twice its natural frequency, the condition known to cause the largest response. When the Mathieu instability occurs, no linear damping can restrain its response; it is known (reference [6]) that small amounts of square law damping limits the oscillation amplitude to a finite value without significantly interfering with the mechanism of the instability, so this was included. It is, conveniently, also representative of the dissipation mechanism of many marine structures.

The square law damping ratio was set to a level of 0.1 and the strength of the Mathieu forcing, f was fixed at 0.5, a substantial value for many typical problems.

Figure 6 shows plots of rms response for a range of values of e and all types of randomness. Here, the fundamental response is not involved in the energy input and is negligible with respect to the subharmonic response at the resonant frequency. White noise and frequency wander have relatively little effect on the amplitude of oscillation; the bandwidth spread destroys the subharmonic resonance very effectively. Figure 7 shows the maxima as functions of the randomness parameters.

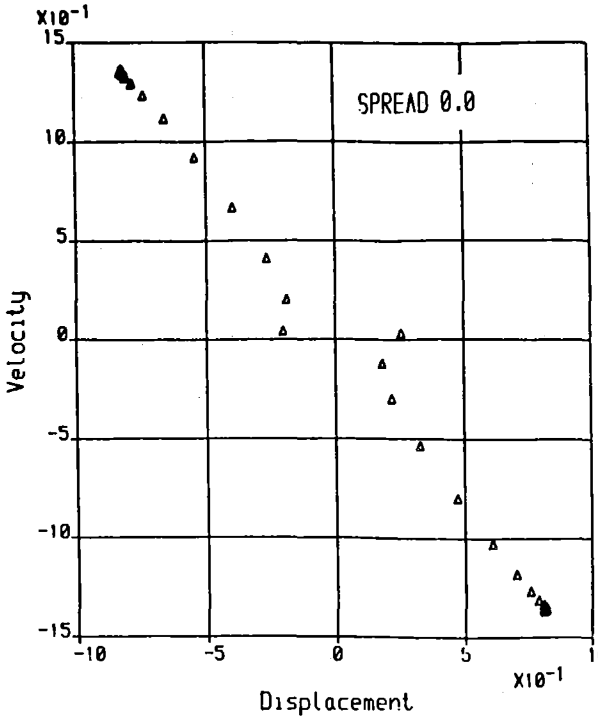
The Poincaré points display unexciting behavior, similar to that shown in Figs. 1 and 2 for the first two types of randomness; the frequency wandering plots are shown here in Fig. 8.

Conclusions

Randomness in the input to a nonlinear system can dramatically increase the amplitudes of subharmonic and chaotic responses. In some marine applications, the filtering effects of distance or vessel dynamics can generate spectra of extreme narrow bandedness, which are unexceptional in amplitude terms yet may be extremely dangerous if they excite a lightly damped resonance.

The average subharmonic response in any sea state may be small, but the extreme may not; some "appropriate" sequence of waves may excite the system through a nonlinear mechanism to generate a dangerous transient motion.

The severity of the nonlinear response is a roughly continuous function of the severity of the nonlinearity; models based on reasonable approximations to linearity will exhibit subharmonic or chaotic behavior, which is either small in amplitude terms or is weak in that an implausibly large number of regular forcing cycles are required to build the motion up to its steady amplitude.



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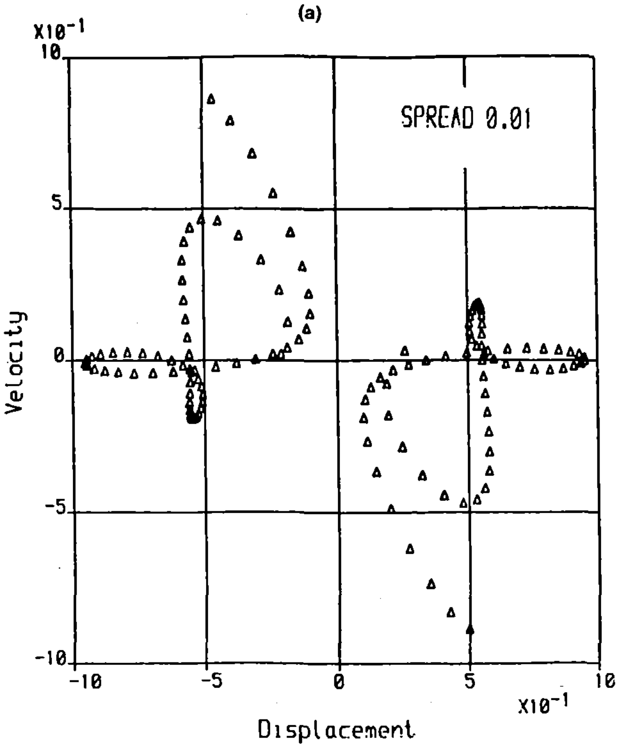

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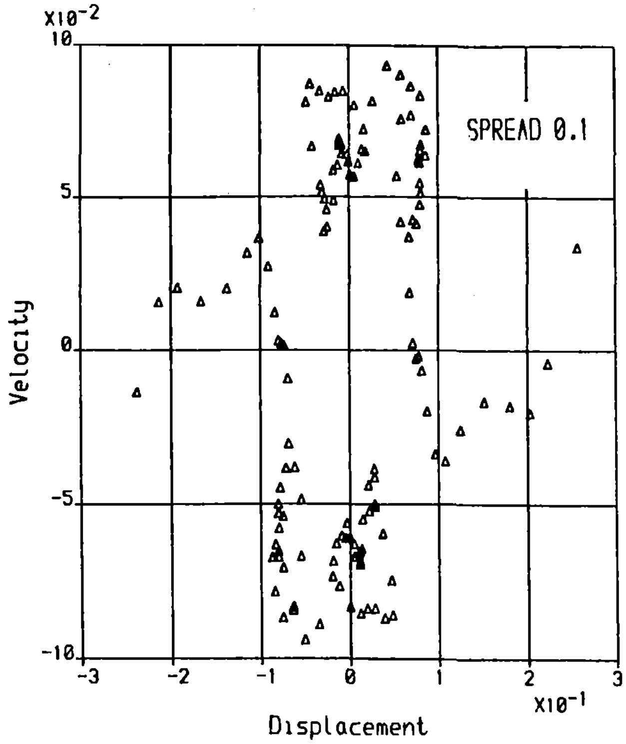

(c)

Fig. 8 Mathieu Poincaré points—bandwidth spread

The system damping is also an important parameter; while the numerical results here are restricted to one value of the damping parameter, it is clear that lightly damped systems will be most susceptible to a fast buildup of subharmonic energy caused by light nonlinear or time-varying forcing.

While this study has been limited to one representative from each of time-varying and nonlinear systems, the conclusions should be broadly applicable as they are based on the fundamental principles of energy conservation.

Acknowledgments

Pop Samra (UCL Mechanical Engineering) and Steve Bishop (UCL Civil Engineering) ran the program and plotted the results; thanks to both of them for their help. This study has been funded by the SERC Marine Technology Directorate and industrial participants in the Compliant Systems program; their support and permission to publish is appreciated.

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