# Uncertainty Evaluation of Wheeler and Reflection Methods for Radiation Efficiency Using Monte Carlo Simulation

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## **1. Introduction**

The radiation efficiency is one of the most important parameters for the antenna built in the mobile communication devices. The Wheeler and reflection methods are known as simple and low-cost method of measuring the radiation efficiency for the small antenna [1],[2]. In these methods, the reflection coefficients in free space and cavity are required. However, the magnitude of the reflection coefficient covered with the cavity is nearly equal to unity so that the uncertainty of the resulting efficiency can deteriorate. Therefore, in order to confirm the reliability of the measurement, the uncertainty of the measurement should be examined. We derived and evaluated the uncertainty expression of the measurement for the Wheeler and reflection methods. In this paper, to validate these expressions, we will evaluate the uncertainty by use of a different method, i.e., Monte Carlo simulation method.

# 2. Two Ways of Evaluating Uncertainty in Wheeler/Reflection Methods

### 2.1 Formula of Uncertainty of Wheeler/Reflection Methods

In the Wheeler method [1], the radiation efficiency of the antenna is given by

$$\eta^{\Gamma} = \frac{|\Gamma_{s}|^{2} - |\Gamma_{f}|^{2}}{1 - |\Gamma_{f}|^{2}},\tag{1}$$

where,  $|\Gamma_f|$  and  $|\Gamma_s|$  donote the magnitude of the reflection coefficients at the input port of the antenna in free space and the cavity as shown in Fig. 1. The uncertainty for  $\eta^{\Gamma}$  is given by

$$\frac{u_c(\eta^{\Gamma})}{\eta^{\Gamma}} = \sqrt{\left(p_f^{\Gamma} \frac{u(|\Gamma_f|)}{|\Gamma_f|}\right)^2 + \left(p_s^{\Gamma} \frac{u(|\Gamma_s|)}{|\Gamma_s|}\right)^2},$$
(2)

where  $u_c(y)$  denotes a combined standard uncertainty for y, and u(x) denotes a standard uncertainty for x, and  $p_f^{\Gamma}$ ,  $p_s^{\Gamma}$  are given in [3].

In the refection method [2], the radiation efficiency of the antenna is given by

$$\eta^{\text{refl}} = \frac{1}{1 - \left|\Gamma_{f}\right|^{2}} \left(r_{c} - \frac{|z_{c}|^{2}}{r_{c}}\right), \qquad (3)$$

where  $z_c$  and  $r_c$  are center and radius of a circle formed by the points  $z_i = \Gamma_{wg,i} - \Gamma_f$ ,  $i = 1, 2, \dots, N$ on the Smith chart.  $\Gamma_{wg,i}$  are the reflection coefficient at the input port of the antenna inserted into the rectangular waveguide with two sliding short as shown in Fig. 2. The uncertainty for  $\eta^{\text{refl}}$  is given by

$$u_{c}^{2}(\eta) = \left\{ \frac{2\left|\Gamma_{f}\right|(r_{c} - |z_{c}|^{2}/r_{c})\right\}^{2}}{(1 - |\Gamma_{f}|^{2})^{2}} \right\}^{2} u^{2}(|\Gamma_{f}|) + \left\{ \frac{2|z_{c}|}{r_{c}(1 - |\Gamma_{f}|^{2})} \right\}^{2} u^{2}(|z_{c}|) + \left\{ \frac{1 + |z_{c}|^{2}/(r_{c}^{2})}{1 - |\Gamma_{f}|^{2}} \right\}^{2} u^{2}(r_{c}),$$
(4)

where  $u(|z_c|)$  and  $u(r_c)$  are given in [4].

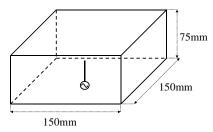
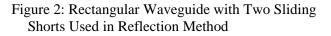


Figure 1: Rectangular Shielding Used in Wheeler Method



### 2.2 Monte Carlo Simulation for Efficiency Based on Wheeler/Reflection Methods

By generating the normal random numbers for the magnitude and phase of measured reflection coefficients, we can estimate the uncertainty of the radiation efficiency. The average and standard deviation of the normal random numbers were selected as the corresponding measured value and corresponding standard uncertainty which can be given by the spreadsheet provided by the manufacturer of a vector network analyzer. Lots of resulting efficiency can lead to an alternative evaluation of the standard uncertainty for the radiation efficiency by calculating their estimated standard derivation.

## 3. Examples of Uncertainty Evaluation

## 3.1 Uncertainty in Wheeler Method

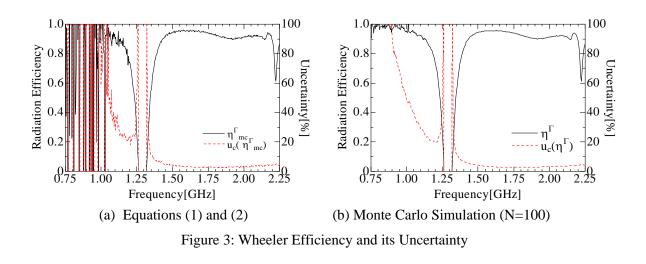
In this paper, a monopole antenna with a length of 40mm on the ground plane was used as shown in Fig. 1. Reflection coefficients were measured in a grounded half space and a metallic shielding as a function of the frequency by use of a vector network analyzer, Agilent 8720ES, with Calc/Meas power of -5dBm, IFBW of 10Hz and Average Factor of 1. The rectangular shielding has a dimension of 150mm  $\times$  150mm  $\times$  75mm as shown in Fig. 1. The antenna under test was inserted at the center on the undersurface of the cavity.

In the Wheeler method, Fig. 3 shows evaluated efficiency and corresponding standard uncertainty by use of the equations (1), (2) and the Monte Carlo simulation. In the Monte Carlo simulation, we generated 100 normal random numbers for the measured reflection coefficients. The cavity has resonant frequencies of 1.00GHz, 1.25GHz and 2.22GHz for TE<sub>101</sub>, TE<sub>102</sub> and TE<sub>103</sub> modes. Note that the estimated efficiency has dips in the neighbourhood of these frequencies. Except the above case, the standard uncertainty evaluated by the equation (2) has a good agreement with the one obtained by the Monte Carlo simulation over 1GHz. This fact confirms the validity of the equation (2) which can evaluate the uncertainty for the radiation efficiency and its standard uncertainty obtained by the equation (1), (2) are quite different from those obtained by the Monte Carlo simulation. This is because  $|\Gamma_f|$  is often larger than  $|\Gamma_s|$  in the Monte Carlo simulation as shown in Fig. 4 so that the efficiency calculated by the equation (1) can be often negative. For reference, over 1GHz, there is no case of  $|\Gamma_f| > |\Gamma_s|$ , as shown in Fig. 5.

### 3.2 Uncertainty in Reflection Method

In the reflection method, the 40mm monopole antenna was inserted along the centerline on the board wall of the rectangular waveguide, whose cross-section has a dimension of 150mm × 75mm as shown in Fig. 2. Reflection coefficients were measured in a grounded half space and a cavity formed by the waveguide with two sliding shorts. Setting parameters of the vector network analyzer were same as those in the above case of the Wheeler method. To avoid the dips of the efficiency, we excluded the measured data with  $|\Gamma_{wg,i}| < 0.9$  in finding a circle. The number of the combination of  $(l_L, l_R)$  is 64, that is,  $l_L$  and  $l_R$  ranged from 60mm to 130mm at intervals of 10mm, where  $l_L$  and  $l_R$  denote the distance from the center of the antenna to the left and right sliding shorts.

Fig. 6 shows evaluated efficiency and corresponding standard uncertainty by use of the equations (3), (4) and the Monte Carlo simulation. In the Monte Carlo simulation, we generated 100 normal random numbers for the measured reflection coefficients. Over 1.25GHz, the standard



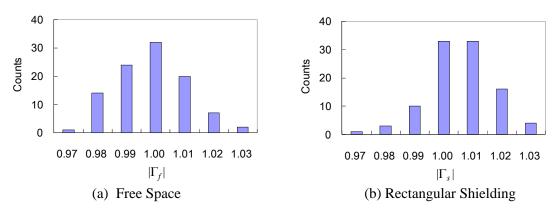


Figure 4: Frequency distribution of reflection coefficients at 0.8GHz in the Monte Carlo simulation of Wheeler efficiency.

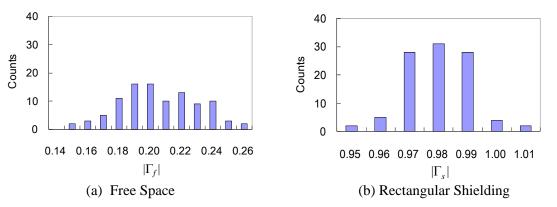


Figure 5: Frequency distribution of reflection coefficients at 1.7GHz in the Monte Carlo simulation of Wheeler efficiency.

uncertainty obtained by the equation (4) has a fairly good agreement with the one obtained by the Monte Carlo simulation. This fact confirms the validity of the equation (4) which can evaluate the uncertainty for the radiation efficiency in the reflection method that we derived in [4]. However, below 1.25GHz, the curves of efficiency and its uncertainty obtained by the equations (1), (2) are quite different from those obtained by the Monte Carlo simulation. This is because the circle required for finding the efficiency cannot be exactly drawn in the Smith chart below 1.25GHz, where the guide wavelength is too long to change  $\Gamma_{wg,i}$  or the guide wavelength is infinite so that  $\Gamma_{wg,i}$  could not vary for moving the sliding short as shown in Figs. 7 (a), (b). For reference, over 1.25GHz,  $\Gamma_{wg,i}$  can vary along the circle as the sliding short is moved as shown in Fig. 7(c).

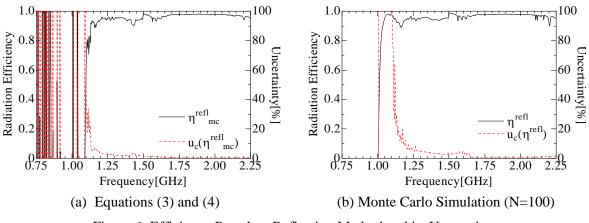


Figure 6: Efficiency Based on Reflection Method and its Uncertainty

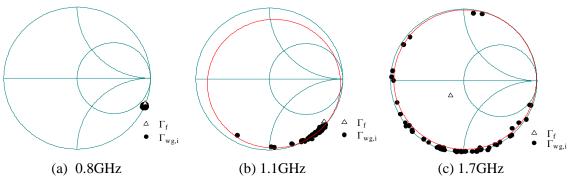


Figure 7:  $\Gamma_{wg,i}$  in the Smith chart for Monte Carlo Simulation

## **5.** Conclusion

By use of the Monte Carlo simulation, we validate the expressions of the uncertainty for the Wheeler and reflection methods that we derived. And we find that the efficiency and its uncertainty cannot be estimated by the Monte Carlo simulation in the neighborhood of the resonant frequencies and below the first resonant frequency of the cavity. Therefore, the use of the expressions for the uncertainty is still powerful way to avoid this difficulty.

## References

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