6 Conclusions

An indirect method of parameter estimation for continuous-time systems from sampled input/output data has been described. Firstly, the ARMA model of the system is identified from the sampled data. Then the order of the reduced model is determined by the dispersion analysis. Finally, the continuous model is obtained by matching the frequency responses of the discrete-time model and the continuous-time model. The proposed estimation procedures have been applied to a power system stabilizer, and satisfactory results are obtained.

Acknowledgment

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Constant Turning Force Adaptive Control Via Sliding Mode Control Design

Chih-Liang Hwang[1] and Bor-Sen Chen[2]

In the constant turning force adaptive control (CTFAC) system, the open-loop gain will vary and the stability cannot be assured when a cutting tool cuts a workpiece at various cutting depths or spindle operates in different speeds. In this paper, the spirit of sliding mode control is extended into discrete-time form to combine with parameter estimation having variable forgetting factor to stabilize the turning system against the variable gain and unmodeled dynamics, such as nonlinear perturbations, inaccurate measurements etc.

1 Introduction

The development of cheaper and reliable digital computers means that the field of adaptive control has been reactivated. The problem of CTFAC has been discussed in the literature [1–3 etc.]. Since in the turning system the cutting variables, such as depth-of-cut, spindle speed, workpiece, and tool material properties, are also the control variables, these cutting variables vary in an unknown fashion. Masory and Koren [1] suggest an integral control with variable gain inversely proportional to the estimation of open-loop gain in every sampling step to keep open-loop gain of adaptive control system to be a constant. The paper discussed by Daneshmend and Pak [2] uses a model reference adaptive control scheme to deal with the feed force in the turning system. Recently, Chang and Chen [3] have successfully developed and implemented a variable structure system controller to the constant turning force system.

The objective of sliding mode control is to drive the state trajectory to a manifold $s=0$, which is generally called a sliding surface. By a suitable choice of the sliding surface, the system can be stabilized or can track a reference input. After determining the sliding surface, we employ a Lyapunov stability requirement to the sliding surface to achieve a sliding mode control law to force the state trajectory to the sliding surface. The parameter estimation with variable forgetting factor [4] can estimate the parameters of sudden change due to a variable cutting depth or a variable spindle speed. Together with the advantageous characteristics of sliding mode control, e.g., invariance properties on the sliding mode [5], order reduction and fast convergence rate [6], this kind of adaptive controller is an easy and effective method [5, 6] to tackle the problem of CTFAC.

2 The Model and Problem Statement

The turning system is composed of a controller, servosystem, cutting process, and dynamometer. The servosystem is a simple structure of servomotor and inertial load of the feed screw presented as [1, 3]:

$$v_f = M(s)u = w_n^2 u/[s^2 + 2\xi w_n s + w_n^2], \tag{1}$$

where ξ, w_n and u are the damping ratio, the natural frequency and input signal of the servosystem, respectively, and v_f is the feedrate of the cutting tool by the servosystem, united by mm/s. Before we introduce the cutting force model, the relation between feedrate v_f and feed f (united by mm/rev) is given by

$$f = 60\, v_f/N, \tag{2}$$

where N is the rotation speed of the spindle in rpm, i.e., rev/min. The cutting process is approximately described as [1, 2, 3]:

$$y = d_c k_f f^g = (d_c k_f f^{g-1}) f, \tag{3}$$

where k_f is the specific cutting force coefficient and g ($g<1$) is a constant, both depending on the workpiece material and tool shape, d_c is the cutting depth, and y (united by Nt) is the cutting force. The conversion factor between y and y_d is k_d, i.e.

$$y_d = k_d y, \tag{4}$$

where y_d is the output of dynamometer. Note that in this paper, k_d is assumed to be 1. If $k_d \neq 1$, the system can be rearranged as unity feedback system. A reference input y_r is a constant cutting force obtained by an off-line optimum analysis constrained by machining ability, certain tool material and geometrical shape of tool. Two samples are placed after a discrete controller C and dynamometer, respectively, and h denotes a sampling interval, ZOH represents a zero order hold (see Fig. 1).

Since the parameter g in cutting process is approximately equal to 1, the nonlinear feature of cutting process is weak [1, 2, 3]. Based on a second order servosystem (1), we assume that servosystem and cutting process is a discrete-time second-

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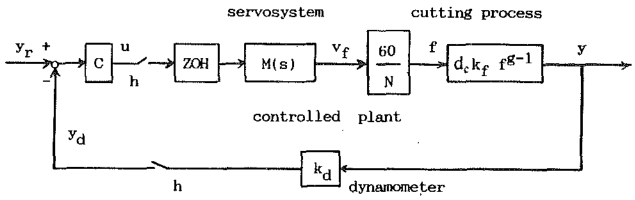

Fig. 1 Block diagram of a turning system

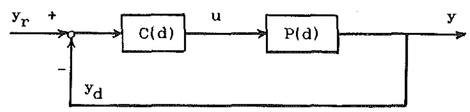

Fig. 2 An equivalent system of Fig. 1

order system (see Astrom and Wittenmark [7]) with variable parameters, i.e.,

$$P(d) = B(d)/A(d), \tag{5}$$

where $d = z^{-1}$ is a backward-time shift operator (i.e., $z^{-1}y(k) \triangleq y(k-1)$, $y(k)$ is a digital signal), $B(d) = b_1 d + b_2 d^2$ and $A(d) = 1 + a_1 d + a_2 d^2$, where a_1, a_2 are unknown plant parameters, and b_1 b_2 are parameters of sudden change. Our problem is to construct an adaptive sliding mode controller to handle a second order discrete-time system with variable parameters (see Fig. 2). This adaptive controller stabilizes the CTFAC system and tracks a constant cutting force for bounded initial condition of system without knowing the system parameters, such as d_c, N, k_f, and g etc.

3 An Introduction of Sliding Mode Control in Discrete-Time Form

In this section, we first introduce the choice of sliding surface by the pole assignment. A sliding mode control law satisfying Lyapunov stability requirement to the sliding surface is derived subsequently.

3.1 The Choice of Sliding Surface by the Pole Assignment. At first, a sliding surface using the pole assignment technique developed by Utkin and Yang [8] is extended to the linear discrete-time system.

Assume that the controlled plant is a second order discrete-time system

$$A(d)y(k) = B(d)u(k), \tag{6}$$

where $A(d) = 1 + a_1 d + a_2 d^2$, $B(d) = b_1 d + b_2 d^2$. It is assumed that $A(d)$ and $B(d)$ are coprime, and $b_1 + b_2 \neq 0$, i.e., the polynomial $(1-d)$ is not a factor of $B(d)$. Equation (6) can also be written as

$$y(k) = \phi(k-1)^T \Theta, \tag{7}$$

where $\Theta^T = [-a_1 - a_2 b_1 b_2]$, $\phi(k-1)^T = [y(k-1) \;\; y(k-2) \;\; u(k-1) \; u(k-2)]$, or in an observable-canonical state form

$$x(k+1) = Ax(k) + bu(k), \tag{8}$$

$$y(k) = c^T x(k), \tag{9}$$

with

$$A = \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \text{ and } \quad c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{10}$$

The system (8), (9) is observable no matter what $A(d)$ and $B(d)$ are coprime or not, and is controllable as well as if $A(d)$ and $B(d)$ are coprime.

Define the coordinate transformation,

$$z(k) = Mx(k), \tag{11}$$

such that $Mb = [0 \;\; q]^T$, where $q \neq 0$ and M is a nonsingular matrix; e.g., if $b_2 \neq 0$,

$$M = \begin{bmatrix} b_2 & -b_1 \\ 0 & 1 \end{bmatrix}. \tag{12}$$

Of course, this coordinate transformation is not unique.

Substituting (11) into (8) gives

$$z(k+1) = \bar{A}z(k) + \bar{b}\,u(k), \tag{13}$$

where

$$\bar{A} = MAM^{-1} =$$

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{bmatrix} = \frac{1}{b_2} \begin{bmatrix} b_1 a_2 - b_2 a_1 & b_1(b_1 a_2 - b_2 a_1) + b_2^2 \\ -a_2 & -a_2 b_1 \end{bmatrix}, \tag{14}$$

$$\bar{b} = Mb = [0 b_2]^T. \tag{15}$$

Rewrite (13) as

$$\begin{bmatrix} z_1(k+1) \\ \hline z_2(k+1) \end{bmatrix} = \begin{bmatrix} \bar{a}_{11} \; \bar{a}_{12} \\ \hline \bar{a}_{21} \; \bar{a}_{22} \end{bmatrix} \begin{bmatrix} z_1(k) \\ \hline z_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \hline b_2 \end{bmatrix} u(k). \tag{16}$$

For single-input single-output system, only one sliding surface is defined, i.e.,

$$s(k) = d^T x(k) = d_1 x_1(k) + d_2 x_2(k). \tag{17}$$

In view of (11), (16), and (17), the motion of sliding surface is governed by

$$z_1(k+1) = \bar{a}_{11} z_1(k) + \bar{a}_{12} z_2(k), \tag{18}$$

$$s(k) = h_1 z_1(k) + h_2 z_2(k) = 0, \tag{19}$$

where h_1 and h_2 are scalars satisfying the relation

$$[h_1 h_2] = d^T M^{-1}. \tag{20}$$

The subsystem (18) may be regarded as an open-loop control system with state vector z_1 and control vector z_2 being determined by (19). That is,

$$z_2(k) = -h_1 z_1(k)/h_2. \tag{21}$$

Without loss of generality, we let $h_2 = 1$. Substituting (21) into (18) gives

$$z_1(k+1) = (\bar{a}_{11} - \bar{a}_{12} h_1) z_1(k). \tag{22}$$

The initial controllability assumption on the pair (A, b), together with (15), implies that the pair $(\bar{a}_{11}, \bar{a}_{12})$ is also controllable. Then the eigenvalue of $(\bar{a}_{11} - \bar{a}_{12} h_1)$ on the sliding mode can be arbitrarily assigned by a suitable choice of the scalar h_1. With $h_2 = 1$, the coefficients of a sliding surface are obtained from (20), i.e.,

$$d^T = [h_1 \vdots 1] M. \tag{23}$$

3.2 Sliding Mode Control Law. At present time, one realizes that the initial state of the controlled plant on the

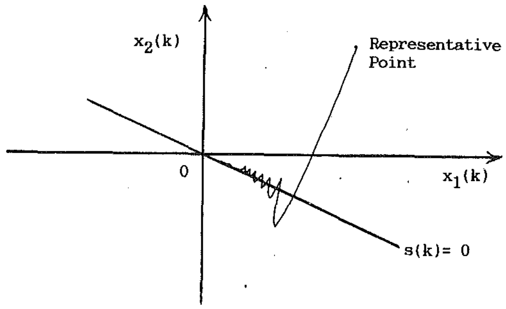

Fig. 3 Sliding mode control

sliding surface is always on it and converges to the equilibrium point by an appropriate eigenvalue assignment of (22) [5, 6, 8]. Consequently, if a control $u(k)$ can be found to guarantee that any initial state deviation from the sliding surface is eventually drived to and then maintained on the sliding surface, the system in Fig. 2 is globally asymptotically stable. Therefore, a Lyapunov function is defined as follows

$$V(k)=s(k)^2>0. \tag{24}$$

It implies that the initial state of the controlled plant (6) is not on the sliding surface. Then the rate of increase of $V(k)$ is made out in the following equation

$$\Delta V(k)=s(k+1)^2-s(k)^2. \tag{25}$$

A control resulting in $\Delta V(k)<0$ can be obtained by assuming $s(k+1)=0$; hence, from this requirement one can achieve the control law. Since

$$s(k+1)=d_1x_1(k+1)+d_2x_2(k+1), \qquad \text{from (17)}$$

$$=-(d_1a_1+d_2a_2)x_1(k)+d_1x_2(k)+(d_1b_1+d_2b_2)u(k), \text{ from (8)}$$

$$=-(d_1a_1+d_2a_2)x_1(k)+d_1x_2(k)+b_2u(k), \text{ from (15), (23), (26)}$$

the control law is accomplished as follows

$$u(k)=[(d_1a_1+d_2a_2)x_1(k)-d_1x_2(k)]/b_2. \tag{27}$$

That is, if the control law (27) is employed to the plant (6), the state $x(k)\rightarrow 0$ (or $y(k)$, $u(k)\rightarrow 0$) as $k\rightarrow\infty$(see Fig. 3, where the path of representative point denotes the state trajectory of the controlled plant). Substituting (8) and (9) into (27) achieves

$$u(k)=\{(d_1a_1+d_2a_2)y(k)-d_1[-a_2y(k-1)+b_2u(k-1)]\}/b_2. \tag{28}$$

Thus, the control law (28) is in an input-output form, one does not need to estimate the state to attain the control law. If we want the plant to track a constant reference input y_r, the control law (28) is modified as follows

$$u(k)=\{(d_1a_1+d_2a_2)y_e(k)-d_1[-a_2y_e(k-1)+b_2u_e(k-1)]\}/b_2+u_0, \tag{29}$$

where

$$y_e(k)=y(k)-y_r, \tag{30}$$

$$u_0=(1+a_1+a_2)y_r/(b_1+b_2), \tag{31}$$

$$u_e(k)=u(k)-u_0. \tag{32}$$

4 Adaptive Control for a Turning System

Because the cutting variables (e.g., cutting depth, spindle speed etc.) change, we adopt a least-squares parameter estimation with variable forgetting factor [4] to treat this CTFAC problem. A smaller value of forgetting factor gives a better responsiveness to change but with a larger steady-state variance, so that a compromise has to be sought. This is the reason for variable forgetting factor. It is assumed that the unknown plant is written in (7). The recursive least-squares parameter estimation with variable forgetting factor is given as

$$\hat{\theta}(k)=\hat{\theta}(k-1)+L(k)e(k), \qquad k\geq 1, \tag{33}$$

and

$$P(k)=[I-L(k)\phi(k-1)^T]P(k-1)/f_f(k), \tag{34}$$

where

$$L(k)=P(k-1)^T\phi(k-1)/[1+\phi(k-1)^TP(k-1)\phi(k-1)], \tag{35}$$

$$e(k)=y(k)-\phi(k-1)^T\hat{\theta}(k-1), \tag{36}$$

$$f_f(k)=1-[1-\phi(k-1)^TL(k)]e(k)^2/s_t, \tag{37}$$

$$s_t=v^2s_a, \tag{38}$$

where v^2 is the expected measurement noise variance based on real knowledge of the controlled plant and s_a will control the speed of adaptation. A smaller value of s_a will give a larger covariance matrix $P(k)$ and a more sensitive system; on the other hand, a larger value of s_a will give a less sensitive estimator and slower adaptation [4]. If a very small value of s_a is chosen, it may result in a very small value or even negative value of $f_f(k)$ (i.e., covariance matrix will "blow up"); so, we set

$$f_f(k)=f_{\min}, \qquad \text{if } f_f(k)\leq f_{\min}, \tag{39}$$

to prevent an unstable control. The selection of $f_{\min}$ is also important; too high a value reduces the speed of adaptation and too low a value may give the stability problem. However, the simulations of the next section have indicated that the selection of s_t is not a critical thing. If $f_f(k)=1$, it reduces to the standard least-squares parameter estimation. Note that circumflex (^) in this paper denotes estimation or computation by estimation.

4.1 Adaptive Control Algorithm. From the above analysis, the adaptive sliding mode control algorithm for a constant turning force system is delineated as follows:

step 1: Update $\hat{A}(d, k)$ and $\hat{B}(d, k)$ using (7) and (33)-(39).

step 2: Calculate $\hat{\hat{A}}(k)$ from (14), then compute $\hat{h}_1(k)$ from (22) for the assigned eigenvalue q, and finally achieve $\hat{d}(k)$ from (23).

step 3: Then the adaptive sliding mode control law for constant turning force system is attained from (29).

The steps 1-3 are repeated in every adaptive step.

Remark 1: In step 2, if $|\hat{b}_2|\leq\epsilon$, where ϵ is a very small positive constant, one can change the coordinate transformation, such as

$$\hat{M}=\begin{bmatrix}\hat{b}_2 & -\hat{b}_1\\ 0 & 1\end{bmatrix} \text{(because } \hat{b}_1,\ \hat{b}_2 \text{ can not equal zero simultaneously),}$$

to avoid a singular control law in (27).

Remark 2: Although the assumption $B(1)\neq 0$, i.e., $b_1+b_2\neq 0$, is made, the freezing technique [9] can be applied to keep safe from a very large control input; e.g., if $|\hat{A}(1)/\hat{B}(1)|\geq L$, where L is a suitable positive constant, then $u(k)=u(k-1)$.

The following two simulation examples are given to il-

Fig. 4(a) Cutting force $y(k)$

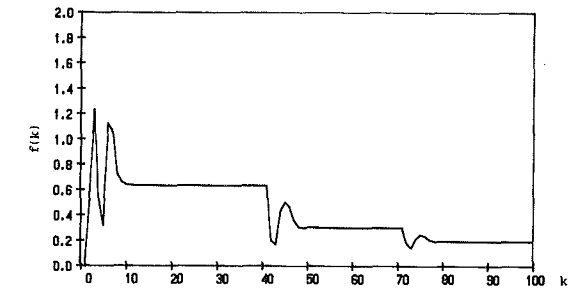

Fig. 4(b) Feed $f(k)$

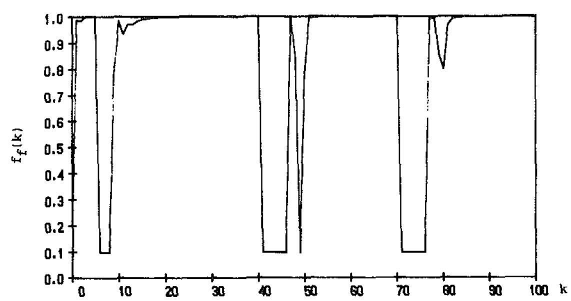

Fig. 4(c) Forgetting factor $f_f(k)$

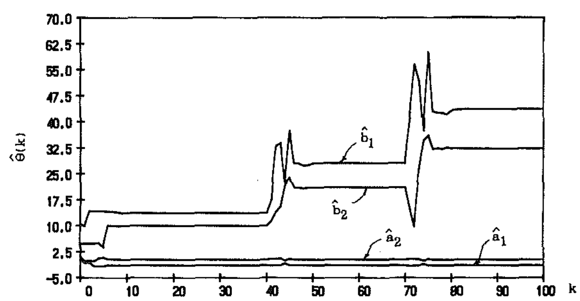

Fig. 4(d) Estimated parameters $\hat{\theta}(k)$

Fig. 4 The responses of Example 1

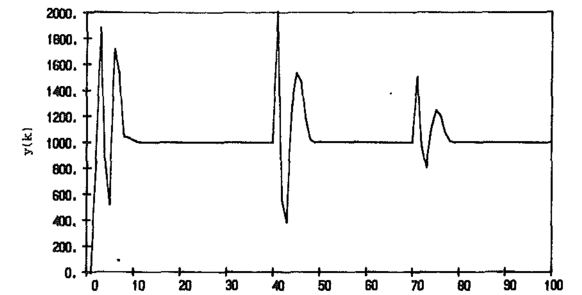

Fig. 5 The cutting force response for $s_t = 200$

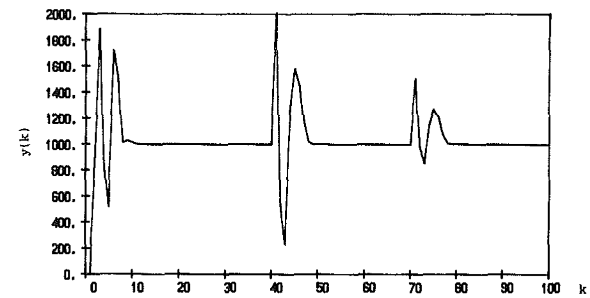

Fig. 6 The cutting force response for $s_t = 0.5$

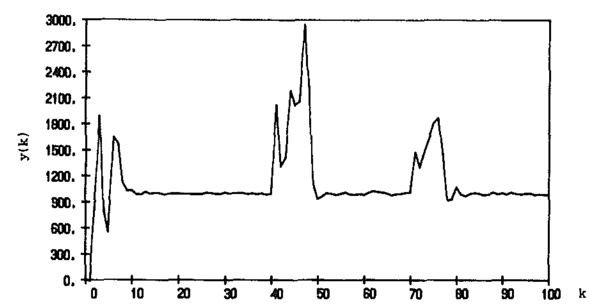

Fig. 7 The cutting force response for inaccurate measurement

lustrate the usefulness of adaptive sliding mode control algorithm to the problem of CTFAC.

5 Simulations and Discussions

5.1 Simulations.

Example 1: A turning system described by Fig. 1 is with $\xi = 0.7$, $w_n = 60$ rad/s, $N = 900$ rpm, $g = 0.95$, $k_f = 1533.3$ Nt rev/mm^2 and $h = 0.01$ s. The cutting depth $d_c = 1$ mm before $k = 40$, $d_c = 2$ mm between $k = 40$ and $k = 70$, $d_c = 3$ mm after $k = 70$. The initial values of estimated parameters are $\hat{a}_1(0) = -0.5$, $\hat{a}_2(0) = 0$, $\hat{b}_1(0) = 10$, $\hat{b}_2(0) = 5$, and the initial values of controlled plant are equal to zero, i.e., $y(-1) = y(-2) = u(-1) = u(-2) = 0$. By applying the adaptive control algorithm in last section with $P(0) = 1000\ I$, $f_{\min} = 0.1$, $q = 0$ and $s_t = 10$, the simulation results are shown in Fig. 4.

Example 2:

Similarly, without changing the above conditions in Example 1 except replacing $s_t = 10$ by $s_t = 200$ and $s_t = 0.5$, the cutting force responses are presented in Figs. 5 and 6, respectively. Consider the Example 1 subject to one percent random inaccurate measurement, i.e., $y(k)$ in the control law (29) is replaced by $y(k)\ [1 + Rn]$, where Rn is a random number with $|Rn| \leq 0.01$. With the above conditions in Example 1, the cutting force response is shown in Fig. 7.

5.2 Discussions

(i) The simulation results show that the proposed adaptive controller can quickly converge the cutting force to the reference input, $y_r = 1000$ Nt, after the cutting depth is changed.

(ii) From Example 2, one realizes that a suitable choice of s_t is not a very difficult task.

(iii) The cutting force response of the plant subject to inaccurate measurement has indicated that the first peak after $k = 40$ and $k = 70$ are caused by the change of cutting depth, and the higher peak are caused by larger inaccurate measurement of cutting force during the change of cutting depth.

(iv) In these simulations, the cutting depth is changed abruptly, i.e., only in one step. In reality, the change rate of cutting depth may be not as fast as these cases; hence, the performances are assumed to be better than these examples.

6 Conclusions

In order to ensure maximum productivity, to eliminate tool breakage, and to stabilize the plant, the adaptive sliding mode control algorithm is proposed to treat the problem of CTFAC. This novel adaptive control algorithm preserves profitable features of sliding mode control, e.g., invariance properties on the sliding mode, order reduction, and fast convergence rate. Because the nonlinear characteristic of cutting process are not very strong, the suggested adaptive controller including a variable forgetting factor can achieve a satisfied performance. The implementation of this constant turning force adaptive control is progressing in our next studies.

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