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ON THE TRANSVERSAL VIBRATIONS OF AN AXIALLY MOVING CONTINUUM WITH A TIME-VARYING VELOCITY: TRANSIENT FROM STRING TO BEAM BEHAVIOR

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ABSTRACT

In this paper an initial-boundary value problem for a linear equation describing an axially moving stretched beam will be considered. The velocity of the beam is assumed to be time-varying. since the order of magnitude of the bending stiffness terms depends on the vibrations modes and the frequencies involved a that combination of two simplified models (a string equation and a beam with string effect equation) will be used to describe the transversal vibrations of the system accurately. Based on the calculations of the natural frequencies the regions of applicability of these models will be determined. A two time-scales perturbation method will be used to construct formal asymptotic approximations of the solutions. It will be shown that the linear axially moving "string to beam" model already has complicated dynamical behavior.

NOMENCLATURE

$U(X,T)$ the displacement of the string in vertical direction,
$V(T)$ the time-varying belt speed,
c the wave speed,
X the coordinate in horizontal direction,
E the modulus of elasticity,
I the moment of inertia with the respect to the X (horizontal) axis,
ρ the mass density of the belt,
A the area of the cross section of the belt,
T the time, and
πL the distance between the pulleys.

INTRODUCTION

Axially moving systems are present in a vast class of engineering problems which arise in industrial, civil, aerospatial, mechanical, electronic, medical, and automotive applications. Serpentine belts, aerial cables, tram and train ways, oil pipelines, magnetic tapes, power transmission belts, band saw blades, chair lifts in skiing resorts, and even models of human DNA are examples of real objects where axial transport of mass can be associated with transverse vibrations. Investigating transverse vibrations of a belt system is a challenging subject which has been studied for many years by many researches and still is of interest today (see references for a recent overview). In the classical analysis of axially moving continua the vibrations are usually classified into two types, i.e. whether it is of a string-like type or of a beam-like type, depending on the bending stiffness of the belt. If the bending stiffness can be neglected then the system is classified as string (wave)-like, otherwise it is classified as beam-like. The transverse vibrations of a belt system (with time-varying velocity $V(t)$) can be modelled mathematically as:

string-like by

$$U_{TT}+2VU_{XT}+V_TU_X+(V^2-c^2)U_{XX}=0, \text{ and} \tag{1}$$

beam-like (with a string effect) by

$$U_{TT}+2VU_{XT}+V_TU_X+(V^2-c^2)U_{XX}+\frac{EI}{\rho A}U_{XXXX}=0, \tag{2}$$

where $c=\sqrt{\frac{T_0}{\rho}}$, in which T_0 is assumed to be the constant tension of the belt. The time-varying belt velocity $V(T)$ is given by $V(T)=\varepsilon(\overline{V}_0+\overline{\alpha}\sin(\overline{\omega}T))$, where $\overline{V}_0$, $\overline{\omega}$ and $\overline{\alpha}$ are some positive constants with $\overline{V}_0>0$ and $\overline{V}_0>|\overline{\alpha}|$, and where ε is a small parameter with $0<\varepsilon\ll 1$. The term $\varepsilon\overline{\alpha}\sin(\overline{\omega}T)$ can be seen as a small perturbation of the main belt speed $\overline{V}_0$, due to different kinds of imperfections of the belt system. The small parameter ε indicates that the belt speed $V(T)$ is small compared to the wave speed c. The condition $\overline{V}_0>|\overline{\alpha}|$ guarantees that the belt always moves forward in one direction.

Due to different kind of imperfections of the belt system such as roll eccentricities and varying belt speed, severe transversal vibrations (due to internal resonances) can occur. The occurrence of resonances should be prevented since they can cause operational and maintenance problems including excessive wear of the belt and the support components, and an increase of energy consumption of belt system. By knowing the natural frequencies of the belt, so called resonance-free belt system can be designed. Although the non-linear models can be more informative, and describe the real conveyer belt systems usually better, it is not meaningless to investigate the linear equations (1) and (2) first.

Equation (1) was studied by G. Seweken and van Horssen in [1]. It was found that there are infinitely many values of ω giving rise to internal resonances in the belt system. It was also shown that the truncation method can not be applied to obtain asymptotic results on long time-scales (that is, on time-scales of order ε^{-1}). On the other hand it was also shown in [2] that for the beam equation (2) the truncation method can be applied, but the dynamic behavior of the belt system is still very complicated. The stability conditions for the belt system were also derived in [2]. From experiments it is known that real dynamic behavior of conveyer belt systems with relatively small bending stiffness is some sort of combination of both models (1)-(2). The first vibration modes look more like string modes and higher order modes (when the bending stiffness becomes more important) look more like beam modes. It is not only interesting but also important from the applicational point of view to investigate such phenomena of transients "from string to beam" behavior. In recent papers [3]- [5] the following attempts to describe these phenomena can be found. When the belt speed is high and has the same order of magnitude as the wave speed c (that is $V(T)=\overline{V}_0+\varepsilon\overline{\alpha}\sin(\overline{\omega}T)$) H. R. Öz, etc. in [3] studied the case for which the bending stiffness is of order ε, and found an approximate analytical expression for the natural frequency and stability regions. E. Özkaya, etc. in [4] used the same assumptions and constructed boundary layer solutions. Approximations of the eigenvalues of the belt system were also presented in [5] by L. Kong and R. G. Parker. All these authors found that the natural frequencies change due to presence of a small bending stiffness, but missed the fact that for the higher order modes the bending stiffness terms are not of order ε anymore, and must be included in the $O(1)$-problem. Moreover, the natural frequencies of the beam model (2) with $V(T)=\overline{V}_0+\varepsilon\overline{\alpha}\sin(\overline{\omega}T)$ can not be found exactly (see for instance [8] and [9]). In this paper for simplicity it will be assumed that $V(t)=\varepsilon(V_0+\alpha\sin(\omega t))$. The idea how and when in this case different simplified models may be applied to construct a more realistic model of the traveling belt system was proposed by I. V. Andrianov and W. T. van Horssen in [10]. Usually it is not possible to calculate the natural frequencies of a real belt system exactly. The bending stiffness, however, is not important for the lower modes of vibrations. And for the higher modes of vibration the bending stiffness terms become more important than the string terms. So, there are at least three simplified models depending on the vibration modes and the corresponding frequencies: a string model for the lower frequencies, a beam-string model for the intermediate frequencies, and a pure beam model for the higher frequencies. A combination of these models can improve the results of the existing models and methods. The proposed method is based on calculating the natural frequencies of each sub-model, and determining the relative errors in it. In this way one can define intervals of applicability of these simplified models with a predefined, desired accuracy.

The paper is organized as follows. In section 2 formulation of the problem will be given. The reregions of applicability of simplified models will be determined. In section 3 the two time-scales perturbation method will be applied in order to avoid secular term in the approximate solution of the problem. Values of ω that give rise to internal resonances will be found. The resonant case and the resonant case will be studied. Finally section 4 some conclusions and remarks will be drawn.

FORMULATION OF THE PROBLEM

In this paper a new method will be proposed to construct a mechanical model for an axially moving continuum, which includes both string type and beam type dynamic behavior. The simplest mechanical model for a traveling belt is a simply supported tensioned Euler-Bernoulli beam (see Figure 1). The equation for this model is given by (see also (2))

$$U_{TT}+2VU_{XT}+V_TU_X+(V^2-c^2)U_{XX}+\frac{EI}{\rho A}U_{XXXX}=0. \tag{3}$$

The speed of the belt is assumed to be time-varying and given by $V(T)=\varepsilon(\overline{V}_0+\overline{\alpha}\sin(\overline{\omega}T))$. The boundary conditions and the initial conditions for (3) are given by

$$U(0,T;\varepsilon) = U(\pi L,T;\varepsilon) = U_{XX}(0,T;\varepsilon) = U_{XX}(\pi L,T;\varepsilon) = 0,\ T \geqslant 0,$$
$$U(X,0;\varepsilon) = f(X),\ \text{and}\ U_T(X,0;\varepsilon) = r(X),\ 0 < X < \pi L, \tag{4}$$

where $f(X)$ represents the initial displacement of the belt, $r(X)$ is the initial velocity of the belt, and where πL is the distance between the pulleys. For simplicity it is assumed that that the cross section of the belt has a rectangular shape, so that $A = hb$ and $I = \frac{bh^3}{12}$, where h is the thickness and b is width of the belt cross-section, respectively (see Figure 2). Following the 3D theory of elasticity additional conditions have to be imposed to the stretched beam equation (3), that is: $\frac{\pi L}{k} \gg h$ and $b \gg h$.

Equation (3) in non-dimensional form becomes:

$$u_{tt} - u_{xx} + \mu u_{xxxx} =$$
$$\varepsilon(-\alpha\omega\cos(\omega t)u_x - 2(V_0 + \alpha\sin(\omega t))u_{xt}) - \varepsilon^2(V_0 + \alpha\sin(\omega t))^2 u_{xx}, \tag{5}$$

where $x = \frac{X}{L}$, $V_0 = \frac{\overline{V}_0}{c}$, $t = \frac{c}{L}T$, $u = \frac{U}{L}$, $\omega = \frac{L}{c}\overline{\omega}$, $\alpha = \frac{\overline{\alpha}}{c}$ and $\mu = \frac{EI}{\rho A c^2 L^2} = \frac{Eh^2}{12\rho c^2 L^2}$. The boundary conditions and the initial conditions for (5) are given by:

$$u(0,t;\varepsilon) = u(\pi,t;\varepsilon) = u_{xx}(0,t;\varepsilon) = u_{xx}(\pi,t;\varepsilon) = 0,\ t \geqslant 0, \tag{6}$$
$$u(x,0;\varepsilon) = \frac{f(x)}{L},\ \text{and}\ u_t(x,0;\varepsilon) = \frac{r(x)}{c},\ 0 < x < \pi. \tag{7}$$

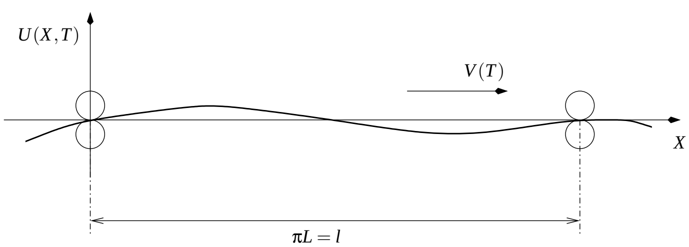

Figure 1. THE TRAVELING BELT SYSTEM.

As it was explained in the introduction, the natural frequencies can usually not be calculated exactly for real problems of traveling belts due to the presence of different sorts of (non-)linearities, such as variable stiffness etc. It is not meaningless to include only string behavior for the lower vibration modes of the belt (when the influence of bending stiffness is very small and can be neglected in the $O(1)$-problem) and the following approach shows how one can define the regions of applicability. Let us first consider the equation:

$$u_{tt} - u_{xx} + \mu u_{xxxx} = 0, \tag{8}$$

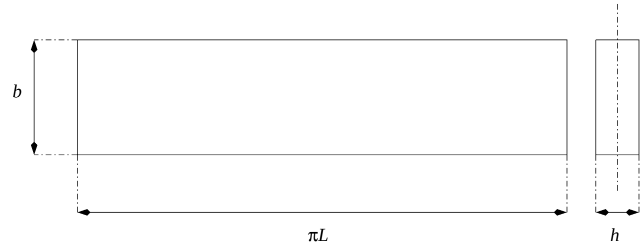

Figure 2. THE TRAVELING BELT CONFIGURATION.

subjected to the boundary conditions (6). To determine the natural frequencies of this problem the method of separation of variables can be used, giving as non-trivial solutions

$$\text{for}\ k = 1,2,3,\ldots:\ e^{i\Omega_k t}\sin(kx), \tag{9}$$

where $i = \sqrt{-1}$, and

$$\Omega_k = k\sqrt{1 + \mu k^2}. \tag{10}$$

Equation (10) gives us exact natural frequencies for equation (8) subjected to the boundary conditions (6). For the string model (i.e. equation (8) without bending stiffness) and for the beam model (i.e. equation (8) without string effect) the natural frequencies also can be found, so that:

$$\begin{aligned} \Omega_k^{(1)} &= k && \text{for the string model,} \\ \Omega_k^{(2)} &= k\sqrt{1+\mu k^2} && \text{for the stretched beam model, and} \\ \Omega_k^{(3)} &= k\sqrt{\mu} && \text{for the beam model.} \end{aligned} \tag{11}$$

It is possible now to find intervals of applicability of these simplified models (for k), based on the natural frequencies (11), with a desired or required accuracy. In table 1 these regions for k are given for μ equal to 0.0001, 0.002, 0.01, 0.1, 1, and 10, and with relative errors of 0,1 %, 1 %, 3 %, and 5 % in the frequencies respectively.

Let us consider a real moving belt, fabricated from rubber, with the following mechanical properties: $E = 1,8\,GPa$, $\rho = 1,5\,g/cm^3$, $h = 0,8\,cm$, $l = 100\,m$, $T_0 = 5\,N/mm$ and $b < 3000\,mm$. This implies that $\mu = 0.002$. From Table 1 it can be seen, that with a relative error of 5 % in the frequencies the following model can be derived (the original problem (5) can now be split by assuming $u(x,t) = \Sigma_{k=1}^{\infty} u_k(t)\sin(kt)$):

For $1 \leqslant k \leqslant 7$ - *string model:*

$$u_{tt} - u_{xx} = \varepsilon(-\alpha\omega\cos(\omega t)u_x - 2(V_0 + \alpha\sin(\omega t))u_{xt} - c_1 u_{xxxx})$$
$$-\varepsilon^2(V_0 + \alpha\sin(\omega t))^2 u_{xx}, \tag{12}$$

μ	**0.0001**	**0.002**	**0.01**	**0.1**	**1**	**10**
rel error; model						
string	$1 \leqslant k \leqslant 4$	$k = 1$	—	—	—	—
0,1%; string-beam	$5 \leqslant k \leqslant 2234$	$2 \leqslant k \leqslant 691$	$1 \leqslant k \leqslant 223$	$1 \leqslant k \leqslant 70$	$1 \leqslant k \leqslant 22$	$1 \leqslant k \leqslant 7$
beam	$2235 \leqslant k < \infty$	$692 \leqslant k < \infty$	$224 \leqslant k < \infty$	$71 \leqslant k < \infty$	$23 \leqslant k < \infty$	$8 \leqslant k < \infty$
string	$1 \leqslant k \leqslant 14$	$1 \leqslant k \leqslant 3$	$k = 1$	—	—	—
1%; string-beam	$15 \leqslant k \leqslant 702$	$4 \leqslant k \leqslant 161$	$2 \leqslant k \leqslant 70$	$1 \leqslant k \leqslant 23$	$1 \leqslant k \leqslant 7$	$1 \leqslant k \leqslant 2$
beam	$703 \leqslant k < \infty$	$162 \leqslant k < \infty$	$71 \leqslant k < \infty$	$24 \leqslant k < \infty$	$8 \leqslant k < \infty$	$3 \leqslant k < \infty$
string	$1 \leqslant k \leqslant 25$	$1 \leqslant k \leqslant 5$	$1 \leqslant \text{k} \leqslant 2$	—	—	—
3%; string-beam	$26 \leqslant k \leqslant 399$	$6 \leqslant k \leqslant 89$	$3 \leqslant k \leqslant 39$	$1 \leqslant k \leqslant 12$	$1 \leqslant k \leqslant 3$	$k = 1$
beam	$394 \leqslant k < \infty$	$90 \leqslant k < \infty$	$40 \leqslant k < \infty$	$13 \leqslant k < \infty$	$4 \leqslant k < \infty$	$2 \leqslant k < \infty$
string	$1 \leqslant k \leqslant 32$	$1 \leqslant k \leqslant 7$	$1 \leqslant \text{k} \leqslant 3$	$k = 1$	—	—
5%; string-beam	$33 \leqslant k \leqslant 304$	$8 \leqslant k \leqslant 68$	$4 \leqslant k \leqslant 30$	$2 \leqslant k \leqslant 9$	$1 \leqslant k \leqslant 3$	—
beam	$305 \leqslant k < \infty$	$69 \leqslant k < \infty$	$31 \leqslant k < \infty$	$10 \leqslant k < \infty$	$4 \leqslant k < \infty$	$1 \leqslant k < \infty$

Table 1. APPLICABILITY REGIONS FOR THE SIMPLIFIED MODELS: STRING, BEAM WITH STRING EFFECT, AND BEAM EQUATIONS.

where it is assumed in (12) that $\mu u_{xxxx} = O(\varepsilon)$, so that $\mu u_{xxxx} = \varepsilon c_1 u_{xxxx}$.

For $8 \leqslant k \leqslant 68$ - *beam with string effect model:*

$$\begin{aligned} &u_{tt} - u_{xx} + \mu u_{xxxx} = \varepsilon(-\alpha\omega\cos(\omega t)u_x - 2(V_0 + \alpha\sin(\omega t))u_{xt}) \\ &-\varepsilon^2(V_0 + \alpha\sin(\omega t))^2 u_{xx}, \end{aligned} \tag{13}$$

where it should be observed that the terms in the left hand side of (13) are of leading order and of the same order of magnitude.

For $69 \leqslant k < \infty$ - *beam model:*

$$\begin{aligned} &u_{tt} + \mu u_{xxxx} = \varepsilon(-\alpha\omega\cos(\omega t)u_x - 2(V_0 + \alpha\sin(\omega t))u_{xt} + \frac{1}{\varepsilon}u_{xx}) \\ &-\varepsilon^2(V_0 + \alpha\sin(\omega t))^2 u_{xx}, \end{aligned} \tag{14}$$

where it should be observed that the terms in the left-hand side of (14) are at least an order of magnitude larger than those terms in the right-hand side of (14).

As it was shown in [1] the truncation method can not be applied to the string equation (12), but for the beam with string effect equation (13) the method can be applied (see [2]) when the internal resonances are taken into account. Based on this observation it is assumed now for simplicity, that the original problem (5) is split up into 2 simple models: for $1 \leqslant k \leqslant 7$ - the string model (12), and for $8 \leqslant k < \infty$ - the beam with string effect model (13). It was shown in [2] that by substituting $u(x,t) = \sum_{n=1}^{\infty} u_n(t;\varepsilon)\sin(nx)$ into (5), by multiplying both sides of so-obtained equation with $\sin(kx)$, and then by integrating with respect to x from $x = 0$ to $x = \pi$ it follows that:

$$\begin{aligned} &\ddot{u}_k + (\mu k^4 + k^2)u_k = \\ &\varepsilon \sum_{n=1}^{\infty\,*} \frac{kn}{(n^2-k^2)\pi}\left(4\alpha\omega\cos(\omega t)u_n + 8(V_0 + \alpha\sin(\omega t))\dot{u}_n\right) + O(\varepsilon^2), \end{aligned} \tag{15}$$

where the * in $\sum_{n=1}^{\infty\,*}$ indicates that the summation is only carried out for $n \pm k$ is odd. For $t = 0$ $u_k(t)$ satisfies: $u_k(0;\varepsilon) = \frac{2}{L\pi}\int_0^{\pi} f(x)\sin(kx)dx$, and $\dot{u}_k(0;\varepsilon) = \frac{2}{c\pi}\int_0^{\pi} r(x)\sin(kx)dx$. In the next section a two time-scales perturbation will be applied.

APPLICATION OF THE TWO TIME-SCALES PERTURBATION METHOD

To avoid secular terms in the approximate solution of (15) a two time-scales perturbation method is used. The two new time scales are $t_0 = t$ and $t_1 = \varepsilon t$, implying that $u_k(t;\varepsilon) = v_k(t_0, t_1;\varepsilon)$. The following transformations are needed for the time derivatives:

$$\begin{aligned} &\frac{du_k}{dt} = \frac{\partial v_k}{\partial t_0} + \varepsilon\frac{\partial v_k}{\partial t_1}, \\ &\frac{d^2u_k}{dt^2} = \frac{\partial^2 v_k}{\partial t_0^2} + 2\varepsilon\frac{\partial^2 v_k}{\partial t_0 \partial t_1} + \varepsilon^2\frac{\partial^2 v}{\partial t_1^2}. \end{aligned} \tag{16}$$

It is assumed that $v_k = v_{k0} + \varepsilon v_{k1} + \ldots$, and $v_k(t_0,t_1;\varepsilon) = v_k^{(1)}(t_0,t_1;\varepsilon)$ for $1 \leqslant k \leqslant 7$ (i.e. time behavior is from the string model), and $v_k(t_0,t_1;\varepsilon) = v_k^{(2)}(t_0,t_1;\varepsilon)$ for $8 \leqslant k < \infty$ (i.e time behavior is from the beam with string effect model). So that, in fact there are two sets of $O(1)$ and $O(\varepsilon)$ problems. By taking together terms of equal powers in ε for $v_k^{(1)}$ and $v_k^{(2)}$ it follows that for $1 \leqslant k \leqslant 7$:

$$
\begin{aligned}
O(1)^{(1)}:\ & \frac{\partial^2 v_{k0}^{(1)}}{\partial t_0^2} + k^2 v_{k0}^{(1)} = 0,\\
O(\varepsilon)^{(1)}:\ & \frac{\partial^2 v_{k1}^{(1)}}{\partial t_0^2} + k^2 v_{k1}^{(1)} = -2\frac{\partial^2 v_{k0}^{(1)}}{\partial t_0 \partial t_1} - c_1 k^4 v_{k0}^{(1)}\\
& + \sum_{n=1}^{7*}\left(\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)v_{n0}^{(1)} + 8(V_0+\alpha\sin(\omega t))\frac{\partial v_{n0}^{(1)}}{\partial t_0}\right)\\
& + \sum_{n=8}^{\infty*}\left(\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)v_{n0}^{(2)} + 8(V_0+\alpha\sin(\omega t))\frac{\partial v_{n0}^{(2)}}{\partial t_0}\right),
\end{aligned}
\tag{17}
$$

and for $8 \leqslant k < \infty$:

$$
\begin{aligned}
O(1)^{(2)}:\ & \frac{\partial^2 v_{k0}^{(2)}}{\partial t_0^2} + (k^2+\mu k^4) v_{k0}^{(2)} = 0,\\
O(\varepsilon)^{(2)}:\ & \frac{\partial^2 v_{k1}^{(2)}}{\partial t_0^2} + (k^2+\mu k^4) v_{k1}^{(2)} = -2\frac{\partial^2 v_{k0}^{(2)}}{\partial t_0 \partial t_1}\\
& + \sum_{n=1}^{7*}\left(\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)v_{n0}^{(1)} + 8(V_0+\alpha\sin(\omega t))\frac{\partial v_{n0}^{(1)}}{\partial t_0}\right)\\
& + \sum_{n=8}^{\infty*}\left(\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)v_{n0}^{(2)} + 8(V_0+\alpha\sin(\omega t))\frac{\partial v_{n0}^{(2)}}{\partial t_0}\right).
\end{aligned}
\tag{18}
$$

Equation (17) represents the time behavior of the main equation (5) for the first seven modes with small bending stiffness terms. The second sum in equation (17) represents the influence of the beam with string effect model. In the first sum in (18) there still is the influence of the string model. So there is an interaction between the two models. In (18) the bending stiffness terms are now of leading order. The solution of the $O(1)^{(1)}$-problem is given by:

$$
v_{ko}^{(1)} = A_{k0}(t_1)\sin(\Omega_k^{(1)}t_0) + B_{k0}(t_1)\cos(\Omega_k^{(1)}t_0),\ k = 1,2,\ldots,7. \tag{19}
$$

The solution of the $O(1)^{(2)}$-problem is given by:

$$
v_{ko}^{(2)} = A_{k0}(t_1)\sin(\Omega_k^{(2)}t_0) + B_{k0}(t_1)\cos(\Omega_k^{(2)}t_0),\ k = 8,9,\ldots. \tag{20}
$$

In (19) and (20) $\Omega_k^{(1)}$ and $\Omega_k^{(2)}$ are given by (11). $A_{k0}(t_1)$ and $B_{k0}(t_1)$ are still arbitrary functions and can be used to avoid secular terms in the solutions of the $O(\varepsilon)^{(1)}$-problem and the $O(\varepsilon)^{(2)}$-problem. The $O(\varepsilon)^{(1)}$ equation now becomes:

$$
\begin{aligned}
&\frac{\partial^2 v_{k1}^{(1)}}{\partial t_0^2} + (\Omega_k^{(1)})^2 v_{k1}^{(1)} = -2\Omega_k^{(1)}\left(\frac{\partial A_{k0}}{\partial t_1}\cos(\Omega_k^{(1)}t_0) - \frac{\partial B_{k0}}{\partial t_1}\sin(\Omega_k^{(1)}t_0)\right)\\
&- c_1 k^4\left(A_{k0}(t_1)\sin(\Omega_k^{(1)}t_0) + B_{k0}(t_1)\cos(\Omega_k^{(1)}t_0)\right)\\
&+ \sum_{n=1}^{7*}\left\{\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)A_{n0}(t_1)\sin(\Omega_n^{(1)}t_0) + B_{n0}(t_1)\cos(\Omega_n^{(1)}t_0)\right.\\
&\left.+8(V_0+\alpha\sin(\omega t))\Omega_n^{(1)}\left(A_{n0}(t_1)\cos(\Omega_n^{(1)}t_0) - B_{n0}(t_1)\sin(\Omega_n^{(1)}t_0)\right)\right\}\\
&+ \sum_{n=8}^{\infty*}\left\{\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)A_{n0}(t_1)\sin(\Omega_n^{(2)}t_0) + B_{n0}(t_1)\cos(\Omega_n^{(2)}t_0)\right.\\
&\left.+8(V_0+\alpha\sin(\omega t))\Omega_n^{(2)}\left(A_{n0}(t_1)\cos(\Omega_n^{(2)}t_0) - B_{n0}(t_1)\sin(\Omega_n^{(2)}t_0)\right)\right\},
\end{aligned}
\tag{21}
$$

and the $O(\varepsilon)^{(2)}$ equation is given by:

$$
\begin{aligned}
&\frac{\partial^2 v_{k1}^{(2)}}{\partial t_0^2} + (\Omega_k^{(2)})^2 v_{k1}^{(1)} = -2\Omega_k^{(2)}\left(\frac{\partial A_{k0}}{\partial t_1}\cos(\Omega_k^{(2)}t_0) - \frac{\partial B_{k0}}{\partial t_1}\sin(\Omega_k^{(2)}t_0)\right)\\
&+ \sum_{n=1}^{7*}\left\{\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)A_{n0}(t_1)\sin(\Omega_n^{(1)}t_0) + B_{n0}(t_1)\cos(\Omega_n^{(1)}t_0)\right.\\
&\left.+8(V_0+\alpha\sin(\omega t))\Omega_n^{(1)}\left(A_{n0}(t_1)\cos(\Omega_n^{(1)}t_0) - B_{n0}(t_1)\sin(\Omega_n^{(1)}t_0)\right)\right\}\\
&+ \sum_{n=8}^{\infty*}\left\{\frac{kn}{(n^2-k^2)\pi}4\omega\alpha\cos(\omega t)A_{n0}(t_1)\sin(\Omega_n^{(2)}t_0) + B_{n0}(t_1)\cos(\Omega_n^{(2)}t_0)\right.\\
&\left.+8(V_0+\alpha\sin(\omega t))\Omega_n^{(2)}\left(A_{n0}(t_1)\cos(\Omega_n^{(2)}t_0) - B_{n0}(t_1)\sin(\Omega_n^{(2)}t_0)\right)\right\}.
\end{aligned}
\tag{22}
$$

From equations (21) and (22) it can readily be seen that there are infinitely many values of ω that can give rise to internal resonances. It fact these values are (in an $O(\varepsilon)$ neighborhood of):

$$
\begin{aligned}
&(i)\quad \omega \pm \Omega_n^{(1)} = \pm\Omega_k^{(1)},\ \text{for } n,k = 1,2,\ldots,7,\\
&(ii)\quad \omega \pm \Omega_n^{(2)} = \pm\Omega_k^{(1)},\ \text{for } k = 1,2,\ldots,7, \text{and } n = 8,9,\ldots,\\
&(iii)\ \omega \pm \Omega_n^{(1)} = \pm\Omega_k^{(2)},\ \text{for } n = 1,2,\ldots,7, \text{and } k = 8,9,\ldots,\\
&(iv)\quad \omega \pm \Omega_n^{(2)} = \pm\Omega_k^{(2)},\ \text{for } n = 8,9,\ldots, \text{and } k = 8,9,\ldots.
\end{aligned}
\tag{23}
$$

For all resonant cases (i)-(iv) the additional condition that $k \pm n$ is an odd number, still holds due to summation in (15). By interchanging n and k, the resonant case (ii) (derived out of the $O(\varepsilon)^{(1)}$-problem) becomes the resonant case (iii) (derived out of the $O(\varepsilon)^{(2)}$-problem). The resonant case (i) is a resonance condition for the string equation, and has been investigated in [1]. The resonant case (iv) is a resonance condition for the beam with string effect equation. The solutions and stability conditions for this case can be found in [2]. Due to the interactions of these two simplified models the model as proposed here, there are additional resonant conditions (ii) and (iii), where ω might be the sum or difference of one natural frequency of the string and one natural frequency of the beam with string effect. It is also necessary to investigate additionally if ω in the resonant case (i) also satisfies the cases (ii), (iii), and (iv), and vice versa.

THE NON-RESONANT CASE.

In this case it is assumed that the frequency ω of the velocity-fluctuations of the axially moving continuum is not equal to any combination of the resonance frequencies as listed in (23). To eliminate the secular terms in the solution of the $O(\varepsilon)^{(1)}$-problem and the $O(\varepsilon)^{(2)}$-problem it follows that A_{k0} and B_{k0} have to satisfy:

$$\begin{cases} \dfrac{dA_{k0}}{dt_1} = -\dfrac{c_1 k^3}{2} B_{k0}, \\ \dfrac{dB_{k0}}{dt_1} = \dfrac{c_1 k^3}{2} A_{k0}, \quad \text{for } 1 \leqslant k \leqslant 7, \end{cases} \tag{24}$$

and

$$\begin{cases} \dfrac{dA_{k0}}{dt_1} = 0, \\ \dfrac{dB_{k0}}{dt_1} = 0, \text{ for } 8 \leqslant k < \infty. \end{cases} \tag{25}$$

In this case system (24) can be seen as some sort of correction on the slow time (t_1) behavior of the solution in the first seven vibration modes due to presence of the small bending stiffness term in the $O(\varepsilon)^{(1)}$-problem.

SOME RESONANT CASES.

The following resonant cases will be investigated in the paper [12]: $\omega = m^*$ (where m^* is any odd number), $\omega = \Omega_9^{(2)} - \Omega_8^{(2)}$ (a difference type of resonance), $\omega = \Omega_1^{(1)} - \Omega_8^{(2)}$ (an additional difference type of resonance), $\omega = \Omega_9^{(2)} + \Omega_8^{(2)}$ (a sum type of resonance), $\omega = \Omega_1^{(1)} + \Omega_8^{(2)}$ (an additional sum type of resonance), $\omega = \Omega_7^{(1)} - \Omega_8^{(2)}$ (an additional difference type of resonance), $\omega = \Omega_7^{(1)} + \Omega_8^{(2)}$ (an additional sum type of resonance), and $\omega = 2\Omega_8^{(2)}$ (a principal parametric resonance). The conclusions about the stability of the axially moving continuum will also be presented for these cases in [12].

CONCLUSIONS AND REMARKS

In this paper an initial-boundary value problem for a linear equation, describing an axially moving stretched beam has been studied. This equation can be used as a model for the transversal vibrations of a conveyor belt system. The axially moving belt is assumed to move in one direction with a non-constant speed $V(t)$, that is, $V(t) = \varepsilon(V_0 + \alpha \sin(\omega t))$, where $0 < \varepsilon \ll 1$, and where V_0, α and ω are positive constants. For V_0 it is assumed that $V_0 > 0$ and $V_0 > |\alpha|$. A new model approach describing the transient "from string to beam" behavior, based on the calculation of the natural frequencies have been proposed. The influence of the bending stiffness on the time behavior of an approximate solution of the problem has been studied. The regions of applicability of the simplified models were found for different values of the bending stiffness parameter and the relative error. It turns out that there are infinitely many values of ω that give rise to internal resonances in the axially moving belt system. In fact, that happens when ω is equal to any sum or difference combination of the natural frequencies of the string and (or) the stretched beam equations. The correction to the natural frequencies has been found in the non-resonant case. The formal approximation of the solution and stability analysis in the resonant case will be presented in the forth coming paper [12].

REFERENCES

[1] Suweken, G., van Horssen, W. T.:On the transversal vibrations of a conveyor belt with a low and time-varying velocity, part I: the string-like case. Journal of Sound and Vibration. 264(1), 117-133 (2003)

[2] Suweken, G., van Horssen, W. T.:On the transversal vibrations of a conveyor belt with a low and time-varying velocity, part II: the beam-like case. Journal of Sound and Vibration. 267(5), 1007-1027 (2003)

[3] Öz, H.R., Pakdemirli, M. and Özkaya, E.: Transition behaviour from string to beam for an axially accelerating material. Journal of Sound and Vibration. 215(3), 571-576 (1998)

[4] Özkaya, E., Pakdemirli, M.: Vibrations of an axially accelerating beam with small flexural stiffness. Journal of Sound and Vibration. 234(3), 521-535 (2000)

[5] Kong, L., Parker, R.G.: Approximate eigensolutions of axially moving beams with small flexural stiffness. Journal of Sound and Vibration. 276(3), 459-469 (2004)

[6] Chen, L.-Q.: Analysis and control of transverse vibrations of axially moving strings. Applied Mechanics Reviews by ASME. 58, 91-115 (2005)

[7] Chen, L.-Q.: Principal parametric resonance of axially accelerating viscoelastic strings with an integral constitutive law. Proceedings of the Royal Society A. 462, 2700-2720 (2005)

[8] Wickert, J. A., Mote, C. D., Jr.: Classical Vibration Analysis of Axially Moving Continua. Journal of Applied Mechanics. 57, 738-744 (1990)

[9] Païdoussis, M.P.: Fluid-structure interactions: slender structures and axial flow. Vol.1. San Diego : Academic Press (1998)

[10] Andrianov, I. V., van Horssen, W. T.: On the transversal vibrations of a conveyor belt: applicability of simplified models. Journal of Sound and Vibration. (submitted for publication) (2006)

[11] van Horssen, W. T., Ponomareva, S. V.: On the construction of the solution of an equation describing an axially moving string. Journal of Sound and Vibration. 287, 359-366 (2005)

[12] van Horssen, W. T., Ponomareva, S. V.: On the transversal vibrations of an axially moving continuum with a time-varying velocity: transient from string to beam behavior. Some resonant cases and stability analysis. (in preparation) (2007).