

Convection in Couple-Stress Magneto-Fluid

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Abstract: Double-diffusive convection in a couple-stress fluid in the presence of uniform vertical magnetic field through porous medium using linearized stability theory and normal mode analysis is studied. For the case of stationary convection, the stable solute gradient, magnetic field and couple-stress are found to have stabilizing effects whereas medium permeability has destabilizing effect on the system. The dispersion relation is also analyzed analytically. Further, it is found that the solute gradient and uniform magnetic field introduce oscillatory modes in the system, which was non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Keywords: couple-stress fluid, double-diffusive convection, linearized stability theory, porous medium, uniform vertical magnetic field,

I. Introduction

The derivation of the basic equations of a layer of fluid heated from below in porous medium, using Boussinesq approximation, has been given by Joseph [1976]. The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering disciplines. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. Lapwood [1948] has studied the stability of convective flow in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [1960]. When the fluid slowly percolates through the pores of the rock, the gross effect is represented by the well-known Darcy's law. An extensive and updated account of convection in porous media has been given by Nield and Bejan [1999].

A detailed account of the theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics has been given in the celebrated monograph by Chandrasekhar [1981]. Veronis [1965] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Thermosolutal convection problems arise in oceanography, limnology and engineering. The investigation of thermosolutal convection is motivated by its interesting complexities as a double diffusion phenomena as well as its direct relevance to geophysics and astrophysics. Stommel et al. [1956] did the pioneering work regarding the investigation of thermosolutal convection. This work was elaborated in different physical situations by Stern [1960] and Nield [1967]. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz [1965]) and some Antarctic lakes (Shirtcliffe [1964]). The physics is quite similar in the stellar case in that Helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motion in double-diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics and several geophysical situation) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The theory of couple-stress fluid has been formulated by Stokes [1966]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. According to the theory of Stokes [1966], couple-stresses appear in noticeable magnitudes in fluids with very large molecules.

Many of the flow problems in fluids with couple-stresses, discussed by Stokes, indicate some possible experiments, which could be used for determining the material constants, and the results are found to differ from those of Newtonian fluid. Couple-stresses are found to appear in noticeable magnitudes in polymer solutions for

force and couple-stresses. This theory is developed in an effort to examine the simplest generalization of the classical theory, which would allow polar effects. The constitutive equations proposed by Stokes [1966] are:

$$T_{(ij)} = (-p + \lambda D_{kk}) \delta_{ij} + 2\mu D_{ij} ,$$

$$T_{[ij]} = -2\eta \bar{W}_{ij,kk} - \frac{\rho}{2} \bar{\epsilon}_{ijs} G_s ,$$

and

$$M_{ij} = 4\eta \bar{\omega}_{j,i} + 4\eta' \bar{\omega}_{i,j} ,$$

where

$$D_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i}), \bar{W}_{ij} = -\frac{1}{2}(V_{i,j} - V_{j,i})$$

and
$$\bar{\omega}_i = \frac{1}{2} \bar{\epsilon}_{ijk} V_{k,j} .$$

Here T_{ij} , $T_{(ij)}$, $T_{[ij]}$, M_{ij} , D_{ij} , $\bar{W}_{i,j}$, $\bar{\omega}_i$, G_s , $\bar{\epsilon}_{ijk}$, V , ρ and λ , μ , η , η' , are stress tensor, symmetric part of T_{ij} , anti-symmetric part of T_{ij} , the couple-stress tensor, deformation tensor, the vorticity tensor, the vorticity vector, body couple, the alternating unit tensor, velocity field, the density and material constants respectively. The dimensions of λ and μ are those of viscosity whereas the dimensions of η and η' are those of momentum.

Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [1999] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with a malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is, due to its content of the hyaluronic acid, a fluid of high viscosity, near to a gel. Goel et al. [1999] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [2002] have considered a couple-stress fluid with suspended particles heated from below. They have found that for stationary convection, couple-stress has a stabilizing effect whereas suspended particles have a destabilizing effect. In another study, Sunil et al. [2002] have considered a couple stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al [2004] have considered the thermal instability of a layer of a couple–stress fluid acted on by a uniform rotation, and have found that for stationary convection, the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the double-diffusive convection in couple-stress fluid in the presence of uniform magnetic field through porous medium has been considered in the present paper.

II. Formulation of the problem and perturbation equations

Here we consider a layer of electrically conducting couple-stress fluid of thickness d in porous medium heated and soluted from below so that the temperatures and solute concentrations at the bottom surface $z = 0$ are T_o and C_o and at the upper surface $z = d$ are T_1 and C_1 respectively, z - axis being taken as vertical. A uniform vertical magnetic field $\vec{H}(0, 0, H)$ pervades the system.

Let $\delta\rho$, δp , θ , γ , $\vec{q}(u, v, w)$ and $\vec{h}(h_x, h_y, h_z)$ denote the perturbations in density ρ , pressure p , temperature T , solute concentration C , velocity $(0,0,0)$ and magnetic field $\vec{H}(0,0,H)$ respectively. Then the linearized hydromagnetic equations relevant to the problem are

$$\frac{1}{\epsilon} \frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p - \bar{g}(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1} \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \vec{q} + \frac{\mu_e}{4\pi\rho_0} (\nabla \times \vec{h}) \times \vec{H}, \tag{1}$$

$$\nabla \cdot \vec{q} = 0, \tag{2}$$

$$E \frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta, \tag{3}$$

$$E \frac{\partial \gamma}{\partial t} = \beta' w + \kappa \nabla^2 \gamma, \tag{4}$$

$$\nabla \cdot \vec{h} = 0, \tag{5}$$

$$\varepsilon \frac{\partial \bar{h}}{\partial t} = (\bar{H} \cdot \nabla) \bar{q} + \varepsilon \eta \nabla^2 \bar{h}, \tag{6}$$

where $E = \varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho c}$.

Here ε is the medium porosity, k_1 is the medium permeability, and $\bar{g}(0, 0, -g)$ is gravitational acceleration respectively. k_1 has the dimension of length squared. ρ, c and ρ_s, c_s stand for density and specific heat of fluid and solid (porous matrix) material respectively.

ν is the kinematic viscosity, μ' is the couple-stress viscosity, κ is the thermal diffusivity, α is the coefficient of thermal expansion, η is the electrical resistivity (all assumed to be constants), and μ_e stands for magnetic permeability.

The equation of state is

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{7}$$

where the suffix zero refers to values at the reference level $z = 0$ and so the change in density $\delta\rho$ caused by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_0 (\alpha\theta - \alpha'\gamma).$$

The equation of state (7) contains a thermal coefficient of expansion α and an analogous solvent coefficient α' . E' is a solute parameter analogous to E .

The steady state solution is

$$\bar{q} = (0, 0, 0), T = T_0 - \beta z, C = C_0 - \beta' z,$$

$$\rho = \rho_0 (1 + \alpha\beta z - \alpha'\beta' z),$$

where $\beta = \frac{T_0 - T_1}{d}$ and $\beta' = \frac{C_0 - C_1}{d}$ are the magnitudes of uniform temperature and concentration gradients

and are both positive as the temperature and concentration decrease upwards.

Now we consider the case in which both the boundaries are free, the medium adjoining the fluid is a perfect electrical conductor and temperature, concentration at a boundary are kept fixed. Then the boundary conditions appropriate to the problem are

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = \gamma = \frac{\partial h_z}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = d. \tag{8}$$

Equations (1)-(6) give

$$\left[\frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{1}{k_1} \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \right] \nabla^2 w - g \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\alpha\theta - \alpha'\gamma) - \frac{\mu_e H}{4\pi\rho_0} \frac{\partial}{\partial z} \nabla^2 h_z = 0, \tag{9}$$

$$\left(E \frac{\partial}{\partial t} - \kappa \nabla^2 \right) \theta = \beta w, \tag{10}$$

$$\left(E \frac{\partial}{\partial t} - \kappa \nabla^2 \right) \gamma = \beta' w, \tag{11}$$

$$\varepsilon \left(\frac{\partial}{\partial t} - \eta \nabla^2 \right) h_z = H \frac{\partial w}{\partial z}. \tag{12}$$

III. Dispersion relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, h_z] = [W(z), \Theta(z), \Gamma(z), K(z)] \exp(ik_x x + ik_y y + nt), \tag{13}$$

where k_x, k_y are the wave numbers along the x - and y - directions respectively, $k (= \sqrt{k_x^2 + k_y^2})$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (13), equations (9)-(12) in non-dimensional form become

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1} - \frac{F}{P_1} (D^2 - a^2) \right] (D^2 - a^2)W + \frac{gd^2 a^2}{\nu} (\alpha\Theta - \alpha\Gamma) - \frac{\mu_e Hd}{4\pi\rho_0\nu} (D^2 - a^2)DK = 0, \quad (14)$$

$$(D^2 - a^2 - Ep_1\sigma)\Theta = -\left(\frac{\beta d^2}{\kappa}\right)W, \quad (15)$$

$$(D^2 - a^2 - E'q\sigma)\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right)W, \quad (16)$$

$$(D^2 - a^2 - p_2\sigma)K = -\left(\frac{Hd}{\varepsilon\eta}\right)DW, \quad (17)$$

where we have put

$$a = kd, \sigma = \frac{nd^2}{\nu}, \frac{x}{d} = x^*, \frac{y}{d} = y^*, \frac{z}{d} = z^* \text{ and } D = \frac{d}{dz^*}.$$

Here

$$p_1 = \frac{\nu}{\kappa} \text{ is the Pr andtl number,}$$

$$p_2 = \frac{\nu}{\eta} \text{ is the magnetic Pr andtl number,}$$

$$q = \frac{\nu}{\kappa'} \text{ is the Schmidt number,}$$

$$P_1 = \frac{k_1}{d^2} \text{ is the dimensionless medium permeability}$$

and

$$F = \frac{\mu'}{\rho_0 d^2 \nu} \text{ is the dimensionless couple - stress parameter.}$$

We shall suppress the star (*) for convenience hereafter.

Eliminating Θ, Γ and K between equations (14)-(17) and using the proper solution $W = W_0 \sin \pi z, W_0$ being constant, we obtain the dispersion relation

$$R_1 = \frac{(1+x) \left(1+x + Ep_1 \frac{\sigma}{\pi^2} \right) \left[\left\{ \frac{\sigma_1}{\varepsilon\pi^2} + \frac{1}{P} + \frac{\pi^2 F}{P} (1+x) \right\} \right]}{x} + S_1 \frac{\left(1+x + Ep_1 \frac{\sigma}{\pi^2} \right)}{\left(1+x + E'q \frac{\sigma}{\pi^2} \right)} + \frac{Q_1}{\varepsilon} \frac{(1+x) \left(1+x + Ep_1 \frac{\sigma}{\pi^2} \right)}{x \left(1+x + p_2 \frac{\sigma}{\pi^2} \right)}, \quad (18)$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4}, S_1 = \frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4}, Q_1 = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\pi^2}, P = \pi^2 P_1, \quad x = \frac{a^2}{\pi^2}.$$

IV. The stationary convection

For the stationary convection $\sigma = 0$ and equation (18) reduces to

$$R_1 = \frac{(1+x)}{x} \left[\frac{(1+x)}{P} \left(1 + \pi^2 F \overbrace{1+x} \right) + \frac{Q_1}{\varepsilon} \right] + S_1. \quad (19)$$

Equation (19) yields

$$\frac{dR_1}{dS_1} = +1, \quad (20)$$

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xP^2} \left(1 + \pi^2 F \overline{1+x} \right), \tag{21}$$

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{\epsilon x}, \tag{22}$$

and

$$\frac{dR_1}{dF} = \frac{\pi^2 (1+x)^3}{xP}, \tag{23}$$

which imply that couple-stress, stable solute gradient and magnetic field have stabilizing effects whereas medium permeability has destabilizing effect on the system.

The dispersion relation (19) is also analyzed numerically for various values of S_1 , P , Q_1 and F . It is also evident from Fig. 1 - 4 that couple-stress, stable solute gradient and magnetic field have stabilizing effects whereas medium permeability has destabilizing effect on the system.

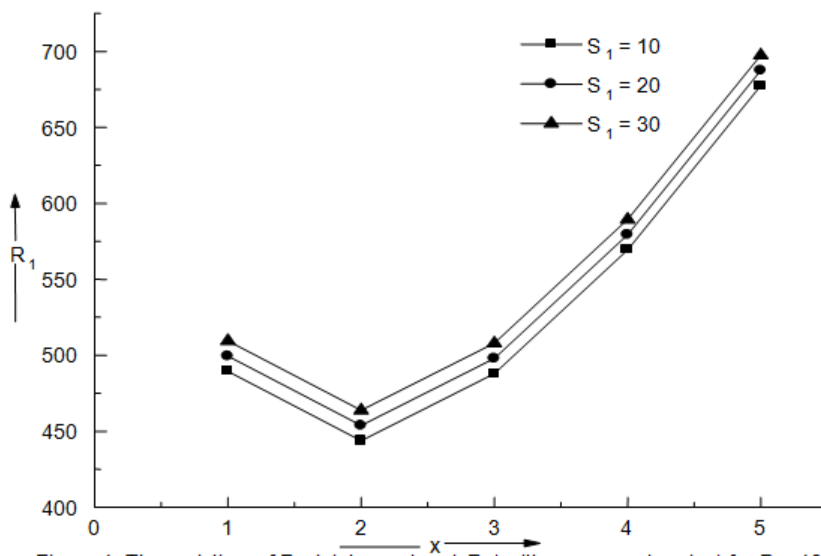


Figure 1: The variation of Rayleigh number (R_1) with wave number (x) for $P = 10$, $F = 10$, $Q_1 = 100$, $\epsilon = 0.5$ and $S_1 = 10, 20, 30$.

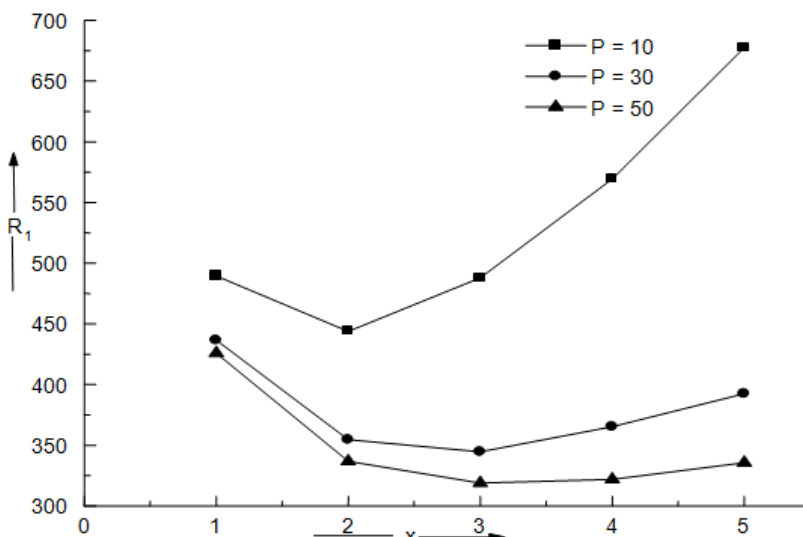


Figure 2: The variation of Rayleigh number (R_1) with wave number (x) for $F = 10$, $Q_1 = 100$, $S_1 = 10$, $\epsilon = 0.5$ and $P = 10, 30, 50$.

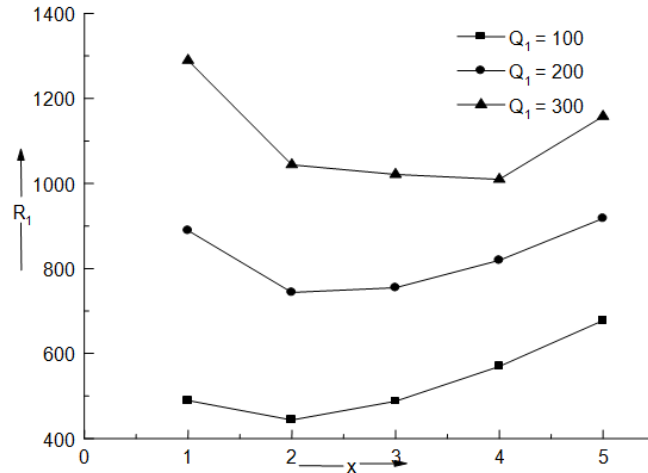


Figure 3: The variation of Rayleigh number (R_1) with wave number (x) for $P = 10$, $F = 10$, $S_1 = 30$, $\epsilon = 0.5$ and $Q_1 = 100, 200, 300$.

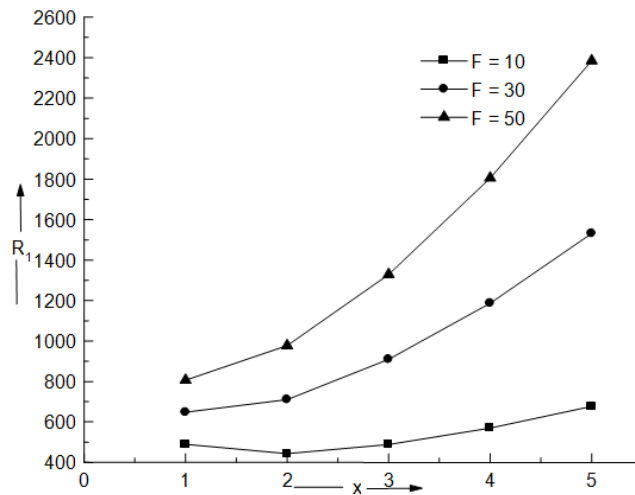


Figure 4: The variation of Rayleigh number (R_1) with wave number (x) for $P = 10$, $Q_1 = 100$, $S_1 = 10$, $\epsilon = 0.5$ and $F = 10, 30, 50$.

Some important theorems

Theorem I: The system is stable or unstable in the presence of the solute gradient, magnetic field and porous medium.

Proof: Multiplying equation (14) by W^* , the complex conjugate of W , integrating over the range of z , and making use of equations (15)- (17) together with boundary conditions, we obtain

$$\left(\frac{\sigma}{\epsilon} + \frac{1}{P_1}\right) I_1 + \frac{g\alpha'\kappa'a^2}{\nu\beta'} (I_4 + E'q\sigma^* I_5) + \frac{\mu_e \epsilon \eta}{4\pi\rho_0\nu} (I_6 + p_2\sigma^* I_7) + \frac{F}{P_1} I_8 = \frac{g\alpha\kappa a^2}{\nu\beta} (I_2 + Ep_1\sigma^* I_3), \tag{24}$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2) dz, \quad I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz,$$

$$I_3 = \int_0^1 (|\Theta|^2) dz, \quad I_4 = \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2) dz,$$

$$I_5 = \int_0^1 (|\Gamma|^2) dz, \quad I_6 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2) dz,$$

$$I_7 = \int_0^1 (|DK|^2 + a^2 |K|^2) dz, \quad I_8 = \int_0^1 (|D^2W|^2 + 2a^2 |DW|^2 + a^4 |W|^2) dz, \tag{25}$$

which are all positive definite. Substituting $\sigma = \sigma_r + i\sigma_i$ and then equating real and imaginary parts of equation (24), we obtain

$$\begin{aligned} \sigma_r & \left(\frac{I_1}{\varepsilon} + \frac{g\alpha'\kappa'a^2}{v\beta'} E'qI_5 + \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu} p_2I_7 - \frac{g\alpha\kappa a^2}{v\beta} Ep_1I_3 \right) \\ & = - \left(\frac{1}{P_l} I_1 + \frac{g\alpha'\kappa'a^2}{v\beta'} I_4 + \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu} I_6 + \frac{F}{P_l} I_8 - \frac{g\alpha\kappa a^2}{v\beta} I_2 \right), \end{aligned} \tag{26}$$

and

$$\sigma_i \left(\frac{I_1}{\varepsilon} - \frac{g\alpha'\kappa'a^2}{v\beta'} E'qI_5 - \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu} p_2I_7 + \frac{g\alpha\kappa a^2}{v\beta} Ep_1I_3 \right) = 0. \tag{27}$$

Equation (26) yields that σ_r may be positive or negative i.e. there may be stability or instability in the presence of solute gradient, magnetic field and porous medium in couple-stress fluid.

Theorem II: The modes may be oscillatory or non-oscillatory in contrast to the case of no magnetic field and in absence of stable solute gradient.

Proof: Equation (27) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that the modes may be non-oscillatory or oscillatory. In the absence of stable solute gradient and magnetic field, equation (27) reduces to

$$\sigma_i \left(\frac{I_1}{\varepsilon} + \frac{g\alpha\kappa a^2}{v\beta} Ep_1I_3 \right) = 0, \tag{28}$$

and terms in brackets are positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for a porous medium in couple-stress fluid in the absence of stable solute gradient and magnetic field. This result is true for the porous as well as non-porous medium. The presence of each, the stable solute gradient and the magnetic field brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence.

V. The Overstable case

Here we discuss the possibility of whether instability may occur as overstability. Put $\frac{\sigma}{\pi^2} = i\sigma_1$, it being remembered that σ may be complex. Since for overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (18) will admit of solutions with σ_1 real. Equation (18) becomes

$$\begin{aligned} R_1 & = \left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P} + \frac{\pi^2 F}{P} \overbrace{1+x} \right) \frac{(1+x)(1+x+iEp_1\sigma_1)}{x} + S_1 \frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)} \\ & \quad + \frac{Q_1}{\varepsilon} \frac{(1+x)(1+x+iEp_1\sigma_1)}{x(1+x+ip_2\sigma_1)}. \end{aligned} \tag{29}$$

It can be easily shown from equation (29) that $\frac{dR_1}{dS_1}, \frac{dR_1}{dQ_1}, \frac{dR_1}{dP}$ and $\frac{dR_1}{dF}$ yield the same values as given in equations (20)-(23). This means that couple-stress, stable solute gradient and magnetic field have stabilizing effects whereas medium permeability has destabilizing effect on the system for overstable case also. Equating real and imaginary parts of equation (29) and eliminating R_1 between them, we obtain

$$\left[\frac{E'^2 q^2 p_2^2 \alpha}{\varepsilon} + \frac{EE'^2 q^2 p_1 p_2^2 (1 + \pi^2 F \alpha)}{P} \right] c_1^2 + \left[(E'^2 q^2 + p_2^2) \left\{ \alpha^2 \left(\frac{\alpha}{\varepsilon} + \frac{Ep_1 1 + \pi^2 F \alpha}{P} \right) \right\} + \frac{E'^2 q^2 \alpha Q_1 (Ep_1 - p_2)}{\varepsilon} + S_1 \overline{\alpha - 1} p_2^2 (Ep_1 - E'q) \right] c_1 + \left[\alpha^4 \left(\frac{\alpha}{\varepsilon} + \frac{Ep_1 1 + \pi^2 F \alpha}{P} \right) + S_1 \alpha^2 (\alpha - 1) (Ep_1 - E'q) + \frac{\alpha^3}{\varepsilon} Q_1 (Ep_1 - p_2) \right] = 0, \quad (30)$$

where $1 + x = \alpha$ and $\sigma_1^2 = c_1$. As σ_1 is real for overstability, the two values of $c_1 (= \sigma_1^2)$ are positive. Equation (30) is quadratic in c_1 and does not involve any of its roots to be positive if

$$\text{and } \left. \begin{array}{l} Ep_1 > p_2 \\ Ep_1 > E'q \end{array} \right\}, \quad (31)$$

i.e.

$$\kappa < \eta \left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho c} \right] \quad (32)$$

and

$$\left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s'}{\rho c'} \right] \kappa < \left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho c} \right] \kappa',$$

where ρc , $\rho_s c_s$ and $\rho c'$, $\rho_s c_s'$ denote respectively the heat capacity of fluid, solid matrix and analogous solute capacity of fluid, solid matrix.

Thus, if conditions (32) are satisfied, overstability is impossible and the principle of exchange of stabilities holds good. Therefore, equations (32) are the sufficient conditions for the non-existence of overstability, the violation of which does not necessary involve occurrence of overstability.

In the absence of solute gradient, the sufficient condition for non-existence of overstability reduces to

$$\kappa < \eta \left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho c} \right],$$

which for non-porous medium ($\varepsilon = 1$) further reduces to $\kappa < \eta$ (Chandrasekhar [1981]), for Newtonian fluid], but the introduction of solute gradient introduces an additional sufficient condition

$$\left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s'}{\rho c'} \right] \kappa < \left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho c} \right] \kappa'.$$

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