Pressure Loss Due to the Tip Clearance of Impeller Blades in Centrifugal and Axial Blowers

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The pressure loss based on the tip clearance of impeller blades consists of the pressure loss induced by the leakage flow through the clearance and the pressure loss for supporting fluid against the pressure gradient in the channels and in the thin annular clearance space between the shroud and the impeller. Equations to evaluate these losses are derived and the predicted efficiency drop is compared with experimental data for two types of centrifugal impellers. Furthermore, the equations are simplified for axial impellers as a special case, and the predicted efficiency drop is compared with the experimental data for seven cases in the literature. Fair agreement demonstrates plausibility of the present model.

Introduction

In many turbomachines impellers are not shrouded and the leakage flow through the tip clearance of blades is an unavoidable factor which deteriorates the performance. Therefore, the impeller geometry should be designed considering the tip clearance effects, but there is no rational method to evaluate the tip clearance loss. Loss equations have been derived from a few experimental data [1] or many assumptions on the loss mechanism [2]. There are several papers [3, 4, 5] which concern tip clearance, but they do not clarify the mechanism of pressure loss.

The leakage flow through the tip clearance of impeller blades modifies the flow pattern in the impeller. Considerable effort has been made to examine the details of the flow pattern [6, 7], but the results have not been utilized to evaluate the pressure loss due to the tip clearance of turbomachines.

Pressure Loss Due to Leakage Through the Clearance

One blade pitch of an impeller which is rotating counterclockwise is taken as a control volume as shown in Fig. 1(*a*). Between the blades the relative velocity is the maximum along the suction surface of the left side blade and it is decreased monotonically toward the pressure surface of the right side blade. If there is a clearance at the blade tip, fluid in the left channel leaks into the central channel through the tip clearance of the left blade. At the same time an equal amount of fluid with identical dynamic condition leaks away from the central channel. As a result it looks as if the tip leakage has no net effect on the flow in the central channel. However, in reality there is a drag due to the leakage.

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Cases Where Blades Are Loaded by Curvature. In a case of ideal two-dimensional flow in a curved channel, the velocity w_p near the concave wall is smaller than the velocity w_s near the convex wall. If a small amount of fluid q is removed from the concave wall and an equal amount of fluid with identical total head is injected in at the convex wall in such a way that the tangential velocity is identical to w_s, there is no loss of total pressure, and the flow downstream in the curved channel is hardly modified by the disturbance due to the injection. That is, although the tangential velocity of the injected fluid at the convex surface is larger than the tangential velocity of the outgoing fluid at the concave surface the flow in the curved channel as a whole gets no net effect.

If the direction of injection at the convex surface is changed so that the tangential velocity component is equal to w_p instead of w_s, the flow in the curved channel suffers a drag force $q(w_s - w_p)$ because the tangential momentum of the injected fluid is smaller by $q(w_s - w_p)$ compared with the value of the last example.

The conditions of leakage through the tip clearance of blades, which are loaded by the curvature of streamlines, is identical to the second case in the above examples, and the drag due to leakage through the tip clearance is $q(w_s - w_p)$.

Cases Where Blades Are Loaded by Rotation of Impeller. If a fluid particle transverses a channel of an impeller which is loaded only by rotation of the impeller, the parallel component of velocity w of the fluid particle is decelerated by $-dw/dt = 2\Omega v dr/dm$ due to the Coriolis acceleration; meanwhile the fluid particle moves to a line which is parallel to the blade and at a circumferential distance of $ds = vdt/\sin\beta$ away from the blade. Since $-dw/ds = -(dw/dt)(dt/ds) = -2\Omega\sin\beta\, dr/dm$, the circumferential reduction rate of the relative velocity w is independent of the transverse velocity, and the reduction rate is identical to the velocity gradient of the main flow. That is, any fluid particle reduces the velocity component parallel to the blade by

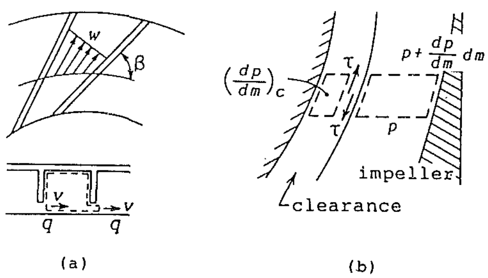

Fig. 1 Models of flow and forces at the clearance

$w_s - w_p$ while it transverses a channel from the suction surface of a blade to the pressure surface of the adjacent blade.

If a small amount of fluid q leaks away from the pressure surface of a channel and an equal amount of fluid leaks into the suction surface of the channel with a velocity component parallel to the blade which is equal to w_p instead of w_s, the drag due to the leakage is $q(w_s - w_p)$.

In general cases the blades of impellers are loaded by curvature of blades as well as rotation of the impeller. Since the drag due to leakage is expressed as $q(w_s - w_p)$ whether the blade is loaded by the Coriolis force or by the curvature of blades, the relation is applicable to any impellers where the blades are loaded by a combination of them.

Influence of Number of Blades. The angle between the two adjacent blades of an impeller is $(2\pi/Z)(dr/dm)$. Therefore, the leakage flow normal to the blade surface must change the momentum in the direction of blades by $qv(2\pi/Z)(dr/dm)$ keeping the momentum qv normal to the blades identical at the clearances of both blades. As a result the fluid in the channel of the impeller receives a negative drag.

Combining these three effects, the drag force D due to the leakage flow q is expressed as

$$D = q(w_s - w_p) - qv(2\pi/Z)(dr/dm)$$

Leakage Rate and Pressure Loss. The velocity of leakage flow normal to the blade is

$$v = \sqrt{2(p_p - p_s)/\rho} = \sqrt{2\bar{w}_0(w_s - w_p)}$$

where w_s, w_p, and $\bar{w}_0$ are the values of an ideal flow with $\sigma = 1.0$. Even when the effectiveness σ of the channel cross-sectional area is reduced by displacement thickness and the velocity $\bar{w}$ is large so that $\bar{w} = \bar{w}_0/\sigma$, as long as the blade loading remains constant the pressure difference across a blade hardly varies; therefore the foregoing equation should be generalized as follows

$$v = \sqrt{2(p_p - p_s)/\rho} = \sqrt{2\sigma\bar{w}(w_s - w_p)} \tag{1}$$

The leakage flow rate per blade length is

$$dq/dl = \alpha c \rho v$$

where α is the contraction factor of leakage flow.

The power loss P' based on the leakage drag force is the product of D and the main flow velocity $\bar{w}$ near the shroud in the impeller.

$$\frac{dP'}{dl} = \frac{dq}{dl}\,\bar{w}\left\{(w_s - w_p) - \frac{2\pi}{Z}\,v\,\frac{dr}{dm}\right\}$$

$$= \alpha c \rho v^2 \left\{\frac{v}{2\sigma} - \frac{2\pi}{Z}\,\bar{w}\frac{dr}{dm}\right\}$$

The power loss is related to the entropy rise $\Delta s'$ (or pressure loss p_l' for the incompressible case) at the exit of the impeller as follows:

$$\rho Q T \Delta s' \,(= Q p_l') = Z\int_{m_1}^{m_2} \alpha \rho c v^2 \left(\frac{v}{2\sigma} - \frac{2\pi}{Z}\,\bar{w}\frac{dr}{dm}\right)\frac{dm}{\sin\beta_b}$$

where $dl = dm/\sin\beta_b$. Replacing $T\Delta s'$ $(= p_l'/\rho) = \psi_l' U^2/2$, $Q = \varphi U 2\pi r_2 b_2$ and $c/b_2 = \lambda$,

$$\psi_l' \doteq \frac{Z}{\rho_2 \pi r_2 \varphi}\int_{m_1}^{m_2} \frac{\rho\alpha\lambda}{\sin\beta_b}\left(\frac{v}{U}\right)^2\left(\frac{v}{2\sigma U} - \frac{2\pi}{Z}\,\frac{\bar{w}}{U}\,\frac{dr}{dm}\right)dm \tag{2}$$

Clearance Loss Due to Pressure Gradient

In a bundle of stream tubes passing through a cross section, if there is a stream tube where the flow does not have enough velocity or momentum to support itself against the adverse pressure gradient, a shear force supports the flow in the stream tube to keep the steady state and a pressure loss results. There are similar problems regarding the flow in the tip clearance region of an impeller.

Annular Tip Clearance Space Along the Shoud. The meridional pressure gradient along the shroud induced by an impeller is usually larger than the centrifugal force of fluid in the impeller. If the fluid in the annular clearance space rotates with a circumferential velocity which is equal to that of the fluid in the impeller, the excess portion of the pressure

Nomenclature

- b = blade height
- c = clearance
- D = drag force due to leakage flow
- k = slip coefficient
- l = distance along blade or chord
- m = meridional distance along shroud
- p = pressure
- p_l = pressure loss of incompressible flow due to clearance
- P = power loss per channel
- q = leakage mass flow per blade
- Q = volume flow rate of impeller
- r = radial distance from the axis
- $\tilde{r}$ = hub tip mean radius
- s = pitch of blades or entropy
- T = absolute temperature
- u = circumferential component of velocity
- U = peripheral speed of impeller outer diameter
- v = component of leakage velocity normal to blade
- w = velocity component parallel to blade
- Z = number of blades
- α = contraction factor of leakage flow
- β = flow angle, from circumference
- β_b = blade angle
- δ = blockage multiplier due to tip clearance
- ϵ = pressure recovery coefficient
- η = efficiency
- λ = tip clearance ratio c/b_2
- ρ = density of fluid
- σ = effectiveness of flow area in impeller, equation (4)
- φ = flow coefficient at impeller exit
- ψ ≐ pressure coefficient of impeller
- ψ_h = pressure loss coefficient due to causes other than tip clearance
- ψ_i ≐ work input coefficient of impeller
- ψ_l ≐ pressure loss coefficient due to tip clearance
- Ω = angular velocity of impeller

Subscripts and Superscripts

- e = experiment
- m = mean value along blade length
- p = pressure surface
- s = suction surface
- u = circumferential component
- 0 = values for zero clearance
- 1 = inlet
- 2 = exit
- $'$ = due to leakage
- $''$ = due to pressure gradient on annular clearance
- $'''$ = due to pressure gradient on blockage in channel
- $\bar{}$ = circumferential average value

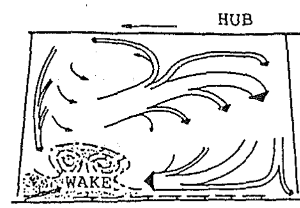

Fig. 2 Secondary flow pattern in the radial part of a centrifugal impeller

gradient exerted by the impeller must be balanced by the shear force along the surface of revolution of blade-edges as shown in Fig. 1(*b*).

If there is no pressure loss in the impeller,

$$\frac{-\kappa}{\kappa-1}\,\frac{p}{\rho}=\frac{w^2}{2}-\frac{r^2\Omega^2}{2}+\text{constant}$$

where κ is the specific heat ratio and it is infinity for incompressible flow. Differentiating the equation and considering the pressure loss due to deceleration,

$$\frac{1}{\rho}\,\frac{dp}{dm}=r\Omega^2\frac{dr}{dm}-\epsilon w\frac{dw}{dm}$$

for compressible as well as for incompressible flows.

If the circumferential velocity of fluid in the annular clearance space is equal to that in the impeller, the pressure gradient which can be supported by the centrifugal force in the annular space is

$$\frac{1}{\rho}\left(\frac{dp}{dm}\right)_c=\frac{1}{r}\,(r\Omega-w\cos\beta)^2\,\frac{dr}{dm}$$

The meridian component of the shear force along the surface of revolution is

$$\tau=c\left\{\frac{dp}{dm}-\left(\frac{dp}{dm}\right)_c\right\}$$

and the power loss P'' due to this drag force is

$$Z\frac{dP''}{dm}=\rho QT\frac{ds''}{dm}\left(=Q\frac{dp_l''}{dm}\right)=2\pi r\tau w\sin\beta$$

Integrating the foregoing equation along the shroud

$$\psi_l''=\frac{2}{r_2\varphi}\int_{m_1}^{m_2}\frac{\rho}{\rho_2}r\lambda\frac{w}{U}\sin\beta\left(-\epsilon\frac{w}{U^2}\frac{dw}{dm}+\frac{2\Omega w}{U^2}\cos\beta\frac{dr}{dm}-\frac{w^2}{U^2 r}\cos^2\beta\frac{dr}{dm}\right)dm \tag{3}$$

The region where the value in the parentheses is negative must be excluded in the integration.

Stagnant Flow Area in the Impeller Due to Clearance. The leakage flow through the tip clearance is intercepted by a secondary flow along the shroud which moves from the pressure side of a blade to the suction side of the adjacent blade, and the flow with low velocity component w near the shroud moves into the channel between blades as shown in Fig. 2. It is presumed that the flow area of an impeller $2\pi r(b+c)\sin\beta_b$ is virtually reduced by the tip clearance, and the decrement of the effective flow area is expressed as $2\pi rc(1+\delta)\sin\beta_b$. That is, the effectiveness σ of the flow area is expressed as

$$\sigma=1-\delta\lambda(b_2/b) \tag{4}$$

The power loss to support the stagnant flow area in the channel between blades due to the tip clearance is expressed as

$$ZP'''=\int_{l_1}^{l_2}-\frac{\rho}{\rho_2}2\pi rc\delta\sin\beta\epsilon w^2\frac{dw}{dm}\,dl=\rho QT\Delta s'''\ (=Qp_l''')$$

$$\psi_l'''=\frac{2}{r_2\varphi}\int_{m_1}^{m_2}\frac{\rho}{\rho_2}r\lambda\frac{w}{U}\sin\beta\left(-\epsilon\delta\frac{w}{U^2}\frac{dw}{dm}\right)dm \tag{5}$$

Integration should be made excluding the region where the value in the parentheses is negative.

The pressure loss due to the tip clearance is the summation of the leakage loss and the pressure gradient losses

$$\psi_l=\psi_l'+\psi_l''+\psi_l''' \tag{6}$$

Working Equations for Change of Performances

The effective flow area in the channel between blades is reduced due to the leakage through the clearance of blades and the velocity is increased. On the other hand, the velocity distribution in the impeller is usually estimated for the case without tip clearance. Therefore, the equations (1, 2, 3, 5) are expressed as follows using the relation $w=w_0/\sigma$, where w_0 is the value at $\sigma=1.0$

$$v=\sqrt{2\bar{w}_0(w_s-w_p)} \tag{7}$$

$$\psi_l'=\frac{Z}{\pi r_2\varphi}\int_{m_1}^{m_2}\frac{\rho}{\rho_2}\,\frac{\alpha\lambda}{\sigma\sin\beta_b}\left(\frac{v}{U}\right)^2\left(\frac{v}{2U}-\frac{2\pi}{Z}\,\frac{\bar{w}_0}{U}\,\frac{dr}{dm}\right)dm \tag{8}$$

$$\psi_l''+\psi_l'''=\frac{2}{r_2\varphi}\int_{m_1}^{m_2}\frac{\rho}{\rho_2}\,\frac{r\lambda\bar{w}_0}{\sigma^3 U}\sin\beta\left\{-\epsilon(1+\delta)\frac{\bar{w}_0}{U^2}\,\frac{d\bar{w}_0}{dm}+\frac{2\sigma\Omega\bar{w}_0}{U^2}\cos\beta\frac{dr}{dm}-\frac{\bar{w}_0^{\,2}}{U^2 r}\cos^2\beta\frac{dr}{dm}\right\}dm \tag{9}$$

For integrating the foregoing equation δ must be assumed zero in the region where $d\bar{w}_0/dm>0$, and if there is a region where the value in the brackets is negative the region must be omitted from the range of integration.

Tip clearance not only induces a pressure loss but also reduces the Euler head of the impeller, and the decrement of the pressure rise due to tip clearance is the sum of the two effects. That is

$$\psi_0-\psi=\psi_{i0}-\psi_i-(\psi_{h0}-\psi_h)+\psi_l \tag{10}$$

where ψ_h is the hydraulic pressure loss other than the tip clearance loss and subscript 0 indicates the values for the case of zero clearance.

Reduction of the efficiency due to the tip clearance is evaluated as the difference of the two equations

$$\eta_0=1-(\psi_{h0}/\psi_{i0}),\quad \eta=1-(\psi_h+\psi_l)/\psi_i$$

$$\frac{\eta_0-\eta}{\lambda_2}=\frac{\psi_l}{\psi_i\lambda_2}-\frac{\psi_{h0}-\psi_h}{\psi_i\lambda_2}+(1-\eta_0)\frac{\psi_{i0}-\psi_i}{\psi_i\lambda_2} \tag{11}$$

$\psi_{h0}-\psi_h$ is almost zero but it may be negative at off-design conditions.

Loading Equations for Centrifugal Impellers

The flow behavior in a centrifugal impeller without tip clearance can be evaluated by means of quasi-three-dimensional flow analysis assuming inviscid flow. In reality the viscous effect is not negligible, but it is known that the predicted work input of an impeller agrees well with the experiment. Therefore, it is expected that the distribution of p_p-p_s along the blade may be predicted well using an inviscid flow analysis. The pressure gradient along the shroud in the meridional plane is reduced by various kinds of pressure losses, and their effects may be expressed in equations (3) and (5) by properly choosing the recovery coefficient ϵ. That is, a quasi-three-dimensional flow analysis in an impeller supplies sufficient information to evaluate the pressure losses due to the tip clearance.

In many cases it is desired to evaluate the pressure losses based on the tip clearance without executing laborious flow analysis. If the meridian component of velocity along the shroud is evaluted by some relatively simple means, the blade loading may be roughly estimated in the following way.

It is assumed that the flow is axisymmetric, and on the surface of revolution along the shroud the direction of flow varies smoothly from the inlet relative flow angle to the blade angle at the throat of the impeller blades, and then it flows parallel to the blades up to a radius of about $(1-2k)r_2$. Near the exit of the impeller the flow deviates from the direction of blades so that it satisfies the slip coefficient k.

The blade loading of an impeller is related to the change of angular momentum of the flow as follows:

$$(p_p - p_s)\,rbZdm = \rho\bar{w}_0(w_s - w_p)\,rbZdm$$
$$= 2\pi rb\sigma\rho\bar{w}\sin\beta\frac{d}{dm}(r^2\Omega - r\bar{w}\cos\beta)\,dm$$

Replacing $\bar{w} = \bar{w}_0/\sigma$, this equation becomes

$$w_s - w_p = \frac{2\pi r}{Z}\sin\beta\Big\{2\Omega\frac{dr}{dm} + \frac{1}{\sigma^2}\,\bar{w}_0\cos\beta\frac{d\sigma}{dm} - \frac{1}{\sigma}\Big(\frac{\bar{w}_0}{r}\cos\beta\frac{dr}{dm} + \cos\beta\frac{d\bar{w}_0}{dm} - \bar{w}_0\sin\beta\frac{d\beta}{dm}\Big)\Big\} \qquad (12)$$

Comparison With Experiments of Centrifugal Impellers

Reliable experimental data on the tip clearance effects of centrifugal impellers are scarce in the literature. In [9] data of two impellers are presented where uncertainty of efficiency and head coefficient are 0.019 and 0.008 respectively.

The tip clearance changes the head coefficient in two ways: one through input head ψ_i and the other through head loss. That is, in order to predict head coefficient, it is necessary to know variation of input head due to tip clearance, but the relation is not well known. Therefore, comparison of predicted head coefficient with experimental data is not possible. On the other hand, variation of efficiency is not much influenced by a little change of the input head. However, uncertainty of 0.019 is not satisfactory for the present purpose because the variation of efficiency is about 0.05 for a change of 0.1 in λ_2. Because of these reasons quantitative comparison between the prediction and experimental data is difficult unless accurate data with respect to efficiency are available. At the present stage only the order of magnitude of efficiency change should be compared between the prediction and the experiments.

According to [9] R-impeller is an impeller of 0.21m in diameter with 20 radial blades and 20 inducers and b_2/r_2 = 0.14. B-impeller is an impeller of 0.51m in diameter with 16 backward-leaning untwisted blades and b_2/r_2 is 0.067. The latter had originally a rotating shroud which was removed for the experiment. They were tested at 4000 and 2000 rpm respectively.

The tip clearances of these impellers were changed by moving the stationary shrouds axially relative to the impellers. In the case of B-impeller, the clearance is constant from the inlet to the exit of the impeller, while the clearance of R-impeller varies from the radial clearance at the inlet to the axial clearance at the exit and the distribution of clearance varies with the dimension of the exit clearance. The details are presented in [9].

The characteristic curves of these impellers demonstrate that the decrement of pressure due to tip clearance is large at the medium flow rate but it is rather small both at a large flow rate and at a small flow rate [9]. The pressure distributions along the wall demonstrate that at off-design conditions the leakage flow through the tip clearance relieves the pressure drop behind the leading edge and reduces the incidence loss, and the decrement of hydraulic loss $\psi_h - \psi_{h0} < 0$ in equation (10) considerably compensates the pressure loss due to the tip clearance c. Therefore, in this paper prediction is limited to three flow rates near the design point where the incidence loss is not significant even when the tip clearance is small.

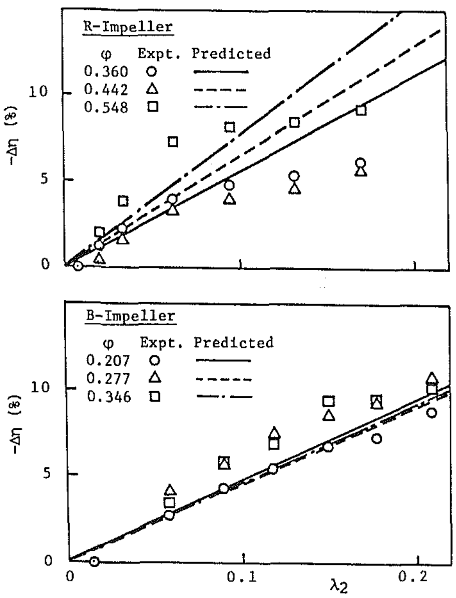

Fig. 3 Decrement of efficiency due to tip clearance ratio, centrifugal impellers

Prediction and Comparison with Experiments. In the case of R-impeller the velocity distribution along the shroud was estimated based on an inviscid quasi-three-dimensional flow analysis at three different flow rates. In the case of B-impeller, the shroud profile is almost straight and the

Table 1 Comparison between experimental and predicted efficiency drops due to tip clearance, centrifugal impellers

	φ	ψ_{io}	$-(\frac{\Delta\psi_i}{\Delta\lambda_2})_e$	η_o	$-\frac{\Delta\eta_e}{\Delta\lambda_2}$	$\frac{\Delta\psi_i}{\Delta\lambda_2}$	$\frac{\Delta\psi_L}{\Delta\lambda_2}$	$-\frac{(1-\eta_o)}{\psi_i}(\frac{\Delta\psi_i}{\Delta\lambda_2})_e$	$-\frac{\Delta\eta}{\Delta\lambda_2}$	$\frac{-\Delta\eta_e/\Delta\lambda_2}{-\Delta\eta/\Delta\lambda_2}$
R-Impeller	0.360	1.76	0.39	0.91	0.55	0.637	0.940	0.020	0.554	0.98
	0.442	1.76	0.43	0.90	0.62	0.715	1.080	0.025	0.638	0.97
	0.548	1.76	0.40	0.90	0.87	0.825	1.300	0.022	0.761	1.14
B-Impeller	0.207	1.35	0.10	0.77	0.44	0.256	0.610	0.017	0.471	0.93
	0.277	1.21	0.04	0.81	0.52	0.172	0.540	0.005	0.454	1.15
	0.346	1.07	0.14	0.81	0.53	0.100	0.468	0.026	0.464	1.15
B'-Impeller	0.207	(1.35)	-	-	-	0.261	0.722	(0.017)	0.553	-
B"-Impeller	0.346	(1.21)	-	-	-	0.178	0.537	(0.005)	0.451	-

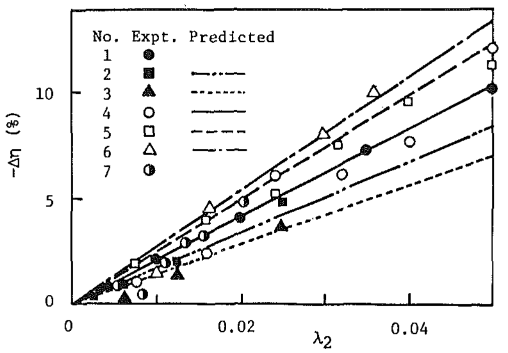

Fig. 4 Decrement of efficiency due to tip clearance ratio, seven axial impellers

Table 2 Comparison between experimental and predicted efficiency drops due to tip clearance, axial impellers

No.	Authors	φ	ψ_e	$\sin\beta_m$	$\frac{\Delta\psi_l'}{\psi_i\Delta\lambda}$	$\frac{\Delta\psi_l}{\psi_i\Delta\lambda} = -\frac{\Delta\eta}{\Delta\lambda}$	$-\frac{\Delta\eta_e}{\Delta\lambda}$	$\frac{-\Delta\eta_e/\Delta\lambda}{-\Delta\eta/\Delta\lambda}$
1	Jefferson & Turner	0.94	0.87	0.9	1.108	1.774	2.0	1.13
2	Williams	0.343	0.5	0.581	0.978	1.681	1.8	1.07
3	Williams	0.500	0.29	0.707	0.670	1.402	1.4	1.00
4	Ruden	0.388	0.225	0.46	0.991	2.086	2.0	0.96
5	Ruden	0.34	0.288	0.40	1.294	2.433	2.4	0.99
6	Kolesh-nikov	0.6	0.6	0.575	1.440	2.689	2.8	1.04
7	Spencer	0.21	0.26	0.246	1.603	2.572	2.3	0.89

meridian component of velocity along the shroud was simply estimated using the continuity equation. The distributions of the flow angle and the velocity were estimated at three flow rates following the simple method mentioned before. The decrement of efficiency due to the tip clearance was calculated for various values of tip clearance λ_2 at the impeller exit assuming that $\alpha = 0.8$, $\epsilon = 0.9$, and $\delta = 0.3$. The results show that the efficiency drops almost in proportion to the tip clearance. The experimental data of these impellers are plotted against the tip clearance λ_2 in Fig. 3. The relation is straight for B-impeller. In the case of R-impeller the relation is not straight, but in the range $0<\lambda_2<0.10$ it is almost straight. The lines in Fig. 3 are the predicted lines.

In order to present the contribution of each term of equations (6) and (11) for the drop of the efficiency, the values of these terms are evaluated for $\lambda_2 = 0$ and $\lambda_2 = 0.10$ and the differences are divided by $\Delta\lambda_2 = 0.10$ to estimate the inclination. The results are presented in Table 1 together with the inclinations of experimental data which are decided by means of the least mean square method. Agreement between prediction and experimental data is fair and satisfactory considering the large uncertainty involved in the experimental data.

Influence of Impeller Geometry on Clearance Losses

Figure 3 and Table 1 clearly show that $-\Delta\eta/\Delta\lambda_2$ becomes larger as the flow rate φ is larger for R-impeller, while it becomes slightly smaller as the flow rate is larger for B-impeller. It is observed in Table 1 that the column $\Delta\psi_l'/\Delta\lambda_2$ or the leakage loss is the item which has opposite tendency for the two impellers. In the case of R-impeller with radial blades, the blade loading is proportional to the flow rate and the leakage loss is increased with the flow rate. In the case of B-impeller with backward leaning blades the blade loading is reduced as the flow rate becomes larger and the leakage loss is also reduced.

B'-impeller is proposed which has the blade geometry and the inlet and exit widths identical to those of B-impeller, but the blade width is narrower at the middle so that the minimum relative velocity does not occur at the middle. The predicted tip clearance loss of B'-impeller is considerably larger than that of B-impeller as listed in Table 1. That is, the tip clearance loss is influenced by the distribution of relative velocity in the impeller.

B''-impeller is designed so that it has the identical pressure rise and the flow rate as those of B-impeller at $\varphi = 0.277$, but the exit width b_2 of B''-impeller is 0.8 times the width of B-impeller. The prediction in Table 1 shows that the ratios $\Delta\psi_l/\Delta\lambda_2$ and $\Delta\eta/\Delta\lambda_2$ are almost equal to those of B-impeller. For the same tip clearance, the clearance ratio λ_2 of B''-impeller is 1.25 times the value of B-impeller; therefore, the clearance loss is also 1.25 times as large. In cases of low specific speed impellers where the tip clearance cannot be made very small, a better efficiency may be achieved by designing a wider impeller with many blades so that the clearance loss is reduced even though the fluid dynamic losses other than the clearance loss are larger.

Working Equations for Axial Impellers

The foregoing equations are derived for mixed flow impellers, which include axial impellers as a special case with $dr/dm = 0$. In cases of axial impellers, equations (2, 3, 5) are respectively

$$\psi_l' = \frac{Z}{\pi\tilde{r}\varphi}\int_{m_1}^{m_2}\frac{\rho}{\rho_2}\frac{\sqrt{2}\sigma\alpha\lambda}{\sin\beta_b}\left(\frac{\bar{w}}{U}\frac{w_s - w_p}{U}\right)^{1.5} dm \quad (13)$$

$$\psi_l'' + \psi_l''' = \frac{-2\epsilon}{\tilde{r}\varphi}\int_{m_1}^{m_2}\frac{\rho}{\rho_2} r\lambda(1+\delta)\sin\beta\,\frac{\bar{w}^2}{U^3}\frac{d\bar{w}}{dm}\,dm \quad (14)$$

where equation (1) is substituted for v, and the flow rate is expressed as $2\pi\tilde{r}b\varphi U = 2\pi r(\tilde{r}/r)b\varphi U$. That is, $(\tilde{r}/r)b$ is an equivalent blade length. Since $\sigma\bar{w}\sin\beta = \varphi U$ and $\rho/\rho_2 \cong 1$, equation (14) becomes

$$\psi_l'' + \psi_l''' = \frac{\epsilon\lambda(1+\delta_m)}{\tilde{r}/r}\frac{w_1^2 - w_2^2}{\sigma U^2} \quad (15)$$

If it is further assumed that the blade loading is uniform from the leading edge to the trailing edge, the lift force L of a blade is

$$L = \rho l w_m (w_s - w_p)_m$$

The work input of a blade is related to the theoretical head as $LU\sin\beta_m = (\rho\psi_i U^2/2)\,s\varphi U$. Substituting for L

$$\frac{w_m(w_s - w_p)_m}{U^2} = \frac{s\varphi\psi_i}{2l\sin\beta_m} \quad (16)$$

On the other hand

$$w_1^2 - w_2^2 = w_{1u}^2 - w_{2u}^2 = 2w_{mu}(w_{1u} - w_{2u})$$

and $U(w_{1u} - w_{2u}) = \psi_i U^2/2$. Therefore,

$$w_1^2 - w_2^2 = w_m\cos\beta_m\psi_i U = (\varphi/\sigma)\psi_i U^2/\tan\beta_m \quad (17)$$

Substituting equations (16) and (17) into equations (13) and (15)

$$\psi_l' = \frac{\alpha\lambda\psi_i}{\tilde{r}/r}\sqrt{\frac{\sigma s}{l}\frac{\varphi\psi_i}{\sin^3\beta_m}} \quad (18)$$

$$\psi_l'' + \psi_l''' = \frac{\epsilon\lambda(1+\delta_m)}{(\tilde{r}/r)\sigma^2}\frac{\varphi\psi_i}{\tan\beta_m} \quad (19)$$

Equation (11) becomes

$$\frac{\eta_0 - \eta}{\lambda} = \frac{\alpha}{\tilde{r}/r}\sqrt{\frac{\sigma s\varphi\psi_i}{l\sin^3\beta_m}} + \frac{\epsilon(1+\delta_m)\varphi}{\sigma^2(\tilde{r}/r)\tan\beta_m}$$

$$-\frac{\psi_{h0}-\psi_h}{\psi_i\lambda}+\frac{1-\eta_0}{\psi_i}\frac{\psi_{i0}-\psi_i}{\lambda} \tag{20}$$

Comparison With Experiments of Axial Impellers

Seven experimental data out of eight data in [2] are compared with the predictions based on equation (20). One case is excluded because it is an experiment on a turbine rotor. Since the pitch chord ratio at the blade tip is not given in [2], it is simply assumed that $s/l = 1.0$. It is also assumed that the hub tip ratio is 0.6 or the ratio $\bar{r}/r = 0.8$ except the case No. 7, where it is assumed that $\bar{r}/r = 1.0$ because the aspect ratio of the blades is 0.47 and a large hub/tip ratio is expected.

The mean flow angle β_m of the original papers is quoted in [2] except two cases, No. 4 and No. 5. In [2] it is estimated that $\sin\beta_m = 0.46$ for the two cases. Since they are the data at two different flow rates of an impeller, $\sin\beta_m$ must vary with the flow rate. Here it is assumed that $\sin\beta_m = 0.46$ for No. 4 following [2], and $\sin\beta_m = 0.40$ is assumed for No. 5 so that they are proportional to the flow coefficients.

It is necessary to estimate a few coefficients in the equations. They are assumed $\alpha = 0.8$, $\epsilon = 0.9$ and $\delta = 0.3$, identical to those for centrifugal impellers in the last examples. Furthermore, the efficiency is assumed $\psi/\psi_i = 0.9$ and the last two terms in equation (20) are disregarded because of lack of relevant data and also because they are very small for axial impellers with $\lambda < 0.05$ and $0.985 < \sigma < 1.0$.

The decrements of efficiency due to the tip clearance at various values of λ_2 in the literature are reproduced in Fig. 4. These data show that the efficiency drop is proportional to λ_2. For comparison the predicted relationships for those impellers are indicated as lines. Good agreement with the experimental data is observed.

Regarding the data in Fig. 4, the conditions of flow in the impellers are presented in Table 2 together with the predicted efficiency drops and clearance losses which consist of the leakage loss and the loss due to the pressure gradient. Comparison with the experimental data is in the right end column. Fair agreement is observed.

Optimum Tip Clearance

According to the present theory there is no optimum dimension of the tip clearance, because all the losses related to the tip clearance are almost proportional to the tip clearance. In a cascade test, it is demonstrated [10] that the secondary flow along the endwall becomes weak by the leakage through the tip clearance and the overall pressure loss can be smaller as the tip clearance is larger.

Also there is a note [2] which experimentally recommends a tip clearance of ~1–1.5% of the blade span as the optimum, but in all seven cases in Table 2, the efficiency is continuously improved by reducing the tip clearance and in cases of No. 2 and No. 3 the minimum clearance was as small as 0.3% of the blade span. Judging from these data and above discussions, it is not easy to estimate the optimum tip clearance for respective cases, and even if it is determined, it is likely that the optimum clearance is too small to make in practice.

Conclusions

The pressure loss and the efficiency drop based on the tip clearance of impellers are examined and the following items are clarified.

1 The pressure loss due to the tip clearance of an impeller is induced by two types of drag forces: One is induced by the leakage flow through the tip clearance and the other is the force to support a layer of fluid against the pressure gradient.

2 Equations of pressure loss and of efficiency drop due to tip clearance are derived for axial impellers as well as for centrifugal impellers. They are not applicable to turbine rotors.

3 A few empirical coefficients are included in these equations. For the time being they were assumed based on the knowledge of conventional hydraulics. Some of them may be examined based on flow measurement at the exit of impellers.

4 The tip clearance loss and the efficiency drop are almost proportional to the tip clearance ratio at the exit of impellers for $\lambda_2 < 0.1$.

5 Prediction is less accurate at off-design conditions, because incidence loss varies with the tip clearance.

6 Tip clearance losses are predicted for three impellers which have identical specifications. The results clearly show that tip clearance ratio is the most important parameter but it is not the only factor which controls the tip clearance losses.

7 Experimental data with good accuracy are required for critical comparison and improvement of the present method.

8 In order to predict the change of discharge pressure due to tip clearance, a good prediction method regarding the change of Euler head due to tip clearance is required.

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