Complexity of Optimal Lobbying in Threshold Aggregation (Extended Abstract)

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ABSTRACT

Optimal Lobbying is the problem a lobbyist or a campaign manager faces in a full-information voting scenario of a multi-issue referendum when trying to influence the result. The Lobby is faced with a profile that specifies for each voter and each issue whether the voter approves or rejects the issue, and seeks to find the smallest set of voters it must influence to change their vote, for a desired outcome to be obtained. We study the computational complexity of Optimal Lobbying when the issues are aggregated using an anonymous monotone function and the family of desired outcomes is an upward-closed family. We analyze this problem with regard to two parameters: the minimal number of supporters needed to pass an issue, and the size of the maximal minterm of the desired set. We show that for extreme values of the parameters, the problem is tractable, and provide algorithms. On the other hand, we prove intractability of the problem for the complementary cases, which are most of the values of the parameters.

Categories and Subject Descriptors

F.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems —*Computations on discrete structures*; J.4 [**Social And Behavioral Sciences**]: Economics

Keywords

optimal lobbying; computational complexity; threshold func-

In many voting scenarios an outcome is an assignment to a set of independent issues (e.g., voting on a series of clauses of a bill in the parliament). In such scenarios the voting process for the chosen outcome — an aggregated outcome for each of the issues — is usually done by running independent elections, one for each of the issues.

The Optimal Lobbying problem is the problem an external actor, the *Lobby*, is facing when it seeks to change the outcome to belong to the *Desired Outcomes* set. The Lobby tries to achieve its goal by influencing voters to change their vote, and seeks to influence the minimal number of voters while still achieving its goal. We define the optimization problem of finding this *optimal coalition* the Lobby should influence and the corresponding decision problem of determining whether there exists such a coalition of size at most k (the *budget* of the Lobby), and study the complexity of these problems.

Optimal Lobbying also describes problems arising in other scenarios of aggregating complex opinions. For instance, principal-agents incentives scheme in a complex combinatorial problem, and bribery and manipulation in Truth-Functional judgement aggregation.

In this paper, we study the computational complexity of Optimal Lobbying for several basic scenarios: The issues are Boolean (a voter either approves or rejects each issue); The desired outcomes set is an upward-closed set (approving more issues can only make the result more desired); And all issues are aggregated using the same anonymous monotone function, that is, an issue passes iff at least t voters approve it, for a predefined *threshold* t. Later, we discuss more general scenarios of Optimal Lobbying, e.g., when the desired outcomes set is "at least one issue should fail to pass" which is equivalent to "prevent the outcome where all issues pass".

The current literature deals mostly with the following simple cases: Each issue passes iff it is approved by a majority of voters and the desired outcomes set consists of a single outcome or is defined by a minimal number of issues that should pass. These cases are known to be computationally hard [2, 1], but it is easy to see that not all cases (all

order to pass — and the Lobby desires all issues to pass, then the problem is linearly solvable by a straightforward greedy algorithm. Similarly, the problem is linearly solvable when aggregating using majority and at least one issue should not pass, by greedily finding the "cheapest" issue.

In many real-life situations one finds non-majority issue aggregation functions or desired outcomes sets consisting of more than one outcome. Hence, we check the sensitivity of the computational complexity of Optimal Lobbying for several voting schemes and desired outcomes sets.

Results

We analyze the time complexity of the problem as a function of two parameters: **(1) t**: The *threshold of the issue-aggregation function* — The minimal number of supporters

Appears in: *Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2013),* Ito, Jonker, Gini, and Shehory (eds.), May 6–10, 2013, Saint Paul, Minnesota, USA.

needed in order to pass an issue — and **(2) z**: The *size of the maximal minterm of the desired set* — The *minterms* are defined to be the (inclusion-wise) minimal desired issues sets. E.g., the minterms of the set represented by $\psi = \left(x^1 \wedge x^2\right) \vee \left(x^2 \wedge x^3 \wedge x^4\right)$ are 1100 and 0111.

We show that the complexity is characterized almost solely by these two parameters. We did not find a discrepancy between the tractability of the search problem and the tractability of the decision problem, and in particular, for all the cases in which we prove the decision problem to be polynomially solvable, we provide algorithms for the search problem.

When $t < n$ (non-unanimity threshold issue aggregation), the problem is tractable if either the threshold of the issue-aggregation function is constant, or if all the minterms of the desired outcomes set are of constant size. On the other hand, when both these parameters are not constant, we show that the problem is intractable, and by that we extend the results shown in Christian et al.[2] and Bredereck et al.[1]. The results for this case are summarized in Table 1a.

When $t = n$ (issue-wise unanimity) the time complexity cannot be characterized solely by the size of the maximal minterm. When the desired outcomes set can be described using $poly\,(m)$ minterms, the problem is tractable. On the other hand, when the desired outcomes set can be described by a threshold (that is, at least z issues should pass) and the threshold is not too extreme (i.e., $superpolylog\,(m)$-far from the boundaries – 0 and m), the problem is intractable. The results for this case are summarized in Table 1b.

Extensions to More Scenarios

Notice that more general scenarios can be reduced trivially to the the above scenarios. Scenarios including downward monotone issue-aggregation function and downward-closed desired outcomes set are reducible to the above scenarios by negating φ and ψ, and the input matrix, e.g., the problem in which at most one issue should pass when aggregating using unanimity is equivalent to the problem in which at least $m-1$ issues should pass when aggregation using threshold of one and with the negated input profile. We can conclude also hardness results of scenarios in which the opinions are not binary but multi-valued, when these problems restricted to two of the values are proven to be hard by our work.

Future Work

In the full version[6] we also analyze the parameterized complexity of this problem w.r.t. the budget. We would like to extend this work and analyze the parameterized complexity w.r.t. other parameters we identify, and extend our analysis to other issue-aggregating functions (φ) and other desired outcomes families (ψ), both monotone and non-monotone. When extending the study to other functions, we expect to see problems that are due to computability problems in φ and ψ beyond the complexity issues of Optimal Lobbying. For example, when φ or ψ is the indicator function of an undecidable problem, the problem of calculating the outcome of the input profile, which is equivalent to the problem of deciding whether influencing is needed, is undecidable.

In cases where we proved the problem to be intractable, a natural question is the approximation question. Currently, the main two approximation works are [3, 4] but both deal with a slightly different framework than ours. We would like to extend these approximability/inapproximability results to the model defined in this paper. An upper bound on the approximability ratio is obtained by two immediate reductions (we conjecture that they'll generate very similar algorithms and hence the same ratio) – to the SET MULTI-COVER problem[7, p. 112] and to the REVERSE COMBINATORIAL AUCTION problem[5].

Acknowledgments

The author would like to thank Yehuda (John) Levy as well as the anonymous referrers for their comments that helped improve this paper. This research was supported by a grant from the Israeli Science Foundation (ISF) and by the Google Inter-university Center for Electronic Markets and Auctions.

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Table 1: Summary: Computational Complexity of Optimal Lobbying $(\mathrm{OLD}(\varphi, \psi))$

(1a) Non-unanimity Issue-Aggregation ($\psi \neq Unan$)

t: The Threshold of φ ($t < n$ Non-Unanimity Issue Aggregation)
z: The size of the maximal minterm of ψ

	$\exists\alpha : t \leqslant \alpha$	$t \in \cap_{\alpha>1}\left[\log^\alpha n,\, n-\log^\alpha n\right]$	$\exists\alpha : t \in \left[n^{1/\alpha},\, n-n^{1/\alpha}\right]$	$\exists\alpha : t \in \left[n-n^{1/\alpha},\, n-1\right]$
$\exists\alpha : z \leqslant \alpha$	P	P	P	P
$z \in \cap_{\alpha>1}\left[\log^\alpha m, m\right]$	P	$\notin P$ (1)	$\notin P$ (1)	$\notin P$ (1)
$\exists\alpha : z \geqslant m^{1/\alpha}$	P	$\notin P$ (1)	*NP-Complete*	*NP-Complete*
$z = m$ (2)	P	$\notin P$ (1)	*NP-Complete*	*NP-Complete*

(1) Assuming $NP \nsubseteq SUBEXP$.
(2) I.e., all issues should pass.

(1b) Unanimity Issue-Aggregation ($\psi = Unan$)

z: The size of the maximal minterm of ψ

If ψ can be defined using at most m^α minterms:		P
If ψ is a threshold function with threshold z and	$\exists\alpha : z \leqslant \alpha$	P
	$\exists\alpha : z \in \left[m^{1/\alpha},\, m-m^{1/\alpha}\right]$	*NP-Complete*
	$z \in \cap_{\alpha>1}\left[\log^\alpha m,\, m-\log^\alpha z\right]$	$\notin P$ (1)
	$\exists\alpha : z \geqslant m-\alpha$	P

(1) Assuming $NP \nsubseteq SUBEXP$.