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OPTIMAL SENSOR PLACEMENT FOR DAMAGE DETECTION IN COMPLEX STRUCTURES

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ABSTRACT

An optimal sensor placement methodology is proposed based on detection theory framework to maximize the detection rate and minimize the false alarm rate. Minimizing the false alarm rate for a given detection rate plays an important role in improving the efficiency of a Structural Health Monitoring (SHM) system as it reduces the number of false alarms. The placement technique is such that the sensor features are as directly correlated and as sensitive to damage as possible. The technique accounts for a number of factors, like actuation frequency and strength, minimum damage size, damage detection scheme, material damping, signal to noise ratio (SNR) and sensing radius. These factors are not independent and affect each other. Optimal sensor placement is done in two steps. First, a sensing radius, which can capture any detectable change caused by a perturbation and above a certain threshold, is calculated. This threshold value is based on Neyman-Pearson detector that maximizes the detection rate for a fixed false alarm rate. To avoid sensor redundancy, a criterion to minimize sensing region overlaps of neighboring sensors is defined. Based on the sensing region and the minimum overlap concept, number of sensors needed on a structural component is calculated. In the second step, a damage distribution pattern, known as probability of failure distribute, is calculated for a structural component using finite element analysis. This failure distribution helps in selecting the most sensitive sensors, thereby removing those making remote contributions to the overall detection scheme.

1 INTRODUCTION

Guided wave based structural health monitoring (SHM) is being widely used in aerospace and civil infrastructure. Piezoelectric transducers are used for guided wave generation and sensing and have great advantage for onboard SHM. In this active sensing approach, user-defined energy is imparted in the structure with an actuator and the responses obtained from the sensors are mined for any useful information. A network of sensors is thus required to monitor the entire area of the structure. This sensor network, if not decided intelligently, involves extra complexity in data management. An optimal sensor placement helps in reducing this complexity by providing important features sensitive to damage.

Sohn et al. [1] and Maul et al. [2] have given lengthy reviews on sensor placement strategies. Gao et al. [3], Guo et al. [4], and Richardson and Abdullah [5] studied sensor placement for vibration based SHM. Although extensive work is done in this area, limited studies on sensor placement for ultrasonic-based structural health monitoring exist in literature. Gao and Rose [6] presented a sensor placement optimization method with covariance matrix adaptation evolutionary strategy (CMAES). Guratzsch and Mahadevan [7] developed a method for optimal sensor placement based on finite element probabilistic models incorporating for uncertainties. Staszewski et al. [8] and Worden and Burrows [9], studied the problem of sensor location optimization for damage detections with selected sensor distribution from a set of predefined possible sensor locations using binary coded simple genetic algorithm (SGA). Das et al. [10] developed a placement strategy in which the sensors are distributed uniformly over the structure to cover maximum detectable area, have minimum overlap between their sensing regions and the sensing radius is such that any change in the signal due to presence of damage above a threshold value is detectable.

In this paper a statistical method based on detection theory is used for optimal sensor placement. A modified test statistics is developed using Generalized Likelihood Ratio Test (GLRT) which provides a threshold energy that a sensor can detect in the presence of noise. This threshold value is based on Neyman-Pearson detector that maximizes the detection rate for a fixed false alarm rate. A minimum overlap criterion of the sensing regions of neighboring sensors is set up based on optimal intersection of three sensing circles. Based on the sensing region and the minimum overlap concept, sensors are distributed uniformly over the structural component. Later, a damage distribution pattern, known as probability of failure distribute, is calculated for the given component using finite element analysis. This failure distribution helps in selecting the most sensitive sensors, thereby removing those making remote contributions to the overall detection scheme.

2 OPTIMAL SENSOR PLACEMENT

In most SHM applications, sensors are placed on the surface of the structure. Hence a two-dimensional optimal sensor placement strategy is developed. This placement strategy is based on several factors which for precise understanding is explained in steps.

Attenuation Coefficient

This is a characteristic of the material of the structure and is calculated through experiment. For a plate like structure, surface-mounted piezoelectric transducers are used both as sensors and actuators. When an actuator is excited, energy is imparted to the structure through their contact. This energy transmission can be characterized through a transfer function (T_{SH}). Similarly the response of the structure obtained at a given sensor can be characterized by a transfer function (T_{HS}) between them. The layout of the sensor network is shown in Fig. 1. Sensor 1 and sensor 2 are symmetrically placed compared to actuator 1 whereas they are at different distances from actuator 2. The transfer function for a given actuator-sensor pair is given in Eqn. 1

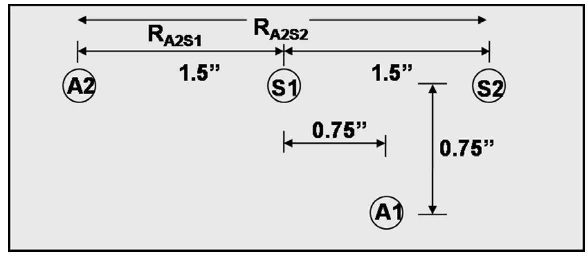

Figure 1. PIEZOELECTRIC TRANSDUCER LAYOUT FOR CALCULATION OF ATTENUATION COEFFICIENT

$$\frac{E_S}{E_A} = (T_{HS}T_{SH})^2 exp(-2\alpha R_{AS}) \tag{1}$$

When actuator 1 is given an excitation, the ratio of the response energies at the sensors gives relative information about their electro-mechanical properties thereby providing details of the structure/host coupling. Since actuator 1 is the source of energy for both the sensors, the effect of its electromechanical properties at the two sensors are nullified (see Eqn. 2)

$$\left(\frac{E_{S_1}}{E_{S_2}}\right)_{A_1} = \left(\frac{T_{S_1H}}{T_{S_2H}}\right)^2 \tag{2}$$

where, E_{S_1} and E_{S_2} are the signal energies received at sensors 1 and 2 respectively. T_{S_1H} is the transfer function between sensor 1 and the host and T_{S_2H} is the transfer function between sensor 2 and the host. When actuator 2 is used to excite the structure, the ratio of the energies at the two sensors is given by Eqn. 3 and the attenuation coefficient (α) thus calculated is given by Eqn. 4.

$$\left(\frac{E_{S_1}}{E_{S_2}}\right)_{A_2} = \left(\frac{T_{S_1H}}{T_{S_2H}}\right)^2 exp[-2\alpha(R_{A_2S_1} - R_{A_2S_2})] \tag{3}$$

$$\alpha = \frac{1}{2(R_{A_2S_2} - R_{A_2S_1})} \log\left[\left(\frac{E_{S_2}}{E_{S_1}}\right)_{A_1}\left(\frac{E_{S_1}}{E_{S_2}}\right)_{A_2}\right] \tag{4}$$

where, $R_{A_2S_2}$ is the distance between sensor 2 and actuator 2 and $R_{A_2S_1}$ is the distance between sensor 1 and actuator 2.

Composite Hypothesis Testing

One of the main challenges for damage detection in structural components using active health monitoring schemes is to

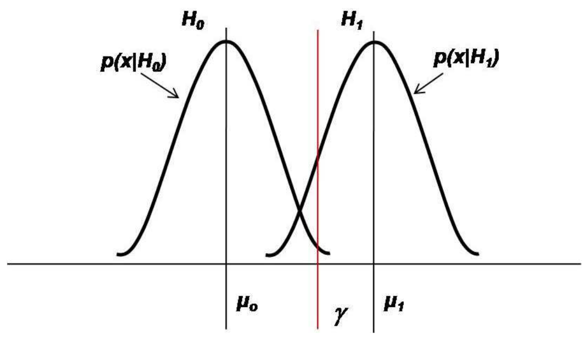

Figure 2. PROBABILITY DENSITY FUNCTIONS FOR HYPOTHESIS TESTING AND DECISION REGIONS

extract salient features from signals embedded in noise. Often weak signals with poor signal-to-noise ratio (SNR) lead to false detection, false characterization and failure of a given damage detection algorithm. Hence, arises the need of an appropriate detector which can maximize the detection rate using the observed sensor signals. Using composite hypothesis testing model, two hypothesis can be defined as follows
$H_0 : x(n) = w(n)$
$H_1 : x(n) = s(n) + w(n)$
where $s(n)$ is the signal to be detected, $w(n)$ is assumed to be white Gaussian noise with known variance σ^2 and n are the data points denoted by $n = 1, 2, \cdots, N-1$. Figure 2 is a simple representation of the probability density functions (PDFs) under hypothesis H_0 and H_1 A modified test statistics $T(x)$ is derived making use of Generalized Likelihood Ratio Test (GLRT) [11] and is written as

$$T(x) = \frac{1}{N}\left[\sum_{n=0}^{N-1} x(n) exp(-j2\pi f_0 n)\right]^2 > V_{th}^2 \tag{5}$$

where j is an imaginary number, f_0 is the excitation frequency and V_{th}^2 is the threshold value. V_{th}^2 is the energy beyond which the signal buried in noise can be detected. The detection performance of the sensor can thus be given as

$$P_{fa} = exp\left(-\frac{V_{th}^2}{\sigma^2}\right) \tag{6}$$

and

$$P_d = Q_{\chi_2'^2(\lambda)}\left(\frac{2V_{th}^2}{\sigma^2}\right) \tag{7}$$

where, P_{fa} is the probability of false alarm, P_d is the probability of detection. The operator $Q_{\chi_2'^2(x)}$ calculates the right-tail probability of a noncentral chi-square PDF of the random variable x with a positive noncentrality parameter $\lambda = NA^2/2\sigma^2$.

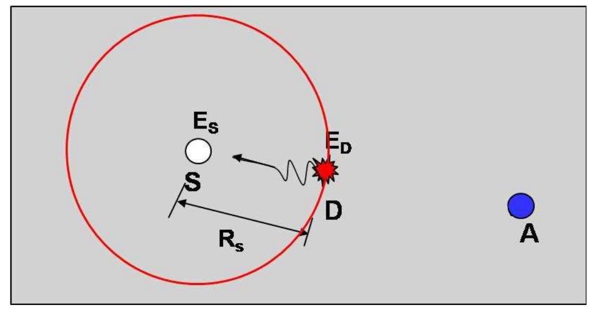

Figure 3. DAMAGE ACTS AS SOURCE OF ENERGY WHEN AN ACTUATION SIGNAL INTERACTS WITH IT. A SENSOR NEEDS TO BE PLACED IN SUCH A WAY THAT THE ENERGY REACHING IT FROM THE DAMAGE SOURCE SHOULD BE MORE THAN THE THRESHOLD VALUE. A CIRCULAR REGION WITH RADIUS R_s REPRESENTS THE SENSING REGION OF THE SENSOR

Sensing Region and Minimum Overlap Criteria

In a structure with defect, the wave interacts with it and the response obtained at the sensor is thus modified/changed from the healthy signal. According to Huygens principle, the defect acts as secondary source of energy and is responsible for this change. Figure 3 explains this principle. If the energy traveling from the secondary source is more than the threshold value at the receiver end, then it can be detected. This can be characterized by the following relation

$$E_s = E_d \; exp(-2\alpha R_s) \tag{8}$$

and to satisfy the detection criteria

$$E_s \geq V_{th}^2 \tag{9}$$

where, E_s is the energy received at the sensor end, E_d is the change in energy due to the presence of defect. R_s is the distance between sensor and the defect, also called as the sensing radius. The sensing region is considered to be circular for isotropic materials with radius R_s and a perturbation/change caused by any defect in this region can be picked up by the sensor.

To make a sensing network fail-safe i.e. if any sensor fails, the neighboring sensor should still be able to sense any changes occurring in the region of the failed sensor, an optimum overlap criteria of the sensing region needs to be developed. In the present study this criteria is based on the optimal intersection of three sensing circles. For two overlapping circles with radius R_s if a third circle passes through them such that they all intersect

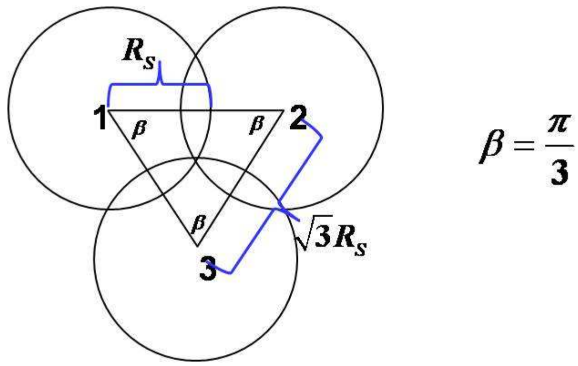

Figure 4. CRITERIA FOR MINIMUM OVERLAP BASED ON OPTIMUM INTERSECTION OF THREE SENSING CIRCLES

at a common point, then the overlap between the circles is optimum. As a result, the distance between the sensors (distance between the centers of the circles) is $\sqrt{3}R_s$ (see Fig. 4) As a first step, an initial sensor placement can be done by uniformly distributing the sensors based on the sensing radius, minimum overlap of the sensing regions and maximum coverage of the area under inspection. However, for a given structure, which is not uniform, not all the sensors have the same performance. As a result, some of the sensors become redundant and hence placement of sensors have to be revised considering the possible locations of the hotspots.

Sensor Placement and Network Performance Based on Probability of Damage Occurrence

For a given geometry and boundary conditions, under the application of load there are areas of high stress/strain concentration. These regions are structural hotspots where probability of damage occurrence is higher. In the present study, the probability of damage (P_{dam}) in each sensing region is calculated as follows:

1. Strain distribution in the entire structure is calculated by carrying out the finite element analysis of the structure
2. The probability of damage in these regions is calculated by taking the ratio of the strains in the given region to the total strain in the structure

Once the probability of damage in each sensing region is determined, the sensor network performance is calculated by solving the optimization problem given in Eqn. 10. Solving this problem gives the optimum location of sensors for a given geometry by removing the redundant sensors.

$$\begin{aligned} &\textbf{minimize: } \text{Number of sensors} = 1^T K \\ &\textbf{subject to: } \text{Sensor network performance} = P_{dam}^T K \geq \xi \\ &\text{and } K \in \{0,1\} \end{aligned} \tag{10}$$

where, the length of the vector is equal to the number of sensors uniformly distributed and ξ is the desired sensor network performance. When a particular sensor is not taken into account, it is given a value of 0 or else 1. This optimization problem is not a convex optimization problem as constraints on K are binary. It is solved using a branch and bound method and the solution of the problem gives the final optimized distribution of sensors.

3 OPTIMAL SENSOR PLACEMENT ON THE LUG JOINT

In this section an example is discussed on optimal sensor placement in lug joints. Lug joints are connector type elements used as structural supports for pin connections. The lug joint sample chosen for this study is made of Al 6061 plate, the dimensions of which are shown in Fig. 5. The steps discussed in the previous section are followed in this example. The transducers used are piezoelectric sensors, used both as sensors and actuators and a 4.5 cycle burst wave with 230KHz central frequency is used as an excitation signal. At this frequency the attenuation coefficient computed is 21.4225 using Eqn. 4. As a second step the threshold value has to be calculated. As it can be seen from Eqn. 6, threshold energy can be calculated once the probability of false alarm and the variance of noise signal is known. The variance of the noise signal is calculated through experiments by picking up the sensor responses even when the actuator is not excited. Fifty sensor readings are acquired (see Fig. 6) to give a statistical distribution of these signals. The variance of the noise signal (σ^2) is then calculated from the sensor responses which is $8.575e-7$. For the variance of the noise signal calculated, the variation of the threshold energy with the probability of false alarm is shown in Fig. 7. A value of $1e-4$ for the probability of false alarm is taken in the current example for which the threshold energy calculated is 7.897e-6 $volt^2.time$.

Once threshold value is known, the sensing radius has to be determined. From Eqn. 8, it can be seen that the sensing radius depends upon the energy released (E_d) by the smallest damage under consideration that can be detected at the sensor end (E_s). E_d is calculated experimentally in the following manner

1. The lug joint sample with sensors attached to it is fatigued under a load of 300lbs (13.9N)to 3000lbs (139N) and sensor readings are taken at every 10000 cycles for burst signal of 4.5 cycles and 230 KHz central frequency
2. The sensor signals are analyzed for any change due to defects with a damage metric developed by the authors in [12] which makes use of Discrete Cosine Transformation (DCT)
3. It was found that first change in the sensor signal was seen at 180056 cycles (see Fig. 8). Hence the energy difference was calculated between the reference signal and the signal at 180056 cycle and was found to be 8.62e-5 $volt^2.time$. This energy change results from the smallest crack detectable

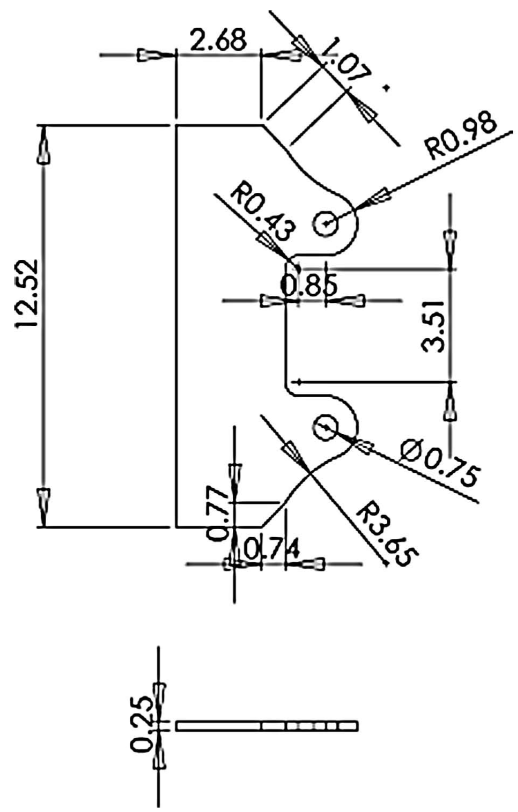

Figure 5. DIMENSIONS OF THE LUG JOINT SAMPLE MADE FROM AL 6061 PLATE IN INCHES

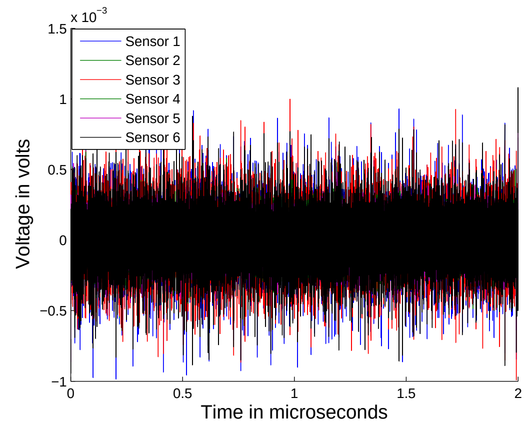

Figure 6. NOISE SIGNALS ACQUIRED AT THE SENSORS FOR NOISE VARIANCE CALCULATION

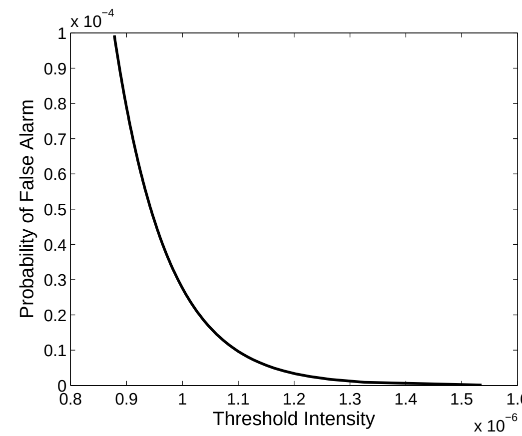

Figure 7. THRESHOLD ENERGY VARIATION WITH PROBABILITY OF DAMAGE

with the current sensors.

The sensing radius is calculated using Eqn. 11 and is 0.056m. The maximum distance between the neighboring sensors calculated using the minimum overlap criteria is $\sqrt{3}R_s = 0.097m$. An initial distribution of the sensors on the lug joint is shown in Fig. 9. A 3D finite element analysis is performed on the lug joint subjected to fatigue load and its strain distribution is calculated. Finally, based on the probability of damage occurrence in different regions of the lug joint, an optimal sensor placement is done as shown in Fig. 10.

$$R_s = \left(\frac{1}{2\alpha}\right)\log\left(\frac{V_{th}^2}{E_d}\right) \tag{11}$$

Another example with a sensing radius of $0.03m$ is demonstrated for the clarity of the algorithm developed. An initial sensor distribution based on sensing radius, minimum overlap criteria and maximum coverage of the area under inspection is shown in Fig. 11. It can be seen that 15 sensors have to be mounted on the surface of the lug joint to monitor damage. However, once the damage distribution in the lug joint is known from the finite element analysis, the optimization problem posed in Eqn. 10 can be solved using branch and bound method. The final optimal sensor placement on the lug joint is shown in Fig. 12. It can be seen that the number of sensors required to monitor damage reduces from 15 to 5.

4 CONCLUDING REMARKS

An important step in the overall structural health monitoring (SHM) scheme is optimal sensor placement on structural components. A methodology for sensor placement was presented which takes into account a number of factors, like actuation frequency and strength, minimum damage size, damage detection scheme, material damping, signal to noise ratio (SNR) and sensing radius. Using this technique a threshold value was calculated

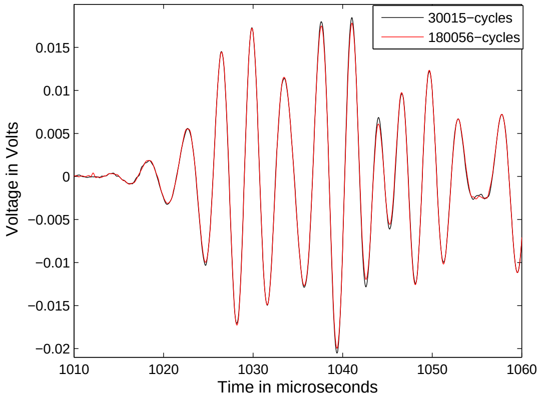

Figure 8. THE SIGNAL AT 30015 CYCLE IS TAKEN AS THE REFERNCE SIGNAL AND THE FIRST CHANGE WAS OBSERVED AT 180057 CYCLE. THE CHANGE IN THE ENERGIES OF THESE TWO SIGNALS IS TAKEN AS THE ENERGY RELEASED BY THE SMALLEST DAMAGE DETECTABLE

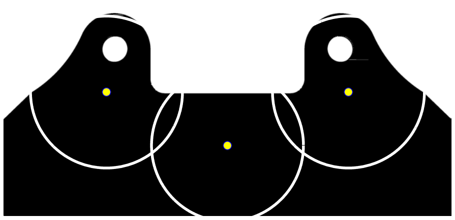

Figure 9. INITIAL SENSOR PLACEMENT BASED ON THE SENSING RADIUS OF 0.056m AND MINIMUM OVERLAP CRITERIA BUT NOT ACCOUNTING FOR STRUCTURAL GEOMETRY AND THE APPLIED LOADING

which maximizes the probability of detection for a given probability of false alarm and below which the output signals are neglected. Sensing region was then defined as the region in which the energy received from a perturbation was above the threshold value. To avoid unnecessary overlap between the sensing regions, a minimum overlap criteria was formulated. Finally it has been demonstrated that a known damage distribution pattern over the structure, based on geometry and loading conditions, in addition to the criteria developed, gives the optimal placement of sensors.

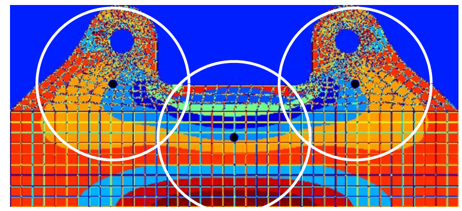

Figure 10. FINAL SENSOR PLACEMENT BASED ON THE SENSING RADIUS OF 0.056m, MINIMUM OVRLAP CRITERIA AND ALSO TAKING INTO ACCOUNT THE DAMAGE DISTRIBUTION PATTERN

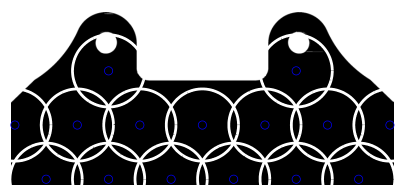

Figure 11. INITIAL SENSOR PLACEMENT BASED ON THE SENSING RADIUS OF 0.03m AND MINIMUM OVERLAP CRITERIA BUT NOT ACCOUNTING FOR STRUCTURAL GEOMETRY AND THE APPLIED LOADING

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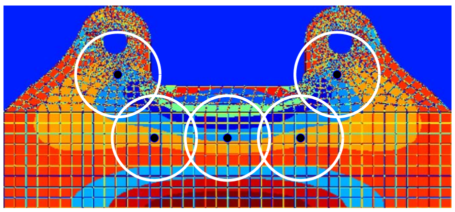

Figure 12. FINAL SENSOR PLACEMENT BASED ON THE SENSING RADIUS OF 0.03m, MINIMUM OVRLAP CRITERIA AND ALSO TAKING INTO ACCOUNT THE DAMAGE DISTRIBUTION PATTERN

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