

Nucleons in Nuclear Matter and the Transition to Quark Matter

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Abstract. We use an effective chiral quark theory to describe the single nucleon, the equations of state (EOS) of nuclear matter (NM) and quark matter (QM), and the phase transition from NM to QM at high density. We pay special attention to the effects of the nucleon quark structure on the EOS of NM, and of scalar diquark condensation (color superconductivity) on the EOS of QM.

INTRODUCTION

The possibility of a phase transition from nuclear matter (NM) to quark matter (QM) at high baryon densities is of great interest in connection with neutron stars and the possible existence of quark stars [1]. In order to describe such a phase transition, one needs a consistent theoretical framework which can account for the properties of single nucleons, NM and QM at the same time. It has been shown recently [2] that the Nambu-Jona-Lasinio (NJL) model [3] is a strong candidate for this purpose: Besides a covariant description of the nucleon as a quark-diquark bound state [4], it also allows a description of a saturating NM equation of state (EOS) [5] and of QM including the effects of color superconductivity [6]. It is therefore of interest to study the conditions under which a transition between NM and QM becomes possible. It is the purpose of this paper to show our results on this phase transition. For details of the model and the formulation we refer to Ref.[2]. The EOS for NM has also been used recently to describe the nuclear structure functions and the EMC effect[7, 8].

DESCRIPTION OF NUCLEAR MATTER AND QUARK MATTER

The NJL model is characterized by a chiral symmetric contact interaction between quarks. Because of its simplicity, the relativistic Faddeev equation for the nucleon can be solved in the ladder approximation, taking into account the interactions in the scalar and axial vector diquark channels[4]. For our present investigations at finite density, however, we will restrict ourselves to the scalar diquark channel, and to a simple approximation to the Faddeev equation, where the momentum dependence of the quark exchange kernel is neglected[5, 6]. Based on this quark-diquark description of the single nucleon, the EOS for NM can be constructed in the mean field approximation, and the phase transition to QM at high densities can be investigated.

The solution of the quark-diquark bound state equation leads to the nucleon mass $M_N(M)$ as a function of the constituent quark mass M . The effective (grand) potential for NM at zero temperature in the mean field approximation then has the form [5]

$$V^{(NM)} = V_{\text{vac}} + V_N + V_\omega, \quad (1)$$

where V_{vac} describes the polarization of the Dirac sea of quarks due to the presence of the valence nucleons, V_N arises from the Fermi motion of the valence nucleons and depends on the function $M_N(M)$, and V_ω is the contribution of the mean vector field (ω_0) in NM. The conditions $\partial V^{(NM)} / \partial M = \partial V^{(NM)} / \partial \omega_0 = 0$ determine M and ω_0 for fixed chemical potential μ .

In effective field theories based on the linear realization of chiral symmetry, one often observes a collapse of the NM EOS because the σ mass decreases too rapidly as a function of the density. It has been shown in Ref.[5], however, that the saturation properties of the NM EOS can be described if the quark structure of the nucleon is taken into account, provided that one eliminates the threshold for the unphysical decay of the nucleon into quarks. This can be done, for example, in the proper time regularization scheme by introducing an infrared cut-off (Λ_{IR}) in addition to the ultraviolet one [9]. The elimination of the unphysical decay threshold then leads to a positive scalar polarizability of the single nucleon, and this in turn gives rise to an effective $NN\sigma\sigma$ interaction which raises the σ meson mass and prevents the collapse.

The EOS of color superconducting QM in the mean field approximation has the form [2]

$$V^{(QM)} = V_{\text{vac}} + V_Q + V_\Delta + V_\omega \quad (2)$$

where V_{vac} describes the polarization of the Dirac sea of quarks due to the presence of the valence quarks, V_Q arises from the Fermi motion of the valence quarks without the effect of quark pairing, V_Δ arises from quark pairing in the scalar diquark channel and depends on the color superconducting gap (Δ), and V_ω is the contribution of the mean vector field (ω_0) in QM. The conditions $\partial V^{(QM)} / \partial M = \partial V^{(QM)} / \partial \Delta = \partial V^{(QM)} / \partial \omega_0 = 0$ determine M , Δ and ω_0 for fixed chemical potential μ .

After the calculation of the grand potential V , the EOS of NM and QM are obtained by applying the usual thermodynamic relationships, and the Gibbs criteria can be used to look for phase transitions. The chemical potential can be eliminated in favor of the baryon density ρ according to $\rho = -\partial V / \partial \mu$.

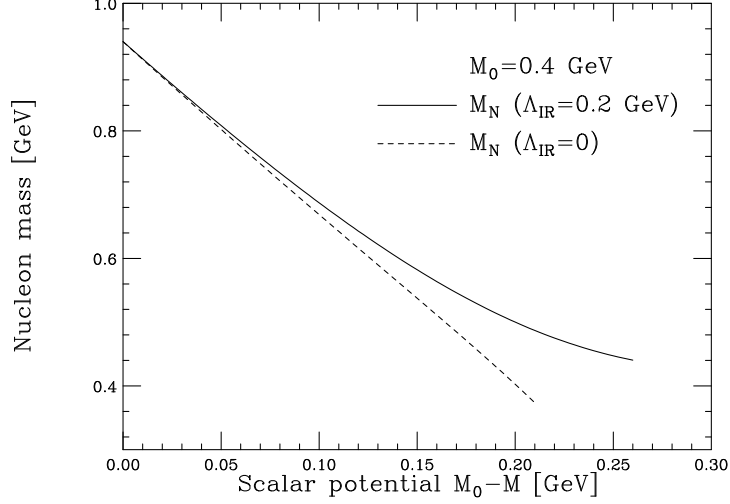


FIGURE 1. Nucleon mass as a function of the scalar potential for $\Lambda_{\text{IR}} = 0$ (dashed line) and $\Lambda_{\text{IR}} = 0.2$ GeV (solid line).

RESULTS

The function $M_N(M)$, which is the solution of the quark-diquark bound state equation for the nucleon, is shown in Fig. 1. The elimination of unphysical quark decay thresholds by an infrared cut-off Λ_{IR} leads to a scalar polarizability (curvature of the function $M_N(M)$), which is clearly seen in Fig. 1. This effect stabilizes the NM EOS, and leads to the saturation of the binding energy per nucleon as a function of density [5].

Since we can simultaneously describe the structure of the nucleon and stable NM, we can investigate the properties of a nucleon bound in the medium. Such an investigation has been carried out for the structure functions (the EMC effect) in Ref.[7, 8]. The main result is that the mean vector field has an important direct effect on the structure function, which leads to a successful description of the EMC effect.

We now ask the question whether there is a phase transition to QM at high baryon densities. Fig. 2 shows the EOS for NM (solid line), and for QM with several values for the strength of the pairing interaction in the scalar diquark channel (r_s). Curve 1 corresponds to normal, i.e., non-color superconducting, QM, and the other curves show the results for increasing strength of the pairing interaction.

It is clear from Fig. 2 that there is no phase transition from NM (solid line) to normal QM (dotted line) in our model. The scalar diquark condensation, however, gives rise to a substantial softening of the QM EOS, and to a phase transition from NM to QM at a transition density which decreases with increasing strength of the pairing interaction. The effective (constituent) quark mass for NM and QM is shown for the same cases as a function of the baryon density in Fig. 3. This figure demonstrates that the tendency toward a chiral phase transition is much stronger in QM than in NM.

We now assume a particular value of the coupling constant in the scalar diquark channel, which leads to reasonable transition densities, and investigate the nature of the phase transition more closely. Fig. 4 shows the resulting pressure of the ground state

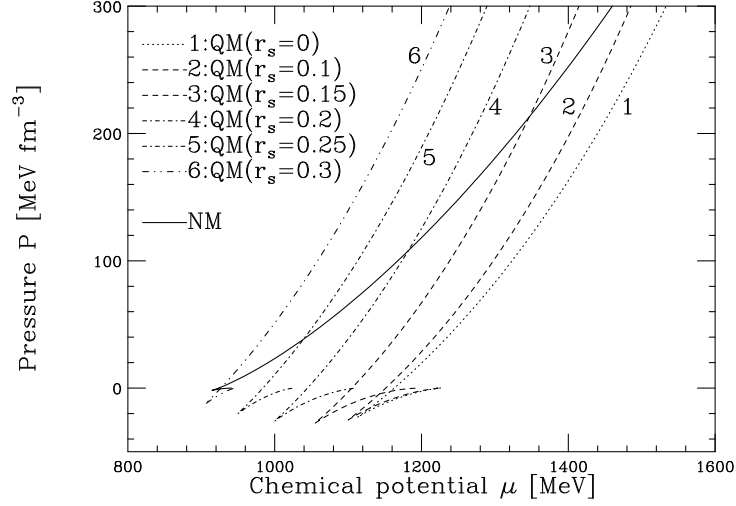


FIGURE 2. Pressure as function of chemical potential in NM (solid line), and in QM for several values of r_s , which is the ratio of the coupling constant in the scalar diquark channel to the one in the pionic channel.

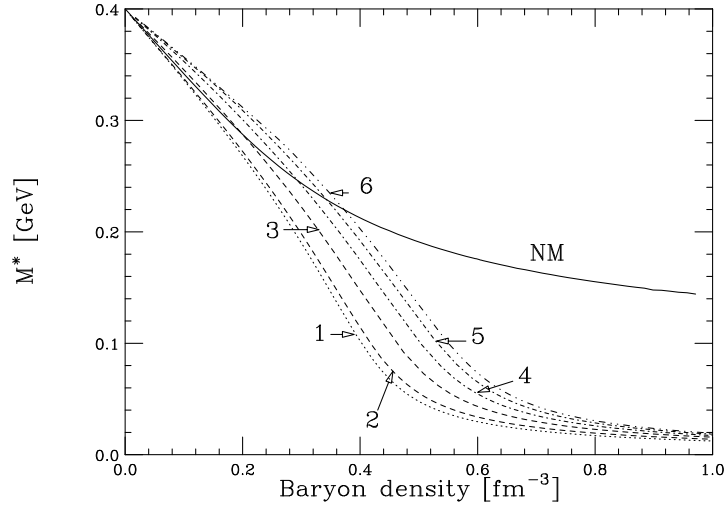


FIGURE 3. Effective quark mass as function of baryon density for NM (solid line) and QM. Labels 1 to 6 correspond to the same values of the pairing strengths in QM as in Fig. 2.

as a function of the baryon density for the case corresponding to $r_s = 0.2$ of Fig. 2.

We obtain first order transitions from the vacuum (VAC) to NM, where in both phases chiral symmetry is broken and color symmetry is intact, and from NM to color superconducting QM, where in the latter phase chiral symmetry is largely restored and color symmetry is broken. The present calculation gives large color superconducting gaps in the QM phase ($\Delta > 200$ MeV).

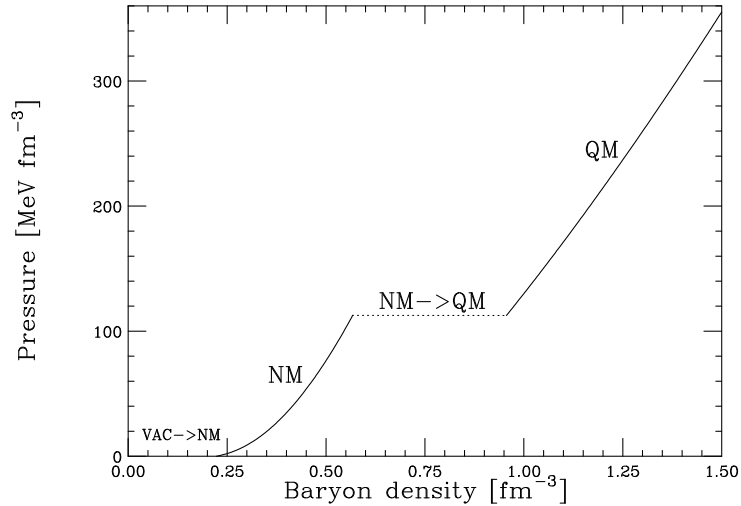


FIGURE 4. Pressure of the ground state of the system - vacuum (VAC), nuclear matter (NM) or superconducting quark matter (QM) - as function of the baryon density for $r_s = 0.2$. The mixed phases are also indicated.

SUMMARY

We used the NJL model as an effective chiral quark theory to describe the single nucleon, the saturation properties of normal NM, and high density QM. We have shown that there is a phase transition from NM to QM, provided that the effects of scalar diquark condensation (color superconductivity) are taken into account. This phase transition is characterized by the restoration of chiral symmetry and the spontaneous breaking of color symmetry in the high density region.

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