

## Surface Constraint of a Rational Interpolation and the Application in Medical Image Processing

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**Abstract :** A new weighted bivariate blending rational spline with parameters is constructed based on function values of a function only. The interpolation is C1 in the whole interpolating region under the condition which free parameters is not limited. This study deals with the bounded property of the interpolation. In order to meet the needs of practical design, an interpolation technique is employed to control the shape of surfaces. This rational interpolation with parameters is used in the medical image enhancement. The value of the interpolating function at any point in the interpolating region can be modified under the condition that the interpolating data are not changed by selecting the suitable parameters. Using the surface control, the local enhancement of the image is implemented. The experimentations show that this algorithm is efficient.

**Keywords:** Rational splines, Interpolation function, surface constraint, Medical image processing, Image enhancement

### INTRODUCTION

The construction method of curve and surface and the mathematical description of them is a key issue in computer-aided geometric design. There are many ways to tackle this problem (Bao *et al.*, 2009; Bezier, 1986, de Boor 2001; Chui 1988; Farin, 1988, Speleers *et al.*, 2007, Wang, 2001), for example, the polynomial spline method, the NURBS method. These methods are effective and applied widely in shape design of industrial products. However, to construct the polynomial spline, usually derivative values are needed as the interpolating data besides the function values. Unfortunately, in many practical problems, such as the description of rainfall in some rainy region and some industrial geometric shapes, derivative values are difficult to obtain. On the other hand, one of the disadvantages of the polynomial spline method is its global property; it is not possible for the local modification for unchanged given data.

In recent years, a univariate rational spline interpolation with the parameters has received attention in the literature (Duan *et al.*, 1999, Goodman and Ong 1998, Schmidt, 1987). Since the parameters in the interpolation function are selective according to the control need, the constrained control of the shape becomes possible. Motivated by the univariate rational spline interpolation, the bivariate rational interpolation with parameters which has simple and explicit mathematical representation has been studied (Duan *et al.*, 2006). The construction of

these kinds of interpolation splines can solve aforementioned difficulties. However, for the bivariate rational interpolation, in order to maintain smoothness of the interpolating surface, the parameters of  $y$  - direction for bivariate interpolating surface must be limited, they can not be selected freely for different interpolating subregion.

Image interpolation is a general research method for image enhancement. Many image interpolation techniques of different tradeoffs between computational complexity and reproduction quality were developed. Popular methods, as commonly used in image software and hardware products, are nearest neighbour interpolation, bilinear interpolation, cubic convolution interpolation, cubic spline interpolation. Those methods are based on a simple polynomial model and work as low-pass filter which restrained the high frequency components of the digital image. The main advantage of these methods is their relatively low complexity. When the magnification is higher. Their common drawback is the inability to adapt to varying pixel structures in a scene. As a result, they are all susceptible to defects such as fuzzy, zigzag stripes or block in the edge region. The edge information are lost. Because the image can be effected by the light, the impact of natural background and the characteristics of its own texture, so the relation between image's adjacent pixels is not linear.

For avoiding the malpractice which exists in the traditional image enhancement, the weighted bivariate

blending rational interpolation is used in this study. This interpolation function has parameters for local image adjusting. The whole shape character of image is considered. The high frequency part of image on the region edge is preserved. Thus, the image not only keeps the contour clear, but also is smooth and continuous. The experiment shows that comparing with traditional algorithms, the quality of image can be improved greatly especially in processing the borders of the source image.

**METHODOLOGY**

**Interpolation:** Let  $\Omega : [a, b, c, d]$  be the plane region and  $\{(x_i, y_i, f_{i,j}, i = 1, 2, \dots, n+1; j = 1, 2, \dots, m+1)\}$  be a given set of data points, where,  $a = x_1 < x_2 < \dots < x_{m+1} = b$  and  $c = y_1 < y_2 < \dots < y_{m+1} = d$  are the knot spacings  $f_{i,j} = f(x_i, y_j)$ . Let  $l_i = y_{i+1} - y_i$  and  $h_i = x_{i+1} - x_i$  and for any point  $(x, y) \in [x_i, x_{i+1}; y_j, y_{j+1}]$  in the  $xy$  - plane, let  $\theta = (x - x_i) / h_i$  and  $\eta = (y - y_j) / l_j$ . First, for each  $y = y_j, j = 1, 2, \dots, m+1$  construct the  $x$  - direction interpolant curve; this is given by

$$\bar{P}_{i,j}^*(x) = \frac{\bar{P}_{i,j}^*(x)}{\bar{q}_{i,j}^*(x)}, i = 1, 2, \dots, n - 1, \tag{1}$$

where,

$$\begin{aligned} \bar{P}_{i,j}^* &= (1 - \theta)^3 \alpha_{i,j} f_{i,j} + \theta(1 - \theta) \bar{V}_{i,j}^* + \theta^2(1 - \theta) \bar{W}_{i,j}^* + \theta^3 f_{i+1,j}, \\ \bar{q}_{i,j}^* &= (1 - \theta) \alpha_{i,j} + \theta, \end{aligned}$$

and

$$\begin{aligned} \bar{V}_{i,j}^* &= (\alpha_{i,j} + 1) f_{i,j} + \alpha_{i,j} f_{i+1,j}, \\ \bar{W}_{i,j}^* &= (\alpha_{i,j} + 2) f_{i,j} - h_i \bar{\Delta}_{i+1,j}^* \end{aligned}$$

with  $\alpha_{i,j} > 0$  and  $\bar{\Delta}_{i,j}^* = (f_{i+1,j} - f_{i,j}) / h_i$

This interpolation is called the rational cubic interpolation based on function values which satisfies

$$\begin{aligned} \bar{P}_{i,j}^*(x_i) &= f_{i,j} \bar{P}_{i,j}^*(x_{i+1}) = f_{i+1,j}, \\ \bar{P}_{i,j}^*(x_i) &= \bar{\Delta}_{i,j}^*, \bar{P}_{i,j}^*(x_{i+1}) = \bar{\Delta}_{i+1,j}^*. \end{aligned}$$

Obviously, the interpolation function  $\bar{P}_{i,j}^*(x)$  on  $[x_i, x_{i+1}]$  is unique for the given data  $\{x_r, f_{r,i} r i, i + 1, i + 2\}$  and positive parameter  $\alpha_{i,s}$ .

For each pair of  $(i, j) i = 1, 2, \dots, n - 1$  and  $j = 1, 2, \dots, m - 1$ , using the  $x$  - direction interpolation function  $\bar{P}_{i,j}^*(x)$ , define the interpolation function  $P(x, y)$  on each rectangular region  $[x_i, x_{i+1}; y_j, y_{j+1}]$  as follows:

$$\begin{aligned} \bar{P}_{i,j}(x, y) &= (1 - \eta)^3 \bar{P}_{i,j}^*(x) + \eta(1 - \eta)^2 (2 \bar{P}_{i,j}(x) \\ &+ \bar{P}_{i,j+1}^*(x)) + \eta^2(1 - \eta) (3 \bar{P}_{i,j+1}^*(x) \\ &+ l_i \bar{\Delta}_{i,j+1}^*(x)) \eta^3 \bar{P}_{i,j+1}^*(x) \end{aligned}$$

where,  $\bar{\Delta}_{i,j} = \bar{P}_{i,j=1}^*(x) - \bar{P}_{i,j}^*(x) l_i$ .  $\bar{P}_{i,j}(x, y)$  is called the bivariate blending interpolation function based on function values which satisfies:

$$\begin{aligned} \bar{P}_{i,j}(x_r, y_s) &= f(x_r, y_s), \frac{\partial \bar{P}_{i,j}(x_r, y_s)}{\partial x} \bar{\Delta}_{r,s}^* \\ \frac{\partial \bar{P}_{i,j}(x_r, y_s)}{\partial y} \bar{\Delta}_{r,s}^* & r = i, i + 1, s = j, j + 1 \end{aligned}$$

This form of interpolation function  $\bar{P}_{i,j}(x, y)$  on  $[x_i, x_{i+1}; y_j, y_{j+1}]$  is unique for the given data  $\{(x_r, y_s, f_{r,i}), r = i, i + 1, i + 2, s = j, j + 1, j + 2$  and parameters  $\alpha_i$  and it is easy to test that the interpolation is  $C^1$  in the whole interpolating region  $[x_1, x_n, y_1, y_m]$ , no matter what the parameters  $\alpha_{i,s}$  might be.

The interpolating scheme above begin in  $x$  - direction first. Now, let the interpolation begins with  $y$  - direction first. For each  $x = x_i, i = 1, 2, \dots, n + 1$  denote the  $y$  - direction interpolation in  $[y_j, y_{j+1}]$  by:

$$\bar{P}_{i,j}(x) = \frac{\bar{P}_{i,j}(x)}{\bar{q}_{i,j}(x)}, i = 1, 2, \dots, n - 1, \tag{2}$$

where,

$$\begin{aligned} \bar{P}_{i,j} &= (1 - \theta)^3 \alpha_{i,j} f_{i,j} + \theta(1 - \theta)^2 \bar{V}_{i,j}^* + \theta^2(1 - \theta) \\ \bar{W}_{i,j} + \theta^3 f_{i+1,j}, \bar{q}_{i,j} &= (1 - \theta) \alpha_{i,j} + \theta \end{aligned}$$

and

$$\begin{aligned} \bar{V}_{i,j} &= (\alpha_{i,j} + 1) f_{i,j} + \alpha_{i,j} f_{i+1,j}, \\ \bar{W}_{i,j} &= (\alpha_{i,j} + 2) f_{i+1,j} - h_i \bar{\Delta}_{i+1,j}^* \end{aligned}$$

with  $\beta_{i,j} > 0$  and  $\bar{\Delta}_{i,j} = (f_{i,j+1} - f_{i,j}) / l_i$ .

For each pair  $(i, j) i = 1, 2, \dots, n - 1$  and  $j = 1, 2, \dots, m - 1$ , using the  $y$  - direction interpolation function  $\bar{P}_{i,j}(y)$ , define the bivariate blending interpolation function  $\bar{P}_{i,j}(x, y)$  on  $[x_i, x_{i+1}, y_j, y_{j+1}]$  as follows:

$$\begin{aligned} \overline{\overline{P}}_{i,j}(x, y) &= (1 - \theta)^3 \overline{\overline{P}}_{i,j}^*(y) + \theta(1 - \theta)^2 (2\overline{\overline{P}}_{i,j}^*(y) \\ &+ \overline{\overline{P}}_{i,j}^*(y)) + \theta^2(1 - \theta) (3\overline{\overline{P}}_{i+1,j}^*(y)) \\ &- h_i \Delta_{i+1,j}(y) + \theta^3 \overline{\overline{P}}_{i+1,j}^*(y), \end{aligned}$$

where,  $\overline{\Delta}_{i,j} = (\overline{P}_{i+1,j}(y) - \overline{P}_{i,j}(y)) / h_i$ . The interpolation function  $\overline{\overline{P}}_{i,j}(x, y)$  satisfies:

$$\begin{aligned} \overline{\overline{P}}_{i,j}(x_r, y_s) &= f(x_r, y_s), \frac{\partial \overline{\overline{P}}_{i,j}(x_r, y_s)}{\partial x} = \overline{\overline{\Delta}}_{r,s}, \\ \frac{\partial \overline{\overline{P}}_{i,j}(x_r, y_s)}{\partial y} &= \overline{\overline{\Delta}}_{r,s}, r = i, i + 1, s = j, j + 1. \end{aligned}$$

Similarly,  $\overline{P}_{i,j}(x, y)$  on  $[x_i, x_{i+1}; y_j, y_{j+1}]$ , is unique for the given data  $\{(x_r, y_s, f_{r,s}), r = i, i + 1, i + 2, s = j, j + 1, j + 2$  and parameters  $\beta_{r,j}$  and it is easy to derive that the interpolation is  $C^1$  in the whole interpolation region  $[x_1, x_n, y_1, y_m]$ , no matter what the parameters  $\beta_{r,j}$  might be.

where, we let:

$$P_{r,j}(x, y) = \lambda \overline{P}_{i,j}(x, y) + (1 - \lambda) \overline{\overline{P}}_{i,j}(x, y)$$

with the weighted coefficient  $\lambda \in [0, 1]$ , then the weighted bivariate blending rational interpolation function  $P_{i,j}(x, y)$  satisfies:

$$\begin{aligned} P_{i,j}(x_r, y_s) &= f(x_r, y_s), \frac{\partial P_{i,j}(x_r, y_s)}{\partial x} = \lambda \overline{\overline{\Delta}}_{r,s}^* + (1 - \lambda) \overline{\overline{\Delta}}_{r,s}, \\ \frac{\partial P_{i,j}(x_r, y_s)}{\partial y} &= \lambda \overline{\overline{\Delta}}_{r,s}^* + (1 - \lambda) \overline{\overline{\Delta}}_{r,s}, r = i, i + 1, s = j, j + 1. \end{aligned}$$

and it is  $C^1$  in whole interpolation region  $[x_1, x_n, y_1, y_m]$ , no matter what the parameters  $\alpha_{i,s}$  and  $\beta_{r,j}$  might be.

**The bases of the interpolation:** Consider the interpolation function  $P_{i,j}(x, y)$  can be written as following:

$$P_{i,j}(x, y) = \sum_{r=0}^2 \sum_{s=0}^2 \omega_{r,s}(\theta, \eta) f_{i+r, j+s}, \quad (3)$$

where,

$$\begin{aligned} \omega_{0,0}(\theta, \eta) &= \frac{(1 - \theta)^2 (\theta + \alpha_{i,j})(1 - \eta)^2 (1 + \eta) \lambda}{(1 - \theta) \alpha_{i,j} + \theta} \\ &+ \frac{(1 - \theta)(1 - \theta)^2 (1 - \eta)^2 (\eta + \beta_{i,j}) \lambda}{(1 - \eta) \beta_{i,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{0,1}(\theta, \eta) &= \frac{(1 - \theta)^2 (\theta + \alpha_{i,j+1}) \eta (1 + 2\eta - 2\eta^2) \lambda}{(1 - \theta) \alpha_{i,j+1} + \theta} \\ &+ \frac{(1 - \theta)^2 (1 + \theta) \eta (3\eta - 2\eta^2 + (1 - \eta) \beta_{i,j}) (1 - \lambda)}{(1 - \eta) \beta_{i,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{0,2}(\theta, \eta) &= - \frac{(1 - \theta)^2 (\theta + \alpha_{i,j}) \eta^2 (1 - \eta)^2}{(1 - \theta) \alpha_{i,j+2} + \theta} \\ &+ \frac{(1 - \theta)(1 - \theta)^2 \eta^2 (1 - \eta)^2 (1 - \lambda)}{(1 - \eta) \beta_{i,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{1,0}(\theta, \eta) &= \frac{\theta(3\theta - 2\theta^2 + (1 - \theta) \alpha_{i,j} (1 - \eta)^2 (1 + \eta) \lambda)}{(1 - \theta) \alpha_{i,j+2} + \theta} \\ &+ \frac{\theta(1 + 2\theta - 2\theta^2)(1 - \eta)^2 (\eta + \beta_{i+1,j}) (1 - \lambda)}{(1 - \eta) \beta_{i+1,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{1,1}(\theta, \eta) &= \frac{\theta(3\theta - 2\theta^2 + (1 - \theta) \alpha_{i,j} \eta (1 + 2\eta - 2\eta^2) \lambda)}{(1 - \theta) \alpha_{i,j+1} + \theta} \\ &+ \frac{\theta(1 + 2\theta - 2\theta^2) \eta (3\eta - 2\eta^2 + (1 + \eta) \beta_{i+1,j}) (1 - \lambda)}{(1 - \eta) \beta_{i+1,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{1,2}(\theta, \eta) &= - \frac{\theta(3\theta - 2\theta^2 + (1 - \theta) \alpha_{i,j+2}) \eta^2 (1 - \lambda)}{(1 - \theta) \alpha_{i,j+2} + \theta} \\ &+ \frac{\theta(1 + 2\theta - 2\theta^2) \eta^2 (1 - \eta) (1 - \lambda)}{(1 - \eta) \beta_{i+1,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{2,0}(\theta, \eta) &= - \frac{\theta^2 (1 - \theta) + (1 - \eta)^2 (1 + \eta) \lambda}{(1 - \theta) \alpha_{i,j+2} + \theta} \\ &+ \frac{\theta^2 (1 - \theta) + (1 - \eta)^2 (\eta + \beta_{i+2,j}) (1 + \lambda)}{(1 - \eta) \beta_{i+2,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{2,1}(\theta, \eta) &= - \frac{\theta^2 (1 - \theta) \eta (1 + 2\eta - 2\eta^2) \lambda}{(1 - \theta) \alpha_{i,j+1} + \theta} \\ &- \frac{\theta^2 (1 - \theta) \eta (3\eta - 2\eta^2 + (1 - \eta) \beta_{i+2,j}) (1 - \lambda)}{(1 - \eta) \beta_{i+2,j} + \eta} \end{aligned}$$

$$\begin{aligned} \omega_{2,2}(\theta, \eta) &= \frac{\theta^2 (1 - \theta) \eta^2 (1 - \eta) \lambda}{(1 - \theta) \alpha_{i,j+2} + \theta} \\ &+ \frac{\theta^2 (1 - \theta) \eta^2 (1 - \eta) (1 - \lambda)}{(1 - \eta) \beta_{i+2,j} + \eta} \end{aligned}$$

and  $\omega_{r,s}(\theta, \eta), (r = i, j + 1, s = j, j + 1)$  are called the basis of the bivariate interpolation defined. These base function satisfy:

$$\sum_{r=0}^2 \sum_{s=0}^2 \omega_{r,s}(\theta, \eta) = 1. \quad (4)$$

**Bounded property of the interpolation:** The weighted bivariate blending interpolation defined is a local interpolation in each subrectangle, which only depends

on the function values. There are some parameters in each interpolating region and when the parameters vary, the interpolating surface changes accordingly. Now, we consider the bounded property of the interpolation.

Note considered the interpolation, assume  $\alpha_{i,j} = \alpha_i$ ,  $\alpha_{i+1,j+1} = \alpha_{i+1}$  and  $\beta_{i,j} = \beta_{i+1,j} = \beta_{i+2,i}$ . Denote

$$N = \max\{|f_{i+r,j+s}|, r = 0, 1, 2, j = 0, 1, 2\}$$

First, discuss the bounded property of the interpolation function  $P_{i,j}(x, y)$ . It is easy to obtain:

$$\begin{aligned} |P_{i,j}(x, y)| &= \left| \sum_{r=0}^2 \sum_{s=0}^2 \omega_{r,s}(\theta, \eta) f_{i+r,j+s} \right| \\ &\leq \sum_{r=0}^2 \sum_{s=0}^2 |\alpha_{r,s}(\theta, \eta)| |f_{i+r,j+s}| \\ &\leq N \frac{(1 + 2\eta^2 - 2\eta^3[(1-\theta)\alpha_{i,j} + \theta(1+2\theta-2\theta^2)])}{(1-\theta)\alpha_{i,j} + \theta} \\ &= N(1 + 2\eta^2 - 2\eta^3 + \frac{2\theta^2(1-\theta)}{(1-\theta)\alpha_{i,j} + \theta}) \leq \frac{97}{54} N \end{aligned}$$

This leads to the following theorem.

**Theorem 1.** For the given interpolation data  $\{f_{r,s}, r = i, i + 1, i + 2, s = j, j + 1, j + 2\}$ , let  $P_{i,j}(x, y)$  be bivariate blending interpolation over the rectangle region  $[x_i, x_{i+1}, y_i, y_{i+1}]$ , then  $|P_{i,j}(x, y)| \leq 97/54 N$

where,  $N = \max\{|f_{i+r,j+s}|, r = 0, 1, 2; j = 0, 1, 2\}$

**Local shape control of the of the interpolation:** The shape of the interpolating surface on the interpolating region depends on the interpolating data. Generally speaking, when the interpolating data are given, the shape of interpolating surface is fixed, this is because of the uniqueness of the interpolation function. But, for the bivariate blending interpolation defined by (3), since there are parameters and weighted coefficient, when they vary, the interpolation function can be changed under the condition that the interpolating data are not changed. Thus, the interpolating surface varies as the parameters and the weighted coefficient vary. Based on this, the shape of the interpolating surface can be modified by selecting suitable parameters and weighted coefficient. For any point  $(x, y)$  in the interpolation region  $[x_i, x_{i+1}, y_i, y_{i+1}]$ , let  $(\theta, \eta)$  be its local coordinate. Now, we consider the following local point control issue: if the practical design require the value of the interpolation function at the point  $(x, y)$  be equal to a real  $M$ , how it can be achieved, Let:

$$M = \sum_{r=0}^2 \sum_{s=0}^2 \mu_{r,s} f_{i+r,j+s},$$

where,  $\mu_{r,s} (r = i, i + 1; s = j, j + 1)$  are real number and

$\sum_{r=0}^2 \sum_{s=0}^2 \mu_{r,s} = 1$ . Then:

$$\sum_{r=0}^2 \sum_{s=0}^2 \omega_{r,s}(\theta, \eta) f_{i+r,j+s} = \sum_{r=0}^2 \sum_{s=0}^2 \mu_{r,s} f_{i+r,j+s}.$$

Eq.(5) is called the value control equation, where the local coordinate  $(\theta, \eta)$  is given, only the parameters  $\alpha_{i,s}, \beta_{r,i}$  and the weighted coefficient  $\lambda$  are unknown. If there exist parameters  $\alpha_{i,s}, \beta_{r,i}$  and the weighted coefficient  $\lambda \in [0,1]$  which satisfy (5), then the problem is solved.

For example, the central point in the interpolation surface is more important than other points for shape control. Note that the interpolation is local, assume  $\alpha_{i,j} = \alpha_{i,j+1} = \alpha$  and denote  $\alpha_i$ ; assume  $\beta_{i,j} = \beta_{i+1,j} = \beta_{i+2,j}$  and denote by  $\beta_i$ . Let the weighted coefficient  $\lambda = 0.5$ . Since  $(x, y)$  is the central point in  $[x_i, x_{i+1}, y_i, y_{i+1}]$   $\theta = 0.5, \eta = 0.5$  In this case Eq.(5) becomes:

$$k_1 \alpha_i + k_2 \beta_j + k_3 \alpha_i \beta_j + k_0 = 0 \tag{5}$$

where,

$$\begin{aligned} k_0 &= 2(3f_{i,j} + 9f_{i,j+1} - 2f_{i,j+2} + 9f_{i+1,j} + 24f_{i+1,j+1} \\ &\quad - 5f_{i+1,j+2} - 2f_{i+2,j} - 5f_{i+2,j+2} + f_{i+2,j+2} - 32M) \end{aligned}$$

$$\begin{aligned} k_1 &= 9f_{i,j} + 24f_{i,j+1} - 5f_{i,j+2} + 12f_{i+1,j} + 36f_{i+1,j+1} \\ &\quad - 8f_{i+1,j+2} - f_{i+2,j} - 4f_{i+2,j+2} - 64M) \end{aligned}$$

$$\begin{aligned} k_2 &= 9f_{i,j} + 12f_{i,j+1} - f_{i,j+2} + 24f_{i+1,j} + 36f_{i+1,j+1} \\ &\quad - 4f_{i+1,j+2} - 5f_{i+2,j} - 8f_{i+2,j+2} + f_{i+2,j+2} - 64M) \end{aligned}$$

$$\begin{aligned} k_3 &= 2(6f_{i,j} + 9f_{i,j+1} - f_{i,j+2} + 9f_{i+1,j} + 12f_{i+1,j+1} \\ &\quad - f_{i+1,j+2} - f_{i+2,j} - f_{i+2,j+1} - 32M) \end{aligned}$$

From Eq. (5), it is interesting that the selection of parameters does not depend on the value of  $\mu_{r,s}$  in point control problem. It must be satisfied control equation.

**Theorem 2:** The sufficient condition for point value control having solution is that the number of the variation sign of the sequence  $\{k_1, k_2, k_3, k_0\}$  are not equal to zero.

**Medical image enhancement:** The theory of Computerized Tomography (CT) is image reconstruction. Based on the character of different body organization

(include normal and abnormal) absorb the X radial is different, we can divide one lay of body data into cubes. X radial pass through a voxel, the density detected or gray value is called a pixel value. All pixels are arranged in matrix to construct the CT image. The detail established process of pixel is: X radial pass through a selected lay, detecting machine receive the sum of attenuation value of all voxels along the direction of X radial after absorbed X radial. This sum value is known and the attenuation value of every voxel is unknown. When X radial emitter and detecting machine moving around human body, iterative method is used to resolve the attenuation value of every voxel. After image reconstruction, a gray image of this lay data can be obtained with different organization density. Based on the above, density value of each pixel is an average value of the voxel. So the value is a sampling value of the voxel region and not an exact point sampling value of this pixel. Suppose that  $P$  is a CT slice which is composed of  $m \times n$  image elements  $P_{i,j}$   $i = 0, 1, \dots, m - 1$ ,  $j = 0, 1, \dots, n - 1$  These image elements are sampling values, which can be regarded as taken from a surface  $P(x, y)$  To enhance the CT slice  $P$ , the best way for making the enhancement image with good quality is to resample from  $P(x, y)$  Then we construct a continuous interpolation surface  $P(x, y)$  based on bivariate rational interpolation method for discrete CT slice.

**Image Interpolation Based on the Rational Function :**

Given a medical  $m \times n$  image  $I_{m,n}$ , let  $f_{i,j} = \{ 0 \leq i \leq m - 1, 0 \leq j \leq n - 1 \}$  be the gray value of the  $i$  line and  $j$  row. The pixel coordinate is  $(i, j)$ . The surface  $P_{i,j}(x, y)$  can be constructed using the points  $f_{i,j}$ ,  $i, j = 0, 1 \dots n - 1$ . matrix  $f_1 = f_{i,j}$ ,  $0 \leq i \leq m - 1, 0 \leq j \leq n - 1$  is expanded into  $f_2 = f_{i,j}$ ,  $0 \leq i \leq m - 1, 0 \leq j \leq n$ , so the values of  $f_{i,j}$  ( $i = m$  or  $j = n$ ) are added. Outer-interpolation method is used, let  $f_{m,j} = 2f_{m-1} - f_{m-2}$ ,  $(0 \leq j \leq n - 1)$   $f_{i,n} = 2f_{i,n-1} - f_{i,n-2}$  ( $0 \leq i \leq m - 1$ ). Then expand  $f_2$  to  $f_3 = \{ f_{i,j} - 1 \leq i \leq m, -1 \leq j \leq n \}$  which is similar to expanding  $f_1$ .  $3 \times 3$  interpolation surface  $P_{i,j}(x, y)$  are constructed based on  $f_3$ . On each interval  $[x_i, x_{i+1}] \times [y_j, y_{j+1}]$ ,  $i, j = 0, 1, \dots, n - 1$ . At last number of  $M \times N$ , a bivariate rational interpolating spline surface  $P_{i,j}(x, y)$  is constructed.

**Medical Image Enhancement Based on the Shape Control:**

The sign of this resample point in the bivariate rational interpolating spline surface of original image can be denoted as  $i = \text{int}(x)$ ;  $j = \text{int}(y)$  where  $\text{int}(\ )$  is the integer function. Based on the interpolation surface  $P_{i,j}(x, y)$  constructed in former section, substitute  $x_0, y_0$  into  $P_{i,j}(x, y)$ . we can get the gray value of the new interpolating pixel. If the medical image is not ideal, it can be adjusted through selecting the suitable parameters. It is the basic principle to modify the interpolation surface using the parameters. Thus the gray of enlarged image is modified, until we get the satisfactory results.

Table 1: the Interpolating Data

$(x,y)$	(0,0)	(0,1)	(0,2)
$f(x,y)$	4	2	3
$(x,y)$	(1,0)	(1,1)	(1,2)
$f(x,y)$	2	3	4
$(x,y)$	(2,0)	(2,1)	(2,2)
$f(x,y)$	3	3	5

**NUMERICAL EXPERIMENTS**

**Shape control :** Let  $\Omega : [0, 2, 0, 2]$  be the plane region and the interpolating data are given in Table 1.

Then the interpolation function  $P_{i,j}(x, y)$  can be constructed in  $[0, 1, 0, 1]$  for the given positive parameters and the coefficient. In general case, one can select any positive real numbers as the values.

Now, let  $\alpha_i = 0.5$ ,  $\beta_j = 2$  and  $\lambda = 0.5$  and for the given interpolation function by  $P_1(x, y)$ , the Fig. 1 shows the graph. In this case, the function value of central point is that.  $P_1(0.5, 0.5) = 2.526$  From the control function, the value of central point can be control through modifying the parameters. For example,  $\alpha_i = 0.25$ ,  $\beta_j = 0.5$ . For those parameters, denote the interpolation function for the given interpolation data by  $P_2(x, y)$ . Figure 2 shows the graph. It can be to test that  $P_2(0.5, 0.5) = 2.5$  Furthermore, for some parameters, denote the interpolation function by  $P_3(x, y)$ .

Figure 3 shows the graph. Then  $P_2(0.5, 0.5) = 2.55$ .

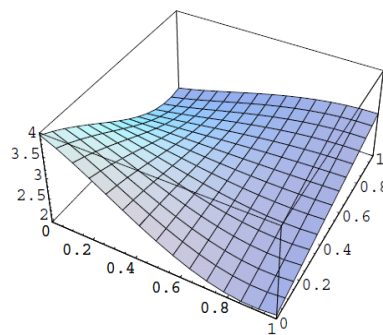


Fig. 1: Graph of  $P_1(x, y)$  with  $\alpha_i = 2$ ,  $\beta_j = 1$  and  $\lambda = 0.5$

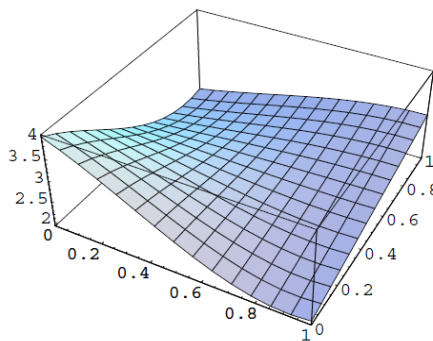


Fig. 2: Graph of  $P_2(x, y)$  with  $\alpha_i = 1.25$ ,  $\beta_j = 0.5$  and  $\lambda = 0.5$

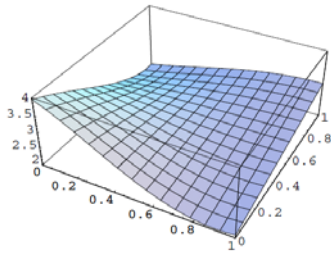


Fig. 3: Graph of  $P_3(x, y)$  with  $\alpha_i = 0.5$ ,  $\beta_j = 7.75$  and  $\lambda = 0.5$



Fig. 4: Bilinear interpolation

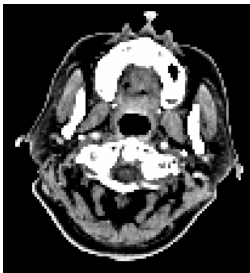


Fig. 5: Bi-cubic interpolation



Fig. 6: Spline interpolation

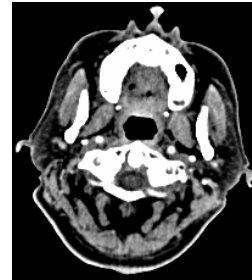


Fig. 7: Aur method

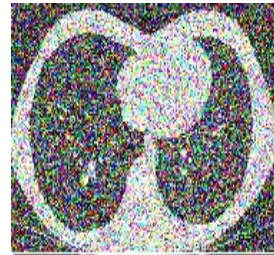


Fig. 8: Nosing image

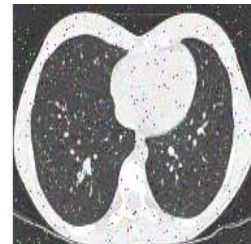


Fig. 9: Neighbourhood averaging

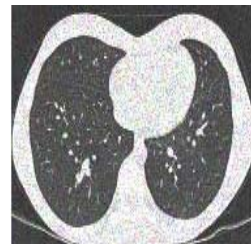


Fig. 10: Adaptive median filter



Fig. 11: Our method

**Image enhancement:** In this experiment, the efficiency of the new method is compared with the three methods which are bilinear, bi-cubic and regular spline interpolation, respectively. The comparison result is shown, where image in Fig. 4 is created by bilinear interpolation to the original image data points, image in Fig. 5 is created by bi-cubic interpolation to the original image data points, the one in Fig. 6 is created by regular spline interpolation to the original image data points, the one in Fig. 7 is created by the new method. To implement

the medical image enhancement, the rational interpolation also can be used for denosing. From the Fig. 8-11 showing, obviously the filtering effect of the neighbourhood averaging is not satisfactory. The adaptive median filter algorithm has a good filtering effect, but the filtered image appears fuzzy. The more effective denosing result is obtained as our algorithm shows. At the same time the visual effect from an image point of view, the method proposed in this study as much as possible retain details of the images. The images showed a clear edge after noise reduction. The simulation results indicate that the shrinkage value is changed adaptively with signals. The algorithm can maintain the details of the image well.

### CONCLUSION

This study gives the explicit expression of a weighted bivariate blending rational interpolation. The interpolation depends only on the function values of the interpolated function. More important is that the smoothness property of the interpolation does not depend on the parameters. this means that the interpolation function is  $C^1$  continuous in the whole interpolating region for any parameters. Positive parameters  $\alpha_{ij}$  and  $\beta_{ij}$  can be freely selected according to the needs of practical design. The point control method of the interpolation is developed in the paper. For any given real number  $M$ , from (5), there are varied expressions, so manifold control scheme can be derived similar to the process for central point value control problem.

As an important non-linear numerical analysis tool, rational interpolation method can describe the value relation of adjacent pixels better. In this paper, bivariate rational spline function is used as interpolation function with functional value, a new enhancement method for CT image zooming is presented. This method considers the whole shape character of image and the high frequency part of image on the region boundary is preserved, which makes the image higher precision and good quality. Comparing with traditional algorithms, the quality of enhancement image can be improved greatly with simple computation.

### ACKNOWLEDGMENT

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### REFERENCES

- Bao, F.X., Q.H. Sun and Q. Duan, 2009. Point control of the interpolating curve with a rational cubic spline. *J. Vis. Commun. Image R.*, 20: 275-280.
- Bezier, P.E., 1986. *The Mathematical Basis of the UNISURF CAD System*. Butterworth, London.
- Chui, C.K., 1988. *Multivariate Spline*. SIAM.
- De Boor, C.A., 2001. *Practical Guide to Splines*. Revised Edn., Springer-Verlag, New York.
- Duan, Q., K. Djidjeli, W.G. Price and E.H. Twizell, 1999. Weighted rational cubic spline interpolation and its application. *J. Comp. Appl. Math.*, 117: 121-135.
- Duan, Q., Y.F. Zhang and E.H. Twizell, 2006. A bivariate rational interpolation and the properties. *Appl. Math. Comput.*, 179: 190-199.
- Farin, G., 1988. *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*. Academic Press.
- Goodman, T.N.T., B.H. Ong and M.L. Sampoli, 1998. Automatic interpolation by fair shape preserving space curves. *Comput. Aided Des.*, 30: 813-822.
- Schmidt, J.W. and W. Hess, 1987. Positive interpolation with rational quadratic spline. *Computing*, 38: 261-267.
- Speleers, H., P. Dierckx and S. Vandewalle, 2007. Weight control for modelling with NURPS surfaces. *Comput. Aided Geom. Des.*, 24: 179-186.
- Wang, R.H., 2001. *Multivariate Spline Functions and Their Applications*. Kluwer Academic Publishers, Beijing/New York/Dordrecht/Boston/London.