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Application of Tornado-flow Fundamental Hydro-dynamic Theory to the Study of Blood Flow in the Heart – Further Development of Tornado-like Jet Technology

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ABSTRACT

It has been proved previously that the Tornado-like swirling flows have strictly ordered hydrodynamic structure which can be exhaustively described by using the exact solution of non-stationary Navier-Stokes and continuity equations for this class of flows [1]. Analysis of the geometry of the flowing channel of the left ventricle (LV) and aorta has shown close correlation between the shape of the cavities and intraventricular trabeculae orientation with the streamlines of Tornado-like flows. LV casts morphometry, MRI tomography and 4D velocimetry of the flow velocity field in the aorta, allowed to prove that the blood flow in the LV and aorta corresponds to this class of flows and may be described using the exact solution [7,8,10].

The current study proposes a method of measurement and calculation of the flow structural parameters derived from the exact solution, using LV cavity casts morphometry in humans and dogs and Multislice computed tomography (MSCT) of LV in two patients without severe cardiac pathology. It has been shown that the dynamic expression of intracardiac trabeculae and instant shape of LV cavity within a complete cardiac cycle correspond closely to the stages of single Tornado-like jet evolution. Since the intraventricular trabeculae profile is streamlined continuously by the blood flow, it should determine the hydrodynamic flow structure as an ensemble of guiding vanes. Therefore it has been concluded that the intraventricular flow dynamics can be analyzed and quantified using the exact solution.

Application of this analysis to the MSCT visualization of LV cavity dynamics has shown the validity of this approach, which may be used for clinical diagnostic purpose.

A realistic mathematical model of intraventricular blood flow has been proposed and evaluated. The results showed a good agreement between the model and known cardiac anatomy and function.

INTRODUCTION

The necessity of blood flow quantitative analysis is obvious since the current knowledge about its hydrodynamics, mechanisms of regulation and biological stability in the bloodstream contains a lot of contradictions and gaps. Despite the fact that the blood flow implementation is the main function of the transport segment of the cardiovascular system and many

relevant studies were conducted, the mechanism of transport of blood remains obscure whereas
the weakness of the heart as a pump, wide range of circulatory statements, and the capacity of circulation to compensate pathological alterations in cardiovascular anatomy and function are well documented.

This work proposes a new successful approach to quantify and model the process of blood transportation using the exact solution of the non-stationary hydrodynamic equations for a class of tornado-like viscous flows. This approach seems to be true because of multiple similarities found between these flows and real blood flow.

It has been previously shown that the blood flow in the human heart and major arteries is twisted and structured as a self-organizing tornado-like swirling flow [7,8,10]. Its hydrodynamic interactions and properties are described by the exact solution of the non-stationary equations for a viscous fluid obtained for this class of flows [1]. These flows were found experimentally when the special dimpled profile on the channel wall flown by a turbulent flux was studied. It has been proved that the swirling jets, generated in the dimples and incorporated in the main turbulent flow do not cause additional energy losses that should inevitably occur due to the increase of flowing surface and unevenness of the channel. Therefore, the exact solution of Navier-Stokes and continuity equations concerns the class of twisted axially symmetric flows that are potential in each plane, containing the longitudinal axis “Z”.

Then the velocity field of such a flow is described by (1),

$$W_r = -C_0(t)r;$$
$$W_z = 2C_0(t)z;$$
$$W_\phi = \frac{\Gamma_0(t)}{2\pi r}\left[1 - \exp\left(-\frac{C_0(t)r^2}{2\nu}\right)\right] \tag{1}$$

where $\boldsymbol{r}$ and $\boldsymbol{z}$ are the values of radial and longitudinal coordinates of the cylindrical coordinate system of the swirling jet, respectively; $\boldsymbol{W_r}$, $\boldsymbol{W_z}$ and $\boldsymbol{W_\varphi}$ are radial, axial and azimuthal velocity components of the swirling flow; $\boldsymbol{C_0(t)}$, arbitrary function of time, reflecting the change in the radial velocity component along the radius of the swirling jet; $\boldsymbol{\Gamma_0(t)}$, arbitrary function of time corresponding to the circulation of the swirling jet; $\boldsymbol{\nu}$, kinematic viscosity of the medium.

Prior to application of this solution, the hydrodynamics of blood flow could only be studied on a phenomenological level. Those studies could not decipher the mechanisms of blood transport, accounting for the high efficiency of the flow despite the usage of a relatively weak pump - the heart. Nevertheless, several papers have suggested the anatomical and hydrodynamic similarities between the blood flow and swirling flows [2-6]. However, those observations lacked appropriate justification needed for application of the swirling flow theory for the purposes of blood flow analysis.

The use of exact solution of non-stationary Navier-Stokes and continuity equations as a hydrodynamic model has allowed the accurate analysis of the blood flow characteristics in the heart and great vessels. It contributes to explaining of the previously misunderstood mechanisms of blood flow generation and evolution, and opens new opportunities for blood flow quantitative characterization for diagnostic purposes, optimization tactics of surgical correction of circulatory disorders, developing new models of implantable devices for cardiac surgery, as well as mathematical and physical modeling of circulation.

In recent studies [7,8,10], it has been proven that, from the structural viewpoint, the blood flow belongs to the class of self-organized tornado flows. However, the data were insufficient for diagnostic use, because the structural parameters needed a statistically reliable assessment. The method of morphometry of corrosive preparations of the heart, which allows visualizing the structure of trabecular layer with sufficient accuracy, was used to analyze the anatomy of the inner surface of the LV. This method has allowed us to detect and quantify the orientation of intracardiac anatomical structures that provide the twisting of intracardiac flow only in a few casts of the LV, where the trabeculae profile was expressed better. However, we could not obtain statistically significant meanings of main quantitative flow characteristics. Therefore, the present study aims to develop a method to determine the values of the structural parameters of blood flow in the LV using anatomical and functional methods with sufficient statistical confidence.

APPLICATION OF THE EXACT SOLUTION OF NON STATIONARY EQUATIONS OF HYDRODYNAMICS TO THE ANALYSIS OF BLOOD FLOW STRUCTURE AND THE ANATOMY OF THE HEART AND BLOOD VESSELS

It is known that the self-organized vortex flows are widespread in nature and act to restore the balance in cases of insufficient throughput capacity of the system, when other types of flow are not sufficiently effective [7]. In these cases, if the necessary and sufficient conditions of the self-organizing of structured axisymmetric Tornado-like flows are fulfilled, the streamlines are directed in a helical converged trajectory.

From the exact solution, one can see that the radial and longitudinal velocities of swirling flow do not depend on the medium viscosity, i.e. in these directions the flow occurs without energy loss. The velocity component directed azimuthally depends on the viscosity only in a narrow axial region, where the degree of reduction of the azimuthal component of velocity is related to the radius (2):

$$R = \sqrt{\frac{2k\nu}{C_0}} \qquad (2)$$

where k is a coefficient reflecting the degree of the azimuthal velocity dissipation, and C_0 is a velocity gradient in the vortex along its radius.

This means that if the medium involved in the tornado-like swirling flow is blood (kinematic viscosity at 37 ° C about 5×10^{-6} m^2/s), the radius of the flow at which the azimuthal component of velocity is reduced by half amounts to ca. 1.8 mm, which is much smaller than the caliber flow channel of the LV and aorta. Therefore, the blood flow in this segment of the circulatory system can be regarded as quasi-potential, i.e. with virtually no loss of energy.

Tornado-like flow's streamlines can be calculated by formula (3) arising from the exact solution (1):

$$z_i \, r_i^2 = Const$$
$$\phi_i = \phi_0 + \frac{W_\phi}{2W_r}\left(\frac{R_0}{r_i} - 1\right) \qquad (3)$$

where z_i, r_i, φ_i are the current values of the coordinates of a swirling jet cylindrical coordinate system, R_0, the maximum transverse dimension of the swirling jet, and φ_0 - the initial value of the angular coordinate in cylindrical coordinate system.

The geometric configuration of the flow channel of LV-aorta and intracardiac trabeculae orientation lines, streamlined by the blood flow in the cavity of the LV, correspond to the ratio (3) as described in earlier studies [7,8].

The degree of unsteadiness of pulsating blood flow is determined by the functions $C_0(t)$ and $\Gamma_0(t)$, which values can only be obtained by direct measurements of the velocity field. This calculation was performed for the flow in the aorta in [10].

CALCULATION OF STRUCTURAL PARAMETERS OF INTRACARDIAC BLOOD FLOW ACCORDING TO THE MORPHOMETRY OF CASTS OF THE LEFT VENTRICLE

Corrosive preparations of the heart allow visualization of the spatial location of intracardiac anatomical structures (trabeculae and papillary muscles) with higher resolution compared to existing methods of intravital imaging of the heart anatomy. However, the morphometry of the heart casts involve significant errors due to postmortem anatomy change. It is possible to fix these errors by increasing the number of measured LV snapshots. This required the development of a new method for calculating the structural parameters of the swirling blood flow on statistically significant data-sets.

A cast is a replica of the instantaneous state of flux. Intracardiac structures visualized on the cast, streamlined by the blood flow, give it an appropriate direction, serving as guiding vanes. Given the heart is a moving organ, the successive contractions of which lead to dynamic changes of the intracardiac trabeculae profile, these trends vary depending on the phases of the cardiac contraction. The cast contains a full range of trabeculae, working in an alternative mode for normal heartbeat. The problem of measurement is to differentiate between the structure responsible for the formation of a stream filling the cavity of the LV during diastole, and the structure which forms the flow ejected from the LV during systole. The identification of these structures is based on the major signs of tornado swirling flows arising from the exact solution:

- the flow is always twisted in the same direction along the trajectory of its movement,
- streamlines of axisymmetric swirling flow are represented by an ensemble of helically oriented non-intersecting lines,
- radius of the flow always decreases along the longitudinal axis
- the angle between the tangent to the streamline and the flow axis, decreases within the flow direction.

In accordance with these criteria the trabeculae were chosen in the cast, schematically depicted in Fig. 1.

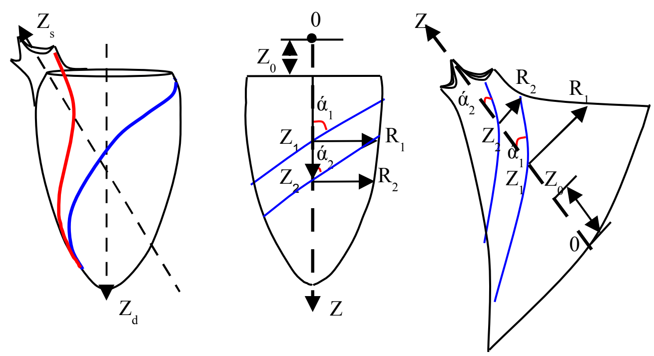

Fig. 1. The principle of identification and measurement of intracardiac structures responsible for the formation of diastolic and systolic phases of the flow evolution in the cavity of LV. Left panel, schematic representation of diastolic [blue] and systolic [red] trabeculae on the surface of the cast; center panel, measuring the angle between the diastolic trabeculae and the axis of the filling flow; right, measuring the angle between systolic trabeculae and the axis of the ejected flow).

Analysis of the blood flow structure in the heart cavity casts is reduced to the analysis of the instantaneous position of the flow streamlines corresponding to the trabeculae position.

Three parameters can be calculated from the cast morphometry that determine the flow structure. The first is the C_0/Γ_0 parameter which reflects the degree of the flow twisting being proportional to the ratio of the radial and/or longitudinal components to the azimuthal velocity component.

The second is the volumetric parameter of the swirling jet ***Q*** which corresponds to *Const* from equation (3) and is equal to the product of the squared radius of the channel and the corresponding value of the longitudinal coordinate.

The third is the Z_0 value which corresponds to the distance from the swirling jet's cylindrical coordinate system starting point to the point where one begins to count the longitudinal coordinate. Here it is necessary to note that the position of the jet origin (the start point of the jet's cylindrical coordinate system) changes during cardiac cycle, whereas the longitudinal coordinate can be counted beginning from an appropriate reference point on the cast. Thus any point along the axis of the flow can be expressed as Z_i+Z_0, where Z_i is the distance measured from reference point to the point of trabecula-axis intersection, and Z_0 is a calculated value serving the constancy of $(Z_i+Z_0)R_i^2$ product.

Under the conditions of non-stationary pulsating flow, values of all three parameters depend on time and change cyclically, reflecting the dynamics of swirling blood flow evolution. In the analysis of the cast, it was assumed that each set of values corresponds to the instantaneous state of a flux.
The principle of calculating the flow structural parameters in the LV casts can be seen from the hodograph of the velocity of an axisymmetric swirling jet (Fig. 2).

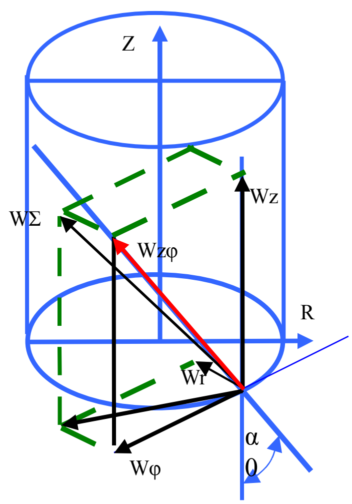

Fig. 2. Hodograph of the velocity of an axisymmetric swirling flow. W_z, W_r and W_φ stand for the longitudinal, radial and azimuthal velocity components, respectively; W_Σ - full velocity vector; $W_{(z,\varphi)}$, the sum of longitudinal and azimuthal velocity components; α_0, angle of $W_{(z,\varphi)}$ relative to the flow axis.

From the exact solution for the swirling jet, which radius is big enough to neglect the exponential value in the expression for $\mathbf{W}_\varphi$, it follows that:

$$\frac{W_z(Z)}{W_\phi(R)}=\frac{C_0(t)}{\Gamma_0(t)}\cdot 4\pi R_i(Z_i+Z_0) \quad (4)$$

where Z_i stands for the current value of the longitudinal coordinate measured from an identifiable reference point on the cast of the LV in the direction of flow (for diastolic casts it was the plane of mitral valve fibrous ring, for systolic casts it was the edge of the left ventricular cavity in the lower third of the free left ventricular wall), and R_i - radius of the flow corresponding to a value of the longitudinal coordinate Z_i measured in the plane perpendicular to the Z axis. Then the value of Z_0 corresponds to the distance from the jet origin to the point of a swirling jet, from which the measuring of the longitudinal coordinates Z_i was started.

Since the twisted flow is symmetrical relative to the axis, any point in the flow can be viewed on the hodograph of the velocity so that it lies in the plane containing the axis of the jet, which is normal to the plane containing velocity vector $W_{(z,\varphi)}$. In this case, the projection of the radial component of velocity is 0, and the projection of the total velocity vector is drawn as the sum of longitudinal and azimuthal components, or:

$$W_{(z,\phi)}=\sqrt{W_z^2+W_{\phi_\phi^2}} \quad (5)$$

Calculation of the Z_0, Q and C_0/Γ_0 parameters for trabeculae allocated on the cast were performed with application of Equations (6) obtained from a consideration of the hodograph in Fig. 2.

$$Z_0=\frac{R_1 Z_1 ctg\alpha_2-R_2 Z_2 ctg\alpha_1}{R_2 ctg\alpha_1-R_1 ctg\alpha_2}$$
$$Q=R_1^2(Z_1+Z_0)=R_2^2(Z_2+Z_0) \quad (6)$$
$$\frac{C_0}{\Gamma_0}=\frac{ctg\alpha_1}{4\pi R_1(Z_1+Z_0)}=\frac{ctg\alpha_2}{4\pi R_2(Z_2+Z_0)}$$

(for notations see the legend for Fig. 2).

Measurements were made for 12 different casts of the human LVs with no apparent disease of the heart. In all 12 casts systolic and/or diastolic trabeculae systems were observed. Besides, 13 casts of canine LVs were measured in the same manner.

Since the flow in the LV belongs to the class of tornado swirling flows, that can be described by exact solution of non-stationary Navier-Stokes and continuity equations, and the trabeculae streamlines correspond to the orientation of the flow [7], we applied the principles of trabeculae relief analysis corresponding to the structural organization of flow arising from the exact solution.

This allowed, on the one hand, to adjust the choice and position of the axis along which relative position of the trabeculae were analyzed, and on the other hand, to refine the positions of trabeculae lines (that were originally registered without reference to the flow). Thus, requirements have been developed, allowing to choose those anatomical elements, which determine the structure of blood flow in a certain phase of the cardiac cycle in the complex structure of the trabeculae layer, seen in the cast of the LV.

In general, these requirements can be formulated as follows: the axis of helical trabeculae oriented system should be located so that everywhere along the axis at the same $\boldsymbol{Z}$ value the slope of all considered trabeculae relative to the axis is the same, the points $\boldsymbol{Z_1}$ and $\boldsymbol{Z_2} > \boldsymbol{Z_1}$ must always run conditions: $\boldsymbol{R_2} < \boldsymbol{R_1}$ and $\boldsymbol{\alpha_2} < \boldsymbol{\alpha_1}$ (Fig. 1).

On the cast, the trabeculae group was chosen, forming a regular spiral bands with respect to the intended direction of movement of blood. In those places where the trabecula line was interrupted or ended, its direction was extrapolated in accordance to the requirements listed above, and pointed a marker at the surface of the cast. The direction of the axis of symmetry of the selected trabeculae group was roughly determined. The axis was marked on the surface of the cast, so that at any position of the cast one could hold the line corresponding to the projection of the axis to the surface of the cast. The cast was attached to the rotary device so that the axis of rotation corresponded to the selected axis of symmetry of trabeculae (Fig. 3). The values of the radii of the cavity in the plane perpendicular to the jet axis, and the angles of inclination of trabeculae to the axis at two longitudinal coordinates, which were counted from the above reference points on the cast, were measured. These values were fixed, rotating the cast of 360-degree increments from 30 to 60 degrees depending on the availability of trabeculae lines. In each position the cast was photographed for flat images (Fig. 3). Angles of at least two trabeculae at fixed values of $\boldsymbol{Z}$ along the selected axis at the intersections of trabeculae and the axis projection to the surface of the cast, and the radii of the cavity at these points were measured on photographs using a protractor and a ruler (Fig. 4). The measurements were repeated several times for various positions of the axis in order to achieve the minimum error in the angle of inclination for each trabeculae with the same value of $\boldsymbol{Z}$. When the optimal position of the axis was found, the values of structural parameters of blood flow $\boldsymbol{C_0/\Gamma_0}$, $\boldsymbol{Q}$ and $\boldsymbol{Z_0}$ were calculated according to (6).

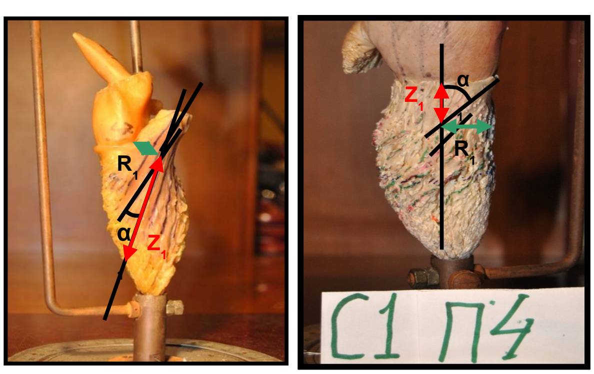

Fig. 3. Sample calculation of the structural parameters of intracardiac blood flow in casts of the LV of the heart for systolic trabeculae (left) and diastolic trabeculae on the cast surface (red marked – the longitudinal axis, green marked – the radius of the swirling jet).

It is clearly seen that the structural parameters of flow highly depend on the channel dimension. In order to understand whether the principle of flow generation and evolution is general and may be extended to the animals of other size, we have performed the same measurement in the casts of the canine LV.

MEASUREMENT OF STRUCTURAL PARAMETERS OF INTRACARDIAC BLOOD FLOW ACCORDING TO THE MULTISLICE COMPUTED TOMOGRAPHY

The method for determining the structural parameters of intracardiac blood flow by the orientation of endocardial trabeculae in the cast of the LV was used to estimate the values of these parameters in the intravital imaging of left ventricular cavity obtained using MSCT (Siemens Definition AS-128 slices, Germany). One cardiac cycle corresponded to approximately 10 images at a frequency rate 80 min^{-1} with the interval between shots about 75 ms. The data for one heart without pronounced cardiac pathology were processed. 3D reconstruction of the cavity allowed visualization of the dynamic changes of the trabeculae profile during the cardiac cycle corresponding to location of the trabeculae in the cast of the LV (Fig. 4).

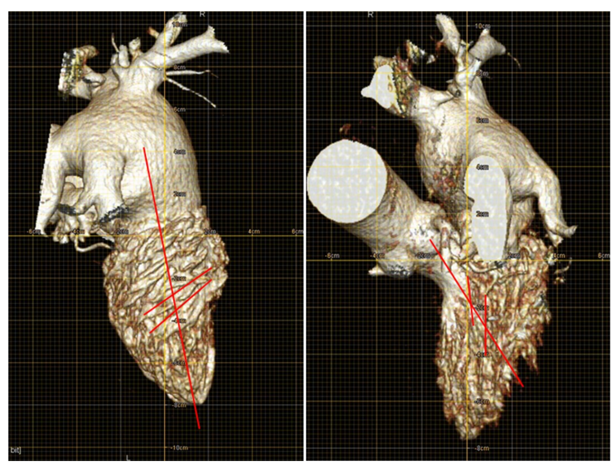

Fig. 4. Visualization of the left ventricular cavity using MSCT. The trabecular profile is clearly visible, mainly oriented relative to the filling flow during diastole (left) and relative to the ejected flow during systole (right). Red lines show the

indicative position of the axes and the direction of the trabeculae.

On the 3D images of the LV, as well as on the photographs of casts, the inferred axes of the incoming and outcoming flows were plotted. The trabeculae profile along these axes complied to the tornado flow constrains (Fig. 4). The structural parameters of the flow $C_0/Γ_0$, Q and Z_0 were calculated in conformance with the scheme in Fig. 1, formulas (6).

MATHEMATICAL MODELING OF INTRACARDIAC BLOOD FLOW ON THE BASIS OF EXACT SOLUTION OF NON-STATIONARY NAVIER-STOKES AND CONTINUITY EQUATIONS

Exact solution of the Navier-Stokes and continuity equations for the class of self-organized tornado-like flows unambiguously determine the magnitude and the direction of the velocity vector at any point in the flow. By specifying a certain mode of change of parameters $C_0(t)$, $Γ_0(t)$ and $Z_0(t)$, it becomes possible to reconstruct in 3D the velocity and trajectory of the medium elements moving along a given curvilinear axis, taking into account that the initial direction of motion corresponds to the exact solution, as determined by necessary and sufficient conditions for self-organization of tornado-like flow [7].

The flow is simulated as a moving set of particles with the same properties. Motion occurs along the curved axis of the flow, which tangent at every point is a direct instantaneous axis of tornado-like jet. Velocity field of the jet with curvilinear axis is obtained by matching exact solution for each pair of adjacent rectilinear axes of the vortices in their common points. This is possible if the angle between two adjacent segments of the axis is big enough to correspond to the condition of big value of radius of curvature of the curvilinear axis compared to the radius of the jet at the point of measurement. Thus, the resulting flow retains a tornado structure, albeit with a curvilinear axis obtained by smoothing the broken line of two straight segments.

The resulting field is parametrized by three functions of time $C_0(t)$, $Γ_0(t)$ and $Z_0(t)$. As we specify these functions, we can change the values of the components of the velocity vector at each point in dependence on time.

The knowledge of the velocity field of flow makes it possible to reconstruct the trajectory of a particle trapped in it, identifying its coordinate by numerical integration of velocity over time. In addition to the time dependencies of $C_0(t)$, $Γ_0(t)$ and $Z_0(t)$ and the form of the flow axis, the model takes into account the initial radius of the flow and viscosity of the medium.

RESULTS AND DISCUSSION

In the current study, it was established that the magnitude of the structural parameters of a twisted intracardiac blood flow $C_0/Γ_0$, Q and Z_0 vary throughout the cardiac cycle. As shown on ventricle casts, each cavity state is characterized by an individually defined set of these parameters. In the studied sample of left ventricular casts, certain phases of the cardiac cycle, or phases of the evolution of intracardiac blood flow were reflected, which could be referred to systole or diastole.

The average values $C_0/Γ_0$ over the circumference of the cast were determined considering several points on several trabeculae of two trabeculae groups. The first belongs to diastolic trabeculae responsible for LV filling and positioned on the free wall of the LV cavity. The second contains long trabeculae of the anterior LV wall and papillary muscles that are responsible for the ejected flow structure. The repeated measurements were performed changing the position of the inlet and outlet flows axes. The result was considered appropriate if the meanings of calculated parameters stayed within 10-15% error at the same value of Z. This improved significantly the accuracy of the parameter measurement and allowed to overcome the measurement error associated with the uncertainty of spatial position of the axis of a swirling jet in the LV cavity. The calculation of the parameter $C_0/Γ_0$ for each trabeculae group allowed to plot the dependence of C_0/G_0 on the value of Q (Fig. 5). The resulting continuous hyperbolic dependence with exponent close to 1 corresponds to the functional relationship between these parameters, following from the exact solution (1), and expressed by (7):

$$\frac{C_0}{Γ_0} = \frac{R_1 \cdot \mathrm{ctg}\, α_1}{4π \cdot Q} \qquad (7)$$

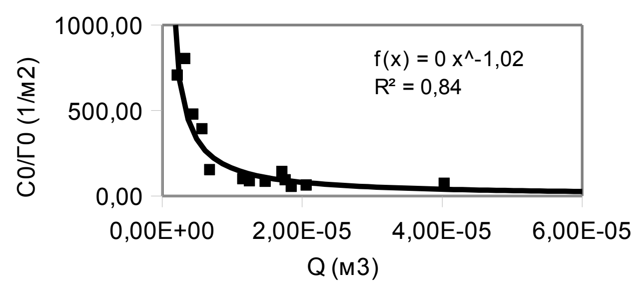

Fig. 5. Dependence of structural flow parameter $C_0/Γ_0$ on the value of volumetric jet parameter Q for 12 LV cavities studied on the casts of the human LV. The meanings for both systolic (5 left points) and diastolic trabeculae (7 right points) were plotted. The approximated line is a power function.

Therefore, the analysis of 12 normal LV casts, corresponding to different stages of cardiac contraction shows that both diastolic or systolic trabeculae streamlined by the flow are always oriented along the streamlines of a single Tornado-like jet.

This suggests an evolution of a single swirling vortex in the cavity of the LV. The vortex moves along the trabeculae, dynamically involved in the process of modulation of flux during the cardiac cycle. This result confirms our previously published data on an alternative mechanism for the LV trabeculae participation in the process of forming the structure of blood flow [3]. One group of trabeculae is located at the free wall of the LV and determines the flow direction, filling the cavity during diastole, another group of trabeculae and papillary muscles forms an ensemble of directing paddles in the left ventricular outflow tract and forms the structure of swirling flow ejected from the LV during systole. In this case, these two processes are related by common law which follows from the exact solution of non-stationary Navier-Stokes and continuity equations.

The same measurement was also performed in the canine hearts. Fig. 6. shows an identical dependence of structural flow parameter C_0/Γ_0 on the value of volumetric jet parameter Q as that obtained for the human heart despite obvious differences in body and heart dimensions, in trabeculae expression, in heart rate and flow velocities. We conclude that the mechanism of flow transportation in humans does not substantially differ from that of other mammals (dog in this case). Apparently, in both cases the blood flow is mediated by the action of specific anatomical structures which function provides for the Tornado-like hydrodynamic structure of the flow.

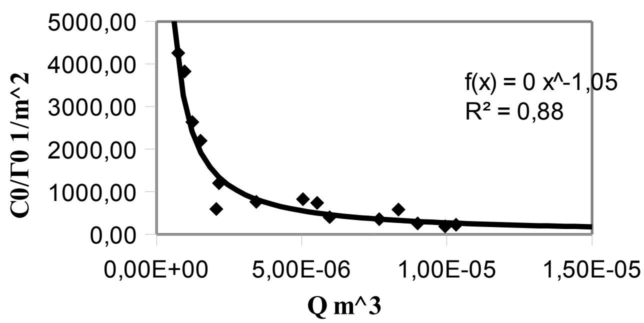

Fig. 6. Dependence of structural flow parameter C_0/Γ_0 on the value of volumetric jet parameter Q (13 casts of canine LV).

The value of the volumetric parameter Q of the swirling jet is always greater in diastolic casts than the corresponding value of this parameter for the systolic position of the trabeculae. The position of the starting point of a cylindrical coordinate system of swirling jet for diastolic casts is determined near the fibrous ring of mitral valve, and for systolic casts near the free wall of the LV. That is, the influence of trabeculae profile of left ventricular inner surface on the jet is realized in such a way that the jet changes its direction from the diastolic to systolic phase. Analysis of the relationship of the parameter Q and the distance from the point of measuring of this parameter to the origin of the jet, which is equal to the sum of values $Z_0 + Z_i$ showed linear relationship both in humans and dogs, as it should be in accordance with the principles of the evolution of tornado-like swirling jet.

We conclude that the evolution of intracardiac blood swirling jet operated by a coordinated change in direction of trabeculae acting as guide paddles, which determine the twisted structure of flow, is in agreement with the exact solution (1) for a single tornado-like jet.

This conclusion admits that the axisymmetric jet is generated in the asymmetric flowing channel. Indeed, some zones always exist in the ventricular cavity where the flow is organized otherwise than in the dominating jet. These may be secondary flows that are the components of the 3D boundary layer which is indispensable for the swirling tornado-like jet initiation and stability. Such a boundary layer was described in hydrodynamics [7], and it may consist of the vortices of Hörtler type having less caliber than the main jet. These vortices appear as a concave surface of the channel wall is flown by liquid medium, and provide the link between the swirling jet and relatively immovable channel wall. By this the jet is rolling along the channel wall over such boundary layer, the wall velocity is equal to 0, and the shear stresses in the near-wall zone are substituted by the rolling stresses. The last consideration explains partially the nature of low hydrodynamic resistance accompanying the tornado-like flows evolution.

The geometrical configuration of the ventricular chamber during cardiac contraction is altered in correspondence with the flow evolution. Both the cavity geometry and trabeculae expression affect the flow evolution so that the origin of the jet moves in the direction of flow from the left atrium, where the swirling jet initially occurs, to the aorta, where it is ejected as a result of cardiac contraction. This mechanism provides a conservative swirling jet in the flow channel, containing two valves on the jet's path.

The calculation of parameters C_0/Γ_0 on 3D images of the left ventricular cavity, obtained using MSCT was performed according to the same protocol as the morphometric measurements in the ventricular casts.

Plotting the values of this parameter vs time during the cardiac cycle which starts at filling of the LV with blood, has a two-phase form as it is shown in Fig. 7. When the LV is being filled during the diastole, the azimuthal velocity component apparently prevails. During contraction, as the ventricular size is reduced, the value C_0/Γ_0 increases, reflecting the dominance of the longitudinal and radial velocity components over the value of the azimuthal component (Fig. 6). The absolute values of this parameter are in agreement with data obtained by morphometry of LV casts.

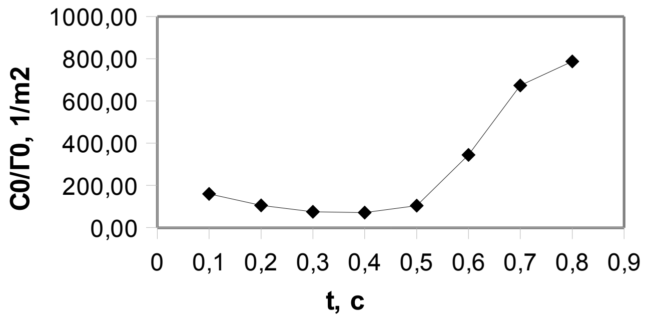

Fig. 7. Dependence of structural parameters of intracardiac blood flow C_0/Γ_0 on time during one cardiac cycle obtained using MSCT.

Therefore, the use of exact solution for the analysis of expression of the anatomical structures responsible for the formation of a swirling tornado-like flow of blood in the LV cavity allows to determine structural parameters of the blood flow C_0/Γ_0, reflecting the attitude of potential longitudinal and radial velocity component to the azimuthal component. This value is the criterion for the instantaneous state of intracardiac blood flow and can be used for clinical diagnostics of heartbeat dis-coordination, decrease of myocardial contractility, the effects of organic changes of intracardiac anatomy on the structure of the flow. This criterion provides additional information about hydrodynamics of intracardiac blood flow, which is important for research purposes - mathematical and physical modeling of flow. The change of this criterion in the evolution of intracardiac flow gives important information about the initial and boundary conditions of the blood flow in the heart, which is necessary for the correct use of numerical modeling techniques of intracardiac hemodynamics.

Structural parameters of intracardiac blood flow obtained from the casts of the LV and CT images were used in the mathematical model of intracardiac blood flow, based on exact solution of Navier-Stokes and Continuity equations.

The model input parameters are the structural parameters of tornado-like flow of $C_0(t)$ and $\Gamma_0(t)$, which dynamics in time is determined by the dynamics of cardiac contraction. The values for these parameters were plotted in such a way that (a) the velocities in the directions of the axes of cylindrical coordinate system did not contradict the known range of normal variation of velocity in the cavity of the heart and (b) changes in the parameters C_0 and Γ_0 during the cardiac cycle were consistent with the experimentally obtained curve in Fig. 5 and Fig. 7. The axis along which the flow was modeled, was built empirically in qualitative agreement with the intended axis of the swirling flow in the heart based on the known configuration of the cavity, studied in the casts of the LV. Parameter Z_0 was specified in a way that, on the one hand, it did not exit the known geometric dimensions of the human LV, and on the other hand, reflected the functioning of the mitral valve. Initial radius of the swirling flow in the area of its origin was taken to be 0.035 m. The kinematic viscosity of the medium was assumed to be $5x10^{-6}m^2s^{-1}$. The resulting set of parameters is shown in Fig. 8.

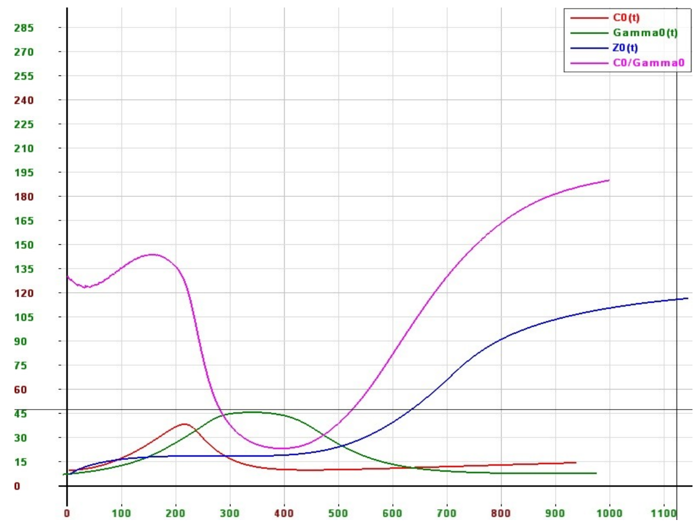

Fig. 8. The plots $C_0(t)$, $\Gamma_0(t)$ and $Z_0(t)$ specified for the simulation of swirling intracardiac blood flow
($C_0 \times 1\ s^{-1}$; $\Gamma_0 \times 10^{-3}\ m^2 s^{-1}$; $Z_0 \times 10^{-3}\ m$; $C_0/\Gamma_0 \times 10\ m^{-2}$).

Calculation of the current lines of swirling flow along a given curved axis is shown in Fig. 9.

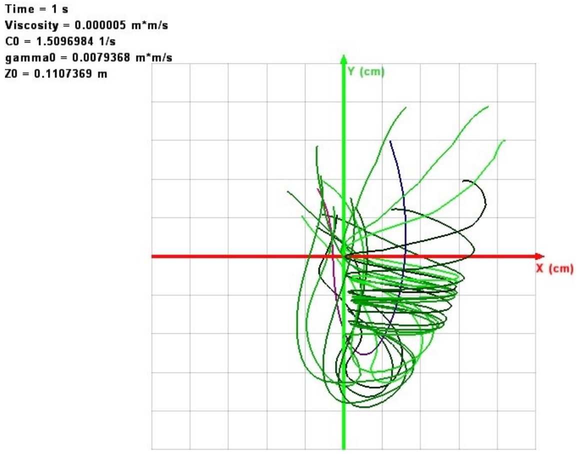

Fig. 9. Three-dimensional reconstruction of the current lines of swirling flow along the curvilinear axis, corresponding to the dynamics of changes of parameters in Fig. 7

Fig. 9 shows that three-dimensional region circumscribed by the streamlines of swirling flow is in a good qualitative agreement with the shape of left ventricular cavity, and the angle of inclination of the streamlines in the inlet portion of the image relative to the flow axis is close to the slope of diastolic trabeculae of the LV free wall. The main stages of flow evolution are reproduced: primary acceleration during the diastolic filling, flow deceleration at the end of diastole, flow turn towards the aortic valve, and flow acceleration during ejection. The full time of the cycle is equal to 1 sec.

CONCLUSION

Thus, the exact solution of non-stationary Navier-Stokes and Continuity equations for a class of self-organized tornado viscous liquid can quantitatively describe and model the flow pattern corresponding to the swirling blood flow in the cavity of the LV. The principle of blood flow structural organization, its generation and evolution appear to be the same in humans and non-human mammals. The knowledge of blood flow hydrodynamics and the possibility of its quantification are important for a better understanding of the mechanisms of generation and evolution of the blood flow in heart. In particular, it should allow for a much more precise determination of the initial and boundary conditions for the blood flow. The magnitude and dynamics of the structural parameters of blood flow used in the exact solution are comprehensive individual characteristics of the blood flow of each particular patient and, therefore, have a potential diagnostic value. The quantitative structural characteristics of flow should be used for the cardiac surgery optimization, for the design of new implantable and paracorporal devices contacting with blood, and for mathematical and physical modeling of circulation.

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