Modeling the Air-Cooled Gas Turbine: Part 2—Coolant Flows and Losses

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This paper is Part II of a study concerned with developing a formal framework for modeling air-cooled gas turbine cycles. It deals with the detailed specification of coolant flowrates and losses. For accurate performance assessment, it is necessary to divide the turbine expansion into individual stages with stator and rotor rows being treated separately. Particular care is needed when deriving the equations for the rotor, and it is shown how all required flow variables can be estimated from minimal data if design values are unavailable. Specification of the cooling flowrates is based on a modified Holland and Thake procedure, which can be formalized in terms of averaged parameters. Thermal barrier coatings can be included if present. The importance of allowing for fluctuations in combustor outlet temperature is stressed and procedures for dealing with end-wall and disk cooling are suggested. There is confusion in the literature concerning cooling losses, and it is shown how these may be defined and subdivided in a consistent way. The importance of representing losses in terms of irreversible entropy creation rather than total pressure loss is stressed. A set of models for the components of the cooling loss are presented and sample calculations are used to illustrate the division and magnitude of the loss. [DOI: 10.1115/1.1415038]

1 Introduction

This paper is Part II of a study concerned with developing a formal framework for modeling air-cooled gas turbine cycles and deals with the detailed specification of cooling flowrates and losses. A general discussion of the thermodynamics of air-cooling was presented in Part I (Young and Wilcock [1]).

Specification of the cooling flowrates and losses for performance predictions involve two essentially separate problems. In the detailed design of a turbine, the cooling flowrates are established by a complex procedure involving correlation of experimental results and semitheoretical calculations. For cycle calculations, this lengthy procedure must be condensed into a simplified scheme, which retains the essential features of the underlying physics and is valid over a range of operating conditions. In this paper, a modified version of the well-known scheme by Holland and Thake [2] is used. Most other theories (Horlock [3], El-Masri [4]) are restricted to a specific geometry or lack generality.

In Part I it was noted that the literature on cooling losses is often confused and that there is no definitive formal analysis available to provide a framework for development. Typically, cycle calculations seem to involve a rather random selection of losses drawn from a list that includes coolant–mainstream mixing, heat transfer through the blades, and coolant throttling. In Part I, a strong case was made for representing the cooling losses as additive entropy creation terms and this will be the approach pursued in Part II. Indeed, one of the main objectives is to provide a baseline set of well-defined expressions for the cooling losses that are sufficiently general for cycle calculations but that can be fine-tuned for more accurate design work.

The paper takes a deliberately detailed approach in specifying the cooling flows and losses but, in order to preserve continuity of the main text, most of the analysis has been relegated to the appendices. These, therefore, embody an important part of the development.

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2 The Cooled Turbine Stage

Any realistic model of a turbine must address the cooling of each blade row separately. Continuous expansion path cooling models (El-Masri [5]), may be acceptable for initial cycle development work but cannot provide the accuracy required once some basic details of the turbine layout are known. The information required to implement the calculation schemes described below is minimal, being little more than a knowledge of the number of stages, the stage pressure ratios or work requirements, and the uncooled stage efficiencies. These data can be obtained in a variety of ways ranging from informed guesswork to an elaborate flow analysis linked to the cooling calculations. A one-dimensional version of the latter based on mean blade angles and empirical loss coefficients is described by Kawaike et al. [6].

Figure 1 is a schematic diagram of a cooled gas turbine stage. Mass flowrate, specific total enthalpy (in a stationary frame of reference) and specific entropy are denoted by m, h_0, and s, respectively. The subscript g is appended to denote mainstream *gas* and c to denote *coolant*. Where necessary, subscripts sc, rc, or dc are used to differentiate between stator, rotor, and disk cooling. The flows crossing any reference plane may be nonuniform and it is understood that h_0 and s represent massflow-averaged values. Reference planes at stator inlet, stator outlet/rotor inlet, rotor outlet, and stage outlet are numbered 1, 2, 3, and 4, respectively.

The mass flowrate and composition of the mainstream changes from one plane to the next because of the addition of coolant. Coolant is drawn from the compressor at state k and enters the blade passages at state i. (The k and i states will generally be different for the stator and rotor blades.) For convection cooling, the coolant leaves from the end of the blade or the trailing edge (states b and t). For combined convection and film cooling, it exits through holes in the blade surfaces or endwalls (states f and e). For each type of flow, a massflow-averaged exit state can be defined. Thus, $h_{0c,f}$ and $s_{c,f}$ are values massflow-averaged over all film cooling holes. For each row, a mean massflow-averaged exit state x can be defined by,

$$m_c h_{0c,x} = m_{c,f} h_{0c,f} + m_{c,e} h_{0c,e} + m_{c,t} h_{0c,t} + m_{c,b} h_{0c,b} \quad (1a)$$

$$m_c s_{c,x} = m_{c,f} s_{c,f} + m_{c,e} s_{c,e} + m_{c,t} s_{c,t} + m_{c,b} s_{c,b} \quad (1b)$$

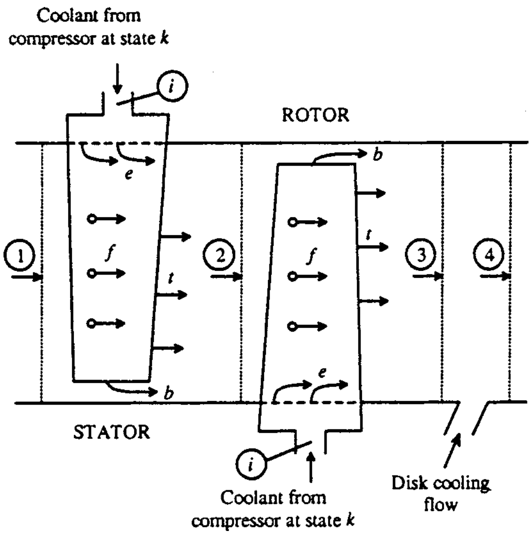

Fig. 1 Schematic diagram of a cooled gas turbine stage

where $m_c = m_{c,f} + m_{c,e} + m_{c,t} + m_{c,b}$. The cooling flows associated with each blade row can therefore be reduced to a single flow drawn from the compressor at state *k*, entering the blade passages at state *i* and exiting into the main flow path (before mixing) at state *x*.

Rotor disk cooling is treated separately. In the present work it is represented by a single injection after each rotor on the assumption that no useful shaft work is obtained from this cooling flow.

3 First Law Analysis

3.1 Stationary Blades. Figure 2 is a schematic diagram illustrating the simplified blade cooling model. Taken together, the mainstream and coolant flows are adiabatic and the steady-flow energy equation for the combination is,

$$m_{g,1}(h_{0g,1} - h_{0g,2}) + m_{sc}(h_{0sc,i} - h_{0g,2}) = 0 \quad (2a)$$

Assuming adiabatic flow for the coolant between compressor bleed point and blade inlet, $h_{0sc,i} = h_{0sc,k}$, $h_{0g,1}$ and $h_{0sc,k}$ are known and hence $h_{0g,2}$ can be determined from Eq. (2*a*) once the coolant-to-mainstream mass flow ratio m_{sc}/m_{g1} is known.

Steady-flow energy equations can also be written separately for the external (mainstream) and internal (coolant) flows,

$$m_{g,1}(h_{0g,1} - h_{0g,2}) + m_{sc}(h_{0sc,x} - h_{0g,2}) = Q_s \quad (2b)$$

$$m_{sc}(h_{0sc,i} - h_{0sc,x}) = -Q_s \quad (2c)$$

where Q_s is the total rate of heat transfer through the blade surface (mainstream to coolant). Clearly, $(2a) = (2b) + (2c)$.

3.2 Rotating Blades. The overall energy equation for the rotor is,

$$m_{g,2}(h_{0g,2} - h_{0g,3}) + m_{rc}(h_{0rc,i} - h_{0g,3}) = P \quad (3a)$$

where $m_{g,2} = m_{g,1} + m_{sc}$ and *P* is the shaft power output. Assuming adiabatic flow between the compressor bleed point and blade inlet, $h_{0rc,i} = h_{0rc,k}$. For the final turbine stage on a spool, *P* is fixed by

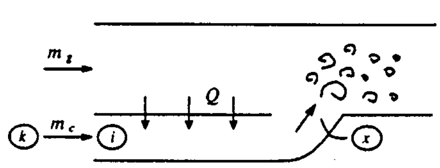

Fig. 2 Simplified blade cooling model

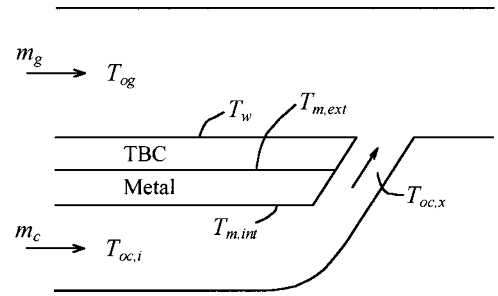

Fig. 3 Notation for the heat transfer model (stators and rotors)

the compressor power requirement and $h_{0g,3}$ can be determined if $m_{rc}/m_{g,2}$ is known. The stage pressure ratio follows once the entropy creation due to the losses has been found. In other cases, the stage pressure ratio is specified, the losses are determined, and *P* and $h_{0g,3}$ follow.

The energy equations for the external and internal flows are,

$$m_{g,2}(h_{0g,2} - h_{0g,3}) + m_{rc}(h_{0rc,x} - h_{0g,3}) = P_{ext} + Q_r \quad (3b)$$

$$m_{rc}(h_{0rc,i} - h_{0rc,x}) = P_{\text{int}} - Q_r \quad (3c)$$

where P_{ext} and P_{int} are the external and internal contributions to the net power ($P = P_{ext} + P_{int}$). Introduction of the Euler turbine equation (see Appendix 1) results in energy equations expressed in terms of the specific rothalpy ($i = h_0 - UV_\theta$),

$$m_{g,2}(i_{g,2} - i_{g,3}) + m_{rc}(i_{rc,i} - i_{g,3}) = 0 \quad (4a)$$

$$m_{g,2}(i_{g,2} - i_{g,3}) + m_{rc}(i_{rc,x} - i_{g,3}) = Q_r \quad (4b)$$

$$m_{rc}(i_{rc,i} - i_{rc,x}) = -Q_r \quad (4c)$$

The application of Eqs. (3) and (4) for the rotating blades requires detailed knowledge of the turbine (particularly the rotor inlet conditions), which may not be available. Simplification is possible, however, by expressing all the unknown quantities in terms of a *stage loading coefficient* ψ. This is defined in the usual way by,

$$\psi = \frac{P}{m_{g,3} U_{mean}^2} \quad (5)$$

It is useful to recall that ψ can be related approximately to the *degree of reaction* ρ. Neglecting coolant addition, assuming constant axial velocity through the rotor and zero exit swirl from the stage, it can be shown that $\psi = 2(1-\rho)$. Thus, for impulse blading $\rho = 0$, $\psi \cong 2$, and for 50 percent reaction blading $\rho = 0.5$, $\psi \cong 1$. The rotor approximations are discussed in detail in Appendix 1.

3.3 Disk Cooling Flows. Disk cooling air is injected both before and after the rotor. The upstream injection does little work in the rotor, however, and it is expedient to combine the two flows into a single flow entering downstream. The energy equation between stations 3 and 4 is then,

$$m_{g,3}(h_{0g,3} - h_{0g,4}) + m_{dc}(h_{0dc,k} - h_{0g,4}) = 0 \quad (6)$$

where $m_{g,3} = m_{g,2} + m_{rc}$ and m_{dc} is the mass flow rate of disk cooling air.

4 Calculation of the Cooling Flowrates

Application of Eqs. (2) and (3) in a performance calculation requires realistic values of the mass flow ratios $m_{sc}/m_{g,1}$ and $m_{rc}/m_{g,2}$. To obtain these, it is first necessary to estimate the minimum cooling flowrates required to maintain the blade temperatures within the safe operating range defined by the blade material properties. The procedure recommended is an extension of the method developed by Holland and Thake [2]. The notation is shown in Fig. 3. The internal and external surface metal temperatures $T_{m,int}$ and $T_{m,ext}$ are assumed uniform over the blade. If a thermal barrier coating (TBC) is present, the outer temperature T_w differs from $T_{m,ext}$.

Details of the heat transfer model and cooling flowrate calculations are given in Appendix 2. The procedure for stator and rotor blades is the same except that the latter is carried out with respect to the rotating co-ordinate system. The calculations require the specification of a number of empirical parameters, notably the *internal flow cooling efficiency*, the *film cooling effectiveness*, and the metal and TBC *Biot numbers*. These parameters (defined in Appendix 2) reflect the level and sophistication of the cooling technology and are introduced in order to bypass the difficult problem of calculating the detailed flow behavior within the blade passages and in the external film. The other important empirical parameter, also defined in Appendix 2, is the *cooling flow factor* K_{cool}. This can be estimated from a knowledge of the blade geometry and the external flow Stanton number.

If the value of T_{0g} corresponding to the mass-averaged value of h_{0g} at blade inlet (absolute or relative as appropriate) is used in the heat transfer calculation procedure, the predicted cooling flowrates are invariably much lower than those found in real engines. This is because the design procedure must make allowance for the possibility of temperature fluctuations (hot spots) in the flow exiting the combustor. A simple way of doing this (Kawaike et al. [6]) is to replace T_{0g} by an estimated maximum temperature,

$$T_{0g}^{\max}=T_{0g}+K_{comb}\Delta T_{comb} \tag{7}$$

where ΔT_{comb} is the temperature rise through the combustor. K_{comb} (sometimes called the *combustion pattern factor*) is an empirical constant which depends on the type of combustor (aero or industrial) and the position of the blade row with respect to the combustor outlet.

The rotor disk cooling flows are more difficult to estimate than the blade cooling flows. Apart from cooling the rotor disks (which receive heat by conduction from the blades), the flows also prevent the ingestion of mainstream gas into the rotor disk cavities. In the present model, values of $m_{dc}/m_{g,3}$ are simply specified by the user.

As an example of the calculation of cooling flowrates, consider a single-stage turbine (without thermal barrier coatings) operating at the conditions given in Table 1. For the purpose of illustration, the stage polytropic efficiency is assumed to include the effects of the cooling losses. The cooling flow factor K_{cool} is based on $\mathrm{St}_g=0.0015$ and $A_{surf}c_{pg}/A_g{*}c_{pc}=30$ (see Appendix 2). Combustion pattern factors K_{comb} of 0.1 for the stator and 0.05 for the rotor were first used to calculate the cooling flowrates. Then, using these flowrates but taking $K_{comb}=0$, the blade cooling effectiveness ε_0 and the metal temperatures $T_{m,ext}$ and $T_{m,int}$ were recalculated to give the values, representative of the mean flow conditions, presented in Table 1.

The coolant to mainstream flow ratios of 14.5 percent for the stator and 4.9 percent for the rotor are typical of those found in real engines. The sensitivity of the *total* stage cooling flowrate to changes in the operating conditions is shown in Fig. 4. Thus, a *decrease* of 1 percentage point in cooling flowrate corresponds to either: (i) a *decrease* in coolant supply temperature of 15°C, or (ii) a *decrease* in combustor outlet temperature of 32°C, or (iii) an *increase* in allowable blade temperature of 10°C, or (iv) an *increase* in film cooling effectiveness of 0.035, or (v) an *increase* in internal cooling efficiency of 0.08, or (vi) a *decrease* in metal Biot number of 0.08.

Table 1 Sample calculation of cooling flowrates

TURBINE DATA		
Combustor outlet temperature	$T_{0g,1}$	= 1700 K
Maximum metal temperature	$T_{m,ext}$	= 1100 K
Coolant supply temperature	$T_{0c,k}$	= 867 K
Turbine inlet total pressure	$p_{0g,1}$	= 34 bar
Coolant supply pressure	$p_{0c,k}$	= 34 bar
Stage pressure ratio	$p_{0g,1}/p_{0g,4}$	= 2.4
Stage loading coefficient	ψ	= 1.0
Stage polytropic efficiency	η_p	= 0.9
Combustion pattern factor (stator)	K_{comb}	= 0.1
Combustion pattern factor (rotor)	K_{comb}	= 0.05
Rotor swirl factor	K_{swirl}	= 0.5
Cooling flow factor (stator/rotor)	K_{cool}	= 0.045
Internal cooling efficiency (stator/rotor)	$\eta_{c,int}$	= 0.7
Film cooling effectiveness (stator/rotor)	ε_f	= 0.4
Metal Biot number (stator/rotor)	Bi_m	= 0.2
STATOR RESULTS		
Coolant/mainstream flowrate ratio	$m_{sc}/m_{g,1}$	= 0.145
Blade cooling effectiveness	ε_0	= 0.75
Rotor inlet temperature (absolute)	$T_{0g,2}$	= 1603 K
Coolant exit temperature	$T_{0c,x}$	= 969 K
External metal temperature	$T_{m,ext}$	= 1078 K
Internal metal temperature	$T_{m,int}$	= 1013 K
ROTOR RESULTS		
Coolant/mainstream flowrate ratio	$m_{rc}/m_{g,2}$	= 0.049
Blade cooling effectiveness	ε_0	= 0.58
Rotor inlet temperature (relative)	$T_{0g,2}$	= 1487 K
Coolant exit temperature (relative)	$T_{0c,x}$	= 966 K
External metal temperature	$T_{m,ext}$	= 1082 K
Internal metal temperature	$T_{m,int}$	= 1043 K

5 Second Law Analysis

5.1 Stationary Blades. With reference to Figs. 1 and 2, the second law of thermodynamics for the combined mainstream and coolant flows is,

$$\Delta\Sigma_s=\Delta\Sigma_{s,basic}+\Delta\Sigma_{s,cool}=m_{g,1}(s_{g,2}-s_{g,1})+m_{sc}(s_{g,2}-s_{sc,k}) \tag{8a}$$

where $\Delta\Sigma_s$ is the total rate of entropy creation due to irreversibilities. (Because of irreversibilities between the compressor bleed point and the inlet to the internal blade passages, $s_{sc,i}>s_{sc,k}$ and this loss has been included in $\Delta\Sigma_s$.) $\Delta\Sigma_s$ is formally subdivided into $\Delta\Sigma_{s,basic}$ (the loss associated with uncooled operation) and $\Delta\Sigma_{s,cool}$ (the extra loss associated with cooling).

Second law statements can also be written for the external mainstream flow, the heat transfer through the TBC and metal, and the internal coolant flow. Thus,

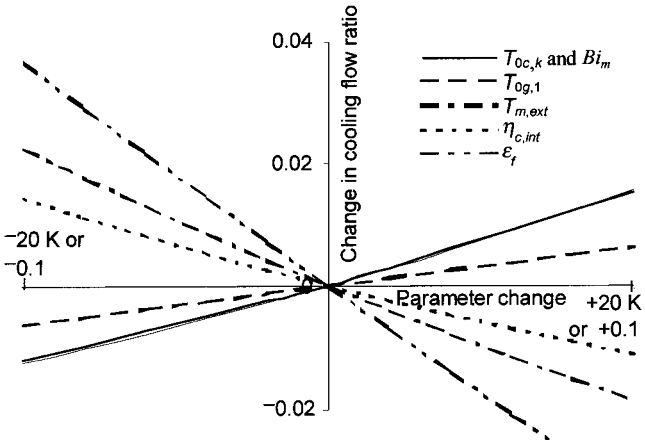

Fig. 4 Sensitivity of cooling flowrate to changes in operating conditions

$$\Delta\Sigma_{s,basic}+\Delta\Sigma_{s,ext}=m_{g,1}(s_{g,2}-s_{g,1})+m_{sc}(s_{g,2}-s_{sc,x})+\frac{Q_s}{T_{sw}} \tag{8b}$$

$$\Delta\Sigma_{s,tbc}=Q_s\left(\frac{1}{T_{sm,ext}}-\frac{1}{T_{sw}}\right) \tag{8c}$$

$$\Delta\Sigma_{s,met}=Q_s\left(\frac{1}{T_{sm,int}}-\frac{1}{T_{sm,ext}}\right) \tag{8d}$$

$$\Delta\Sigma_{s,int}=m_{cs}(s_{sc,x}-s_{sc,k})-\frac{Q_s}{T_{sm,int}} \tag{8e}$$

where subscripts sm and sw denote stator metal and external wall, respectively, and $\Delta\Sigma_{s,ext}$ refers solely to the cooling losses of the external flow. Adding Eqs. (8*b*)–(8*e*) and comparing with Eq. (8*a*) allows the cooling losses to be subdivided into four components,

$$\Delta\Sigma_{s,cool}=\Delta\Sigma_{s,ext}+\Delta\Sigma_{s,tbc}+\Delta\Sigma_{s,met}+\Delta\Sigma_{s,int} \tag{8f}$$

$\Delta\Sigma_{s,tbc}$ and $\Delta\Sigma_{s,met}$ can be calculated directly because Q_s, T_{sw}, $T_{sm,ext}$ and $T_{sm,int}$ are known from the heat transfer analysis. However, $\Delta\Sigma_{s,ext}$ and $\Delta\Sigma_{s,int}$ require separate empirical loss models. Then, $s_{g,2}$ can be found from Eq. (8*a*) and, with $h_{0g,2}$ known from the first law analysis and the gas composition from continuity, state 2 is completely specified.

If each $\Delta\Sigma$ term is multiplied by the *dead state temperature*, the resulting *lost power* term is identical to that which would arise from a formal exergy analysis. The introduction of exergy tends to cloud the issues, however, and it is felt that the straightforward second law analysis presented above provides a better physical interpretation.

5.2 Rotating Blades. The second law analysis for a rotor blade row is identical. Indeed, the entropy creation expressions can be obtained from Eq. (8) simply by an appropriate change of subscript. Thus,

$$\Delta\Sigma_r=\Delta\Sigma_{r,basic}+\Delta\Sigma_{r,cool}=m_{g,2}(s_{g,3}-s_{g,2})+m_{rc}(s_{g,3}-s_{rc,k}) \tag{9a}$$

$$\Delta\Sigma_{r,basic}+\Delta\Sigma_{r,ext}=m_{g,2}(s_{g,3}-s_{g,2})+m_{rc}(s_{g,3}-s_{rc,x})+\frac{Q_r}{T_{rw}} \tag{9b}$$

$$\Delta\Sigma_{r,tbc}=Q_r\left(\frac{1}{T_{rm,ext}}-\frac{1}{T_{rw}}\right) \tag{9c}$$

$$\Delta\Sigma_{r,met}=Q_r\left(\frac{1}{T_{rm,int}}-\frac{1}{T_{rm,ext}}\right) \tag{9d}$$

$$\Delta\Sigma_{r,int}=m_{rc}(s_{rc,x}-s_{rc,k})-\frac{Q_r}{T_{rm,int}} \tag{9e}$$

$$\Delta\Sigma_{r,cool}=\Delta\Sigma_{r,ext}+\Delta\Sigma_{r,tbc}+\Delta\Sigma_{r,met}+\Delta\Sigma_{r,int} \tag{9f}$$

5.3 Disk Cooling Flows. Applying the second law between planes 3 and 4 of Fig. 1 gives,

$$\Delta\Sigma_{dc}=m_{g,3}(s_{g,4}-s_{g,3})+m_{dc}(s_{g,4}-s_{dc,k}) \tag{10}$$

If $\Delta\Sigma_{dc}$ is specified by an empirical model, $s_{g,4}$ can be calculated from Eq. (10). Knowing $h_{0g,4}$ and the gas composition fixes state 4.

6 Specification of Losses

6.1 Basic (Uncooled) Loss. The *basic* rate of entropy creation is related to the uncooled stage polytropic efficiency η_{basic} (assumed known) by the expression,

$$\Delta\Sigma_{basic}=m_{g,1}R_g(1-\eta_{basic})\ln\left(\frac{p_{0g,1}}{p_{0g,4}}\right) \tag{11}$$

where p_{0g} and R_g are the mainstream total pressure and specific gas constant respectively. $\Delta\Sigma_{basic}$ is distributed between the stator and rotor in proportions deemed appropriate. It is a major assumption of the approach that $\Delta\Sigma_{basic}$ is unchanged by the presence of cooling.

6.2 Internal Friction and Heat Transfer Losses. Introducing Eq. (*A*2.1) for the heat transfer Q and assuming the coolant specific heat capacity c_{pc} to be temperature independent, Eqs. (8*e*) and (9*e*) can be written as a single equation,

$$\Delta\Sigma_{int}=m_c c_{pc}\left[\ln\left(\frac{T_{0c,x}}{T_{0c,k}}\right)-\left(\frac{T_{0c,x}-T_{0c,i}}{T_{m,int}}\right)\right]-m_c R_c\ln\left(\frac{p_{0c,x}}{p_{0c,k}}\right) \tag{12}$$

In Eq. (12), the coolant supply conditions $p_{0c,k}$ and $T_{0c,k}$ are expressed in the stationary frame of reference for both stator and rotor blade rows. For rotors, however, $T_{0c,i}$, $T_{0c,x}$ and $p_{0c,x}$, and must each be expressed in the rotating coordinate system.

There are two unknowns in Eq. (12), $\Delta\Sigma_{int}$ and $p_{0c,x}$. $\Delta\Sigma_{int}$ is the rate of entropy creation between the coolant supply and the blade exit holes, and $p_{0c,x}$ is the total pressure at these holes (absolute or relative as appropriate). One approach is to develop an expression for $\Delta\Sigma_{int}$ by modeling the friction and heat transfer losses. The internal flow is very complex, however, and it is actually easier to devise a method to specify $p_{0c,x}$. (This parallels the first law analysis in Appendix 2 where $h_{0c,x}$ was found by a suitable choice of $\eta_{c,int}$.)

Now, ignoring any streamline curvature effects, the coolant static pressure must equal the mainstream static pressure at the exit hole, $p_{c,x}=p_{g,x}$. If, in addition, some information about the coolant exit velocity is supplied, this is sufficient to fix $p_{0c,x}$. The most convenient parameter to specify is the coolant/mainstream momentum flux ratio,

$$I=\frac{\rho_{c,x}V_{c,x}^2}{\rho_{g,x}V_{g,x}^2}=\frac{\gamma_c M_{c,x}^2}{\gamma_g M_{g,x}^2} \tag{13a}$$

where V is velocity (absolute for a stator and relative for a rotor), and M is the corresponding Mach number. The coolant/mainstream total pressure ratio is then,

$$\frac{p_{0c,x}}{p_{0g,x}}=\frac{[1+0.5(\gamma_c-1)M_{c,x}^2]^{\gamma_c/(\gamma_c-1)}}{[1+0.5(\gamma_g-1)M_{g,x}^2]^{\gamma_g/(\gamma_g-1)}}=f(I,M_{g,x},\gamma_c,\gamma_g) \tag{13b}$$

$p_{0g,x}$ is approximately equal to the total pressure at blade inlet (absolute or relative as appropriate) and is therefore known. Thus, specifying I and $M_{g,x}$ allows the calculation of $p_{0c,x}$. $\Delta\Sigma_{int}$ then

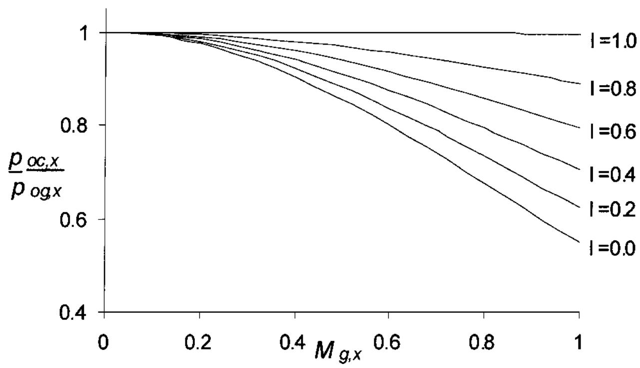

Fig. 5 Coolant/mainstream total pressure ratio as a function of mainstream Mach number and momentum flux ratio *I*

follows from Eq. (12). Figure 5 shows the variation of $p_{0c,x}/p_{0g,x}$ with mainstream Mach number $M_{g,x}$ for different values of the momentum flux ratio *I*.

This analysis shows that the internal losses are fixed by the pressure ratio $p_{0c,k}/p_{c,x}$, the coolant velocity at the exit hole and, to a lesser extent, the heat transfer to the coolant. Once these conditions are specified, no amount of redesign of the internal flow path will affect the loss. If, in practice, the exit velocity were higher than design, it would be necessary to introduce extra throttling somewhere along the flow path in order to achieve the chosen exit condition.

If desired, $\Delta\Sigma_{int}$ can (with minor approximation) be subdivided into contributions associated separately with heat transfer and fluid friction. The analysis, described in Appendix 3, is based on the one-dimensional control volume model illustrated in Fig. 6 and leads to,

$$\Delta\Sigma_{int,Q}=m_c c_{pc}\left[K_{int}\ln\left(\frac{T_{0c,x}}{T_{0c,i}}\right)-\left(\frac{T_{0c,x}-T_{0c,i}}{T_{m,int}}\right)\right] \quad (14a)$$

$$\Delta\Sigma_{int,F}=m_c c_{pc}\left[\ln\left(\frac{T_{0c,x}}{T_{0c,k}}\right)-K_{int}\ln\left(\frac{T_{0c,x}}{T_{0c,i}}\right)\right]-m_c R_c\ln\left(\frac{p_{0c,x}}{p_{0c,k}}\right) \quad (14b)$$

where K_{int} is defined by Eq. (*A*3.6). Clearly, addition of Eqs. (14*a*) and (14*b*) gives Eq. (12).

6.3 External Heat Transfer and Mixing Losses. The external losses result from: (*i*) boundary layer friction and heat transfer and, (*ii*) the mixing of the coolant with the mainstream. The friction loss is already accounted for in the basic loss of Eq. (11) and is assumed unchanged in the presence of cooling. The additional *cooling* losses are therefore the sum of the heat transfer and mixing losses,

$$\Delta\Sigma_{ext}=\Delta\Sigma_{ext,Q}+\Delta\Sigma_{mix} \quad (15)$$

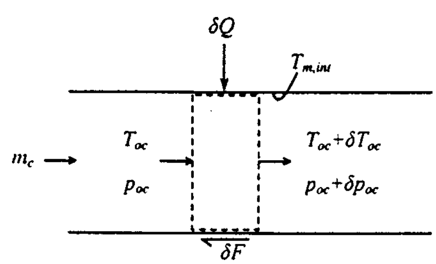

Fig. 6 Fluid friction and heat transfer in a one-dimensional flow

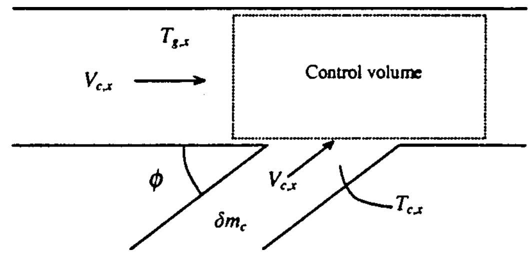

Fig. 7 Definition sketch for mixing analysis

$\Delta\Sigma_{ext,Q}$ can be estimated using the same approach as for the internal coolant flow (see Appendix 3). The resulting expression is,

$$\Delta\Sigma_{ext,Q}\cong\left(\frac{1}{T_w}-\frac{K_{ext}}{T_{0g}}\right)Q \quad (16)$$

where K_{ext} is defined by Eq. (*A*3.8).

The coolant-mainstream mixing loss is usually estimated using the method of Hartsell [7]. This is based on the Hawthorne–Shapiro theory of one-dimensional flow with mass addition and the result is usually expressed in terms of the change in mainstream total pressure. Change in p_0 is a measurable quantity but it can be misleading because, in a mixing process, it does not represent the irreversible loss. This has been the source of much confusion in the literature. In flows with mixing and heat transfer, it is important to work directly with expressions for the entropy creation rates.

The mixing process is shown in Fig. 7. It involves the injection of a differential coolant mass flowrate δm_c at *static* temperature $T_{c,x}$, velocity $V_{c,x}$, and angle ϕ, into the mainstream at local *static* temperature $T_{g,x}$ and velocity $V_{g,x}$. The differential analysis is general and is not restricted to mixing at either constant area or constant pressure. In Appendix 3 it is shown that the rate of entropy creation $\delta\Sigma_{mix}$ can be subdivided into separate contributions representing the dissipation of thermal energy and kinetic energy,

$$\delta\Sigma_{mix}=\delta\Sigma_{mix,Q}+\delta\Sigma_{mix,KE} \quad (17a)$$

$$\delta\Sigma_{mix,Q}=\delta m_c\int_{T_{c,x}}^{T_{g,x}}c_{pc}\left(\frac{1}{T}-\frac{1}{T_{g,x}}\right)dT \quad (17b)$$

$$\delta\Sigma_{mix,KE}=\delta m_c\left[\frac{(V_{g,x}-V_{c,x}\cos\phi)^2}{2T_{g,x}}+\frac{(V_{c,x}\sin\phi)^2}{2T_{g,x}}\right] \quad (17c)$$

Equation (17*b*) represents the thermal dissipation as the coolant mixes with the mainstream flow and their *static* temperatures equilibrate. Thus, $\Delta\Sigma_{mix,Q}$ multiplied by the mainstream temperature is exactly equal to the power that could theoretically be obtained from a Carnot engine coupled between the mainstream flow at constant temperature $T_{g,x}$ and the coolant, as the temperature of the latter increases from $T_{c,x}$ to $T_{g,x}$ due to the heat rejection from the engine. Equation (17*c*) represents the dissipation of bulk kinetic energy as the mainstream and coolant velocities equilibrate. The first term refers to velocity equilibration tangential to the blade surface. The second term shows that, subject to the assumptions of the theory, all the coolant kinetic energy normal to the blade surface is lost.

Following the discussion in Part I (Young and Wilcock [1]), the exergy loss arising from the diffusional mixing of the mainstream and coolant gases is ignored.

Integration of Eqs. (17*b*) and (17*c*) over the complete blade surface is difficult because of the variation of $T_{g,x}$ and $V_{g,x}$. One possible approach is described by Hartsell [7], but this requires comparatively detailed knowledge of the flow in the blade passage. If this is unavailable, it is necessary to adopt suitable average values and integrate approximately to give,

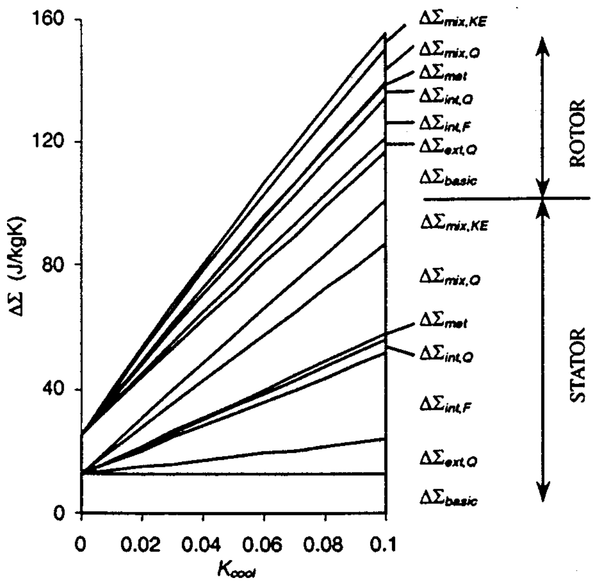

Fig. 8 Variation of cooling losses with cooling flowrate

$$\Delta\Sigma_{mix,Q} \cong m_c c_{pc}\left[\ln\left(\frac{T_{g,x}}{T_{c,x}}\right) - \left(1 - \frac{T_{c,x}}{T_{g,x}}\right)\right] \qquad (18a)$$

$$\Delta\Sigma_{mix,KE} = m_c\left[\frac{(V_{g,x} - V_{c,x}\cos\phi)^2}{2T_{g,x}} + \frac{(V_{c,x}\sin\phi)^2}{2T_{g,x}}\right] \qquad (18b)$$

6.4 Example: Cooling Loss Calculation. As an example, consider again the single stage turbine operating at the conditions shown in Table 1. The extra parameters required to estimate the losses were taken to be $K_{int}=1.01$, $K_{ext}=1.07$ and $\phi=30$ deg. Figure 8 shows how the stator and rotor losses vary with the cooling flow factor K_{cool} around its base value of 0.045. This corresponds to varying the coolant flowrate keeping the combustor flowrate constant. The magnitude of the cooling loss compared with the basic loss is particularly notable, indicating the possibility of improvements to cycle efficiency by careful design.

8 Conclusions

The paper has described a self-consistent approach for modeling the air-cooled gas turbine that is particularly suitable for thermodynamic cycle calculations. The procedure divides naturally into first law and second law analyses, which are almost independent. The former requires a heat transfer model to estimate the cooling flows and an extended Holland and Thake procedure is recommended. It is particularly important to acknowledge the nonuniformity of the combustor exit flow to avoid underestimating the cooling flowrates. Rotating blade rows also require careful consideration.

The second law analysis requires user-specified models for the cooling losses and it is strongly recommended that these are expressed in terms of irreversible entropy creation rates rather than loss of total pressure or modified stage efficiency. For flows with heat transfer, it is crucially important to distinguish clearly between reversible and irreversible entropy changes. Simple models have been proposed for each component of the loss and this allows the source and magnitude of the major cooling irreversibilities to be identified.

It is hoped that the analysis presented in this paper provides a firm foundation on which to proceed to improved and more accurate thermodynamic models of the air-cooled gas turbine.

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Nomenclature

A = area
Bi = Biot number
h, i = specific enthalpy, rothalpy
m = mass flowrate
P = power
p = pressure
Q = heat transfer rate
St = Stanton number
s = specific entropy
t = thickness
T = temperature
U = blade speed
V = absolute flow velocity
α = heat transfer coefficient
ε_0 = blade cooling effectiveness
ε_f = film cooling effectiveness
η_c = internal flow cooling efficiency
λ = thermal conductivity
ψ = stage loading coefficient
$\Delta\Sigma$ = rate of entropy creation by irreversible processes

Subscripts

0 = total (as opposed to static) quantities
1, 2, 3, 4 = stator inlet, stator outlet/rotor inlet, rotor outlet, stage outlet
c, g = coolant, mainstream gas
i = coolant condition at inlet to blade passages
k = coolant condition at compressor bleed point
m = metal
w = wall (outer surface of blade)
x = coolant condition at exit from blade (before mixing)
r, s = rotor, stator (subscripts preceding c, m, w)
tbc = thermal barrier coating
$*$ = blade throat

Appendix 1

Rotor Approximations. The Euler turbine equations are,

$$P = m_{g,2}[(UV_\theta)_{g,2} - (UV_\theta)_{g,3}] + m_{rc}[(UV_\theta)_{rc,i} - (UV_\theta)_{g,3}] \qquad (A1.1a)$$

$$P_{ext} = m_{g,2}[(UV_\theta)_{g,2} - (UV_\theta)_{g,3}] + m_{rc}[(UV_\theta)_{rc,x} - (UV_\theta)_{g,3}] \qquad (A1.1b)$$

$$P_{int} = m_{rc}[(UV_\theta)_{rc,i} - (UV_\theta)_{rc,x}] \qquad (A1.1c)$$

where UV_θ is the massflow-averaged product of the blade speed U and the fluid absolute swirl velocity V_θ. Clearly $P = P_{ext} + P_{int}$.

Coolant Centrifuging Power Requirement. P_{int} represents the negative of the power required to increase the angular momentum of the coolant. This is *not* an irreversible power loss because, in principle, it is recoverable further downstream. P_{int} is given by Eq. ($A1.1c$) but the change in UV_θ of the coolant between passage inlet and outlet is unlikely to be known exactly.

It is assumed: (i) coolant enters the rotor with zero relative swirl $[(V_\theta)_{rc,i} = U_{rc,i}]$, ($ii$) the coolant relative swirl velocity at exit from the rotor is zero $[(V_\theta)_{rc,x}=U_{rc,x}]$ and, (*iii*) $U_{rc,x}=U_{mean}$. Combining Eqs. (*A*1.1*c*) and (5) with a minor approximation then gives,

$$\frac{P_{int}}{P}\cong-\frac{1}{\psi}\frac{L}{R}\frac{m_{rc}}{m_{g,2}} \tag{A1.2}$$

where L/R is the ratio of blade length to mean blade radius. If $\psi=1$ and $L/R=0.2$ then $P_{int}/P\cong-0.2(m_{rc}/m_{g,2})$. Hence, for $m_{rc}/m_{g,2}=0.05$ about 1 percent of the gross power is used in centrifuging the coolant.

Coolant Centrifugal Temperature Change. Noting that $i=h_0^{rel}-U^2/2$, Eq. (4*c*) can be written,

$$-Q_r=m_{rc}\left[(h_{0rc,i}^{rel}-h_{0rc,x}^{rel})-\frac{(U_{rc,i}^2-U_{rc,x}^2)}{2}\right] \tag{A1.3}$$

The final term divided by c_{pc} represents the *centrifugal temperature change* and is typically 10–20°C. A typical value for $(T_{rc,i}-T_{rc,x})$ is 200°C. Given the approximate nature of the cooling model, it is expedient to neglect the centrifugal temperature change. Thus,

$$-Q_r\cong m_{rc}(h_{0rc,i}^{rel}-h_{0rc,x}^{rel}) \tag{A1.4}$$

Rotor Inlet Relative Total Enthalpy (Gas and Coolant). The heat transfer model requires a value of the *relative* total enthalpy of the mainstream gas at rotor inlet. This can be estimated from the stage inlet and outlet conditions and an assumed stage loading coefficient. First note that, without approximation,

$$h_{0g,2}^{rel}=h_{0g,2}+\frac{U_{mean}^2}{2}\left(1-\frac{2(V_\theta)_{g,2}}{U_{mean}}\right) \tag{A1.5}$$

where the enthalpies include the contribution from coolant injected in the stator. Assuming zero swirl at stage outlet and neglecting any cooling flow to the rotor, Eqs. (3*a*), (5), and (*A*1.1*a*) give,

$$\psi=\frac{(V_\theta)_{g,2}}{U_{mean}}=\frac{h_{0g,2}-h_{0g,3}}{U_{mean}^2} \tag{A1.6}$$

Combining Eqs. (*A*1.5) and (*A*1.6) gives the approximating expression,

$$h_{0g,2}^{rel}=\left(\frac{1}{2\psi}\right)h_{0g,2}+\left(1-\frac{1}{2\psi}\right)h_{0g,3} \tag{A1.7}$$

It is also necessary to relate the coolant relative total enthalpy at rotor inlet to the absolute value. The exact expression,

$$h_{0rc,i}-h_{0rc,i}^{rel}=(UV_\theta)_{rc,i}-\frac{U_{rc,i}^2}{2} \tag{A1.8}$$

shows that, if $(V_\theta)_{rc,i}>U_{rc,i}/2$, the relative total enthalpy of the coolant entering the rotor passage will be less than the absolute total enthalpy. From the rotor viewpoint, therefore, a considerable cooling effect is realized by a high coolant absolute swirl velocity $(V_\theta)_{rc,i}$. To this end, stationary swirl vanes are usually mounted upstream of the coolant inlet to the rotor and, in this way, it is possible to reduce the relative total temperature by as much as 100°C. For calculations, it is convenient to introduce a constant K_{swirl} (the *rotor swirl factor*) defined by $(V_\theta)_{rc,i}=K_{swirl}U_{rc,i}$. In practice, $0<K_{swirl}<2.5$. Using the definition of ψ, Eq. (*A*1.8) then becomes,

$$h_{0rc,i}-h_{0rc,i}^{rel}=\frac{(K_{swirl}-0.5)P}{\psi m_{g,3}} \tag{A1.9}$$

Appendix 2

Heat Transfer Calculations. The heat transfer rates are given by Eq. (2*c*) for stators and (*A*1.4) for rotors. Dropping the subscripts *s* and *r*,

$$Q\cong m_c c_{pc}(T_{0c,x}-T_{0c,i}) \tag{A2.1}$$

where T_{0c} denotes the coolant absolute total temperature for a stator and relative total temperature for a rotor. Calculation of $T_{0c,x}$ requires a knowledge of the mean internal heat transfer coefficient, which is difficult to predict accurately. The problem is bypassed by introducing an *internal flow cooling efficiency* $\eta_{c,int}$ defined by,

$$\eta_{c,int}=\frac{T_{0c,x}-T_{0c,i}}{T_{m,int}-T_{0c,i}} \tag{A2.2}$$

$\eta_{c,int}$ is treated as a known empirical parameter whose value (typically 0.6–0.8) reflects the level of the internal cooling technology.

In terms of the mainstream flow, Q can also be expressed by,

$$Q=\alpha_g A_{surf}(T_{aw}-T_w) \tag{A2.3}$$

where α_g is the mean external heat transfer coefficient and A_{surf} is the total cooled external surface area (including the endwalls). T_{aw} is the mean adiabatic wall temperature, which, in the absence of film cooling, equals the mainstream recovery temperature. This, in turn, is approximately equal to the inlet absolute total temperature for stators and the inlet relative total temperature for rotors, both of which will be denoted by T_{0g}. When film cooling is present, T_{aw} is related to T_{0g} and $T_{0c,x}$ by the mean *film cooling effectiveness* ε_f defined by,

$$\varepsilon_f=\frac{T_{0g}-T_{aw}}{T_{0g}-T_{0c,x}} \tag{A2.4}$$

ε_f is treated as a known empirical parameter whose value (typically 0.2–0.4) reflects the level of film cooling technology.

In terms of the heat conduction, Q can also be written,

$$Q=\frac{\lambda_{tbc}}{t_{tbc}}A_{surf}(T_w-T_{m,ext}) \tag{A2.5}$$

$$Q=\frac{\lambda_m}{t_m}A_{surf}(T_{m,ext}-T_{m,int}) \tag{A2.6}$$

where λ and t are the thermal conductivity and material thickness.

Using Eq. (*A*2.3) to eliminate Q from (*A*2.1), (*A*2.5) and (*A*2.6) gives equations for m_{c+} a dimensionless coolant mass flowrate, Bi_{tbc} a TBC Biot number, and Bi_m a metal Biot number:

$$m_{c+}=\frac{m_c c_{pc}}{\alpha_g A_{surf}}=\frac{T_{aw}-T_w}{T_{0c,x}-T_{0c,i}} \tag{A2.7}$$

$$\mathrm{Bi}_{tbc}=\frac{\alpha_g t_{tbc}}{\lambda_{tbc}}=\frac{T_w-T_{m,ext}}{T_{aw}-T_w} \tag{A2.8}$$

$$\mathrm{Bi}_m=\frac{\alpha_g t_m}{\lambda_m}=\frac{T_{m,ext}-T_{m,int}}{T_{aw}-T_w} \tag{A2.9}$$

Finally, the *blade cooling effectiveness* ε_0 is defined by,

$$\varepsilon_0=\frac{T_{0g}-T_{m,ext}}{T_{0g}-T_{0c,i}} \tag{A2.10}$$

ε_0 is fixed by the inlet total temperatures of the mainstream gas and coolant, and the desired external metal temperature.

Elimination of all temperature differences between Eqs. (*A*2.2), (*A*2.4) and (*A*2.7)–(*A*2.9) results in the following expression for m_{c+},

$$(1+\mathrm{Bi}_{tbc})m_{c+}=\frac{\varepsilon_0}{\eta_{c,ext}(1-\varepsilon_0)}-\varepsilon_f\left[\frac{1}{\eta_{c,ext}(1-\varepsilon_0)}-1\right] \quad (A2.11a)$$

where $\eta_{c,ext}$ is a cooling efficiency defined in terms of the external, rather than the internal, metal temperature,

$$\eta_{c,ext}=\frac{T_{0c,x}-T_{0c,i}}{T_{m,ext}-T_{0c,i}}=\frac{\eta_{c,int}}{1+m_{c+}\eta_{c,int}\mathrm{Bi}_m} \quad (A2.11b)$$

The first term of Eq. ($A2.11a$) gives the value of m_{c+} when internal convection is the only cooling mechanism. The second term gives the reduction when film cooling is also used. When a thermal barrier coating is present, m_{c+} is reduced by the factor $(1+\mathrm{Bi}_{tbc})$. Clearly m_{c+} can be determined once ε_0, $\eta_{c,int}$, ε_f, Bi_m, and Bi_{tbc} have been specified.

m_{c+} is now related to the actual coolant/mainstream mass flow ratio. Neglecting the coolant injected upstream of the throat,

$$m_g \cong \rho_{g*}V_{g*}A_{g*} \quad (A2.12)$$

where ρ_{g*}, V_{g*} and A_{g*} are the gas density, velocity, and flow cross-sectional area at the blade throat. Introducing Eq. ($A2.7$) gives,

$$\frac{m_c}{m_g}=\frac{A_{surf}}{A_{g*}}\frac{c_{pg}}{c_{pc}}\mathrm{St}_g m_{c+}=K_{cool}m_{c+} \quad (A2.13)$$

where $\mathrm{St}_g=\alpha_g/c_{pg}\rho_{g*}V_{g*}$ is a Stanton number based on the mean external heat transfer coefficient and the flow properties at the throat. Either α_g or St_g must be estimated from a suitable correlation. The *cooling flow factor* K_{cool} is defined by Eq. ($A2.13$).

Appendix 3

Entropy Creation

Internal Heat Transfer and Friction Losses. Figure 6 shows an idealized onedimensional coolant flow. The momentum, energy, and second law equations are,

$$m_c\left(\frac{\delta p_c}{\rho_c}+V_c\delta V_c\right)=-V_c\delta F \quad (A3.1a)$$

$$m_c(\delta h_c+V_c\delta V_c)=\delta Q \quad (A3.1b)$$

$$\delta\Sigma_{int}=m_c\delta s_c-\frac{\delta Q}{T_{m,int}} \quad (A3.1c)$$

Combining Eqs. ($A3.1$) and using $T_c\delta s_c=\delta h_c-\delta p_c/\rho_c$ gives,

$$\delta\Sigma_{int}=\left(\frac{1}{T_c}-\frac{1}{T_{m,int}}\right)\delta Q+\frac{V_c}{T_c}\delta F=\delta\Sigma_{int,Q}+\delta\Sigma_{int,F} \quad (A3.2)$$

This shows how $\delta\Sigma_{int}$ can be divided into two entropy creation terms, one associated with heat transfer and the other with fluid friction. Expressing δs_c in terms of changes in total temperature and pressure,

$$\delta s_c=c_{pc}\frac{\delta T_{0c}}{T_{0c}}-R_c\frac{\delta p_{0c}}{p_{0c}} \quad (A3.3)$$

and noting that $\delta Q=m_c c_{pc}\delta T_{0c}$, Eq. ($A3.1c$) can be written,

$$\delta\Sigma_{int}=\left(\frac{1}{T_{0c}}-\frac{1}{T_{m,int}}\right)m_c c_{pc}\delta T_{0c}-m_c R_c\frac{\delta p_{0c}}{p_{0c}} \quad (A3.4)$$

Taken together, Eqs. ($A3.2$) and ($A3.4$) give,

$$\delta\Sigma_{int,Q}=\left(\frac{1}{T_c}-\frac{1}{T_{m,int}}\right)\delta Q=\left(\frac{1}{T_c}-\frac{1}{T_{m,int}}\right)m_c c_{pc}\delta T_{0c} \quad (A3.5a)$$

$$\delta\Sigma_{int,F}=\frac{V_c}{T_c}\delta F=\left(\frac{1}{T_{0c}}-\frac{1}{T_c}\right)m_c c_{pc}\delta T_{0c}-m_c R_c\frac{\delta p_{0c}}{p_{0c}} \quad (A3.5b)$$

where R_c is the specific gas constant and T_c is the *static* temperature. Equations ($A3.5$) cannot be integrated analytically. One approximate method of attack is to write,

$$\frac{1}{T_c}=\left(1+\frac{\gamma_c-1}{2}M_c^2\right)\frac{1}{T_{0c}}=K_{int}\frac{1}{T_{0c}} \quad (A3.6)$$

and assume a constant mean value for the coolant Mach number M_c. Integration of Eqs. ($A3.5$) then gives Eqs. (14).

External Heat Transfer Loss. Rewriting Eq. ($A3.5a$) for the external flow,

$$\delta\Sigma_{ext,Q}\cong\left(\frac{1}{T_w}-\frac{1}{T_g}\right)\delta Q \quad (A3.7)$$

where T_g is the mainstream *static* temperature and δQ is the heat transfer to the blade. The variation of T_g is accommodated by writing,

$$\frac{1}{T_g}=\left(1+\frac{\gamma_g-1}{2}M_g^2\right)\frac{1}{T_{0g}}=K_{ext}\frac{1}{T_{0g}} \quad (A3.8)$$

and then assuming a constant mean value for the mainstream Mach number M_g. Integration of Eq. ($A3.7$) leads to Eq. (16).

External Mixing Process. The momentum and energy equations written for the control volume of Fig. 7 are (dropping the subscript x),

$$m_g\left(\frac{\delta p_g}{\rho_g}+V_g\delta V_g\right)+\delta m_c V_g(V_g-V_c\cos\phi)=0 \quad (A3.9a)$$

$$m_g(\delta h_g+V_g\delta V_g)+\delta m_c\left\{h_c(T_g)-h_c(T_c)+\frac{(V_g^2-V_c^2)}{2}\right\}=0 \quad (A3.9b)$$

The second law of thermodynamics is,

$$\delta\Sigma_{mix}=m_g\delta s_g+\delta m_c[s_c(T_g,p_g)-s_c(T_c,p_g)] \quad (A3.9c)$$

Combining Eqs. ($A3.9$) and using $T_g\delta s_g=\delta h_g-\delta p_g/\rho_g$ gives,

$$\delta\Sigma_{mix}=\delta m_c\left\{s_c(T_g,p_g)-s_c(T_c,p_g)-\frac{h_c(T_g)-h_c(T_c)}{T_g}\right\}+\delta m_c\left\{\frac{(V_g-V_c\cos\phi)^2}{2T_g}+\frac{(V_c\sin\phi)^2}{2T_g}\right\} \quad (A3.10)$$

For an ideal gas, Eq. ($A3.10$) is equivalent to Eq. (17).

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