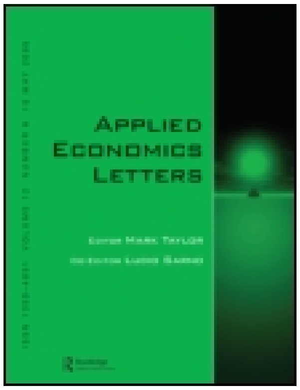

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Demand stochastics, supply adaptation, and the distribution of film earnings

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Demand stochastics, supply adaptation, and the distribution of film earnings

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A market is analysed in which demand is a stochastic process and supply is contingent on the expected level of demand – a model that provides a realistic depiction of the motion picture market where consumer demand is a process of discovery and information sharing, and the supply of theatre screens expands through contingent contracts to accommodate demand. This model predicts that motion picture earnings will deviate from a power law and instead be distributed according to an exponential of a power law due to finite-size effects in demand. Empirical analysis on a large sample of motion pictures finds significant deviation from the power law distribution and a remarkably good fit for the stretched exponential distribution.

I. Introduction

The market for motion pictures is difficult to understand quantitatively, though the intuition is transparent. Filmgoers discover the films they like by consuming them, and through the exchange of information the demand for motion pictures evolves over a time. Supply adjusts as the available screens respond to demand through flexible state-contingent exhibition contracts.[1] And to complicate matters, competing films are entering and exiting theatrical exhibition during the lifetime of any given film. But how can movie demand be quantified in this context?

In order to make headway on the quantification of the process of demand, it is first necessary to have an underlying behavioural model that leads to a particular distribution of outcomes. This paper sets out to do just this: A behavioural model is applied to the film industry where the demand for movies is characterized by random shocks, and this is equivalent to a statistical model based on a multiplicative stochastic process. The multiplicative process with extreme deviations leads to a distribution of movie outcomes that has tails that are distinct from the tails of power-law distribution, such as the Pareto, and also distinct from tails that decline exponentially. In fact, the tails follow a distribution in which an exponential wraps a power law, yielding tails that are heavier than exponential; this distribution is the Weibull, but is frequently referred to as the stretched exponential in the statistical mechanics literature. The stretched exponential (Weibull) model is estimated on a large sample of motion picture data and it is found to fit remarkably well. The alternative models based on Pareto power-law tails and on exponential tails can be rejected. The estimates indicate that on average there are 1.4 levels in the multiplicative cascade model, providing an indication of the degree of information transmission in the demand process.

In the following section the stochastic process that leads to a stretched exponential distribution of outcomes is set out and the Pareto and Weibull distributions discussed. The stretched exponential model is then estimated and compared with the

[1] De Vany and Walls (1996) provide a thorough analysis of role of adaptive exhibition contracts in the movie business.

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simple exponential, power law, and parabolic power law models. In Section IV some useful quantities are calculated based on the stretched exponential estimates. Conclusions are made in Section V.

II. Demand Stochastics and the Distribution of Equilibrium Outcomes

Recent papers on the entertainment industry have focused on the types of demand processes that are consistent with the observed distribution of product success. De Vany and Walls (1999, 2002) find that box-office revenue is asymptotically power law or Pareto distributed. In particular they fit a form of the cumulative distribution function for the Pareto distribution, given by $F(x) = 1 - (\beta/x)^{\alpha}$. De Vany and Walls (1996), Walls (1997) and Hand (2001) find concavity in log–log plots of size against rank, which is known as a parabolic power law, and they interpret the parabolic power law as evidence of increasing returns to information in the demand for motion pictures.[2] In this paper, a distribution is investigated that is related to the power-law models, though it differs in that it provides a superior fit to data on motion-picture box-office revenue and it can be derived from a multiplicative stochastic process that has a behavioural interpretation.

Frisch and Sornette (1997) propose a multiplicative stochastic process that can explain the deviation of the data relative to a power law distribution.[3] Sornette (1998) provides rigorous technical details on multiplicative processes leading to power laws and stretched exponentials. Let the probability density function of i.i.d. random variables m be $p(m)$. Now consider the product $X_n = m_1 m_2 \ldots m_n$ which has probability density function $P_n(X) \sim (p(X^{1/2}))^n$ for $X \to \infty$ and finite n.[4] The tail of $P_n(X)$ is controlled by the realizations where all terms in the product are of the same order, so that $P_n(X)$ is to a first approximation the product of the probability density functions of the n terms in the product $m_1, \ldots, m_n$. When the density function $p(\cdot)$ is exponential, then the density function of the product is a stretched exponential for large n, and follows the Weibull cumulative distribution given by $F(x) = 1 - \exp[-(x/\beta)^{\alpha}]$. When the exponent α in the Weibull distribution is unity, it is the exponential distribution; when the exponent is less than unity, the tails are heavier than exponential tails and the distribution is often referred to as the stretched exponential.

In the empirical application of the next section, the theoretical Pareto and Weibull distribution functions is transformed into their rank-order equivalents to analyse their fit to the data in a familiar regression analysis framework.

III. Statistical Evidence from North American Film Revenues

The sample of data includes 2015 movies that were released from 1985 to 1996, inclusive. The data, obtained from ACNielson EDI, Inc.'s historical database, were compiled from distributor-reported box-office figures for the North American theatrical market.[5] These data are the standard industry source for published information on motion picture theatrical revenues and are used by many major industry publications including *Daily Variety* and *Weekly Variety*. The data are described in detail, including numerous descriptive statistics and cross-tabulations, by De Vany and Walls (1999).

Figure 1 shows a plot of box-office revenue against rank on logarithmic axes. If a power law described the data set over its entire range, this plot would be linear. Consistent with results found in a number of physical and social science settings, the log–log size-rank plot deviates from linearity: The concavity in the plot implies that, while the tails may be appropriately described by a power law – or asymptotically Pareto distributed – the entire distribution deviates from a power law. Ijiri and Simon (1974) show that the concave (with respect to the origin) deviation from the power-law distribution is equivalent to auto-correlated growth, and De Vany and Walls (1996) interpret this as increasing returns to information transmission in the context of the film business.[6]

[2] Sornette (2002) looks at importance of blockbusters when tails follow a power law. Much of the economics literature relating to power laws and deviations from them has been developed in the context of the distribution of firm sizes (Steindl, 1965; Ijiri and Simon, 1971, 1974; Vining, 1976).

[3] Laherrere and Sornette (1998) show that the stretched exponential distribution accounts for many deviations from power law distributions in physical and social sciences.

[4] From the relation $X_{n+1} = X_n x_{n+1}$ the equation for the density function of X_{n+1} can be written in terms of the density functions of x_{n+1} and X_n and the pdf derived by the formal application of Laplaces' method. See Frisch and Sornette (1997) for a detailed derivation of the stretched exponential distribution.

[5] Appreciation is extended to David T. of EDI for his generous assistance in providing the data.

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[6] The empirical result of concave deviation from the power law distribution has been independently verified using different data sets by Walls (1997), Ghosh (2000), and Hand (2001).

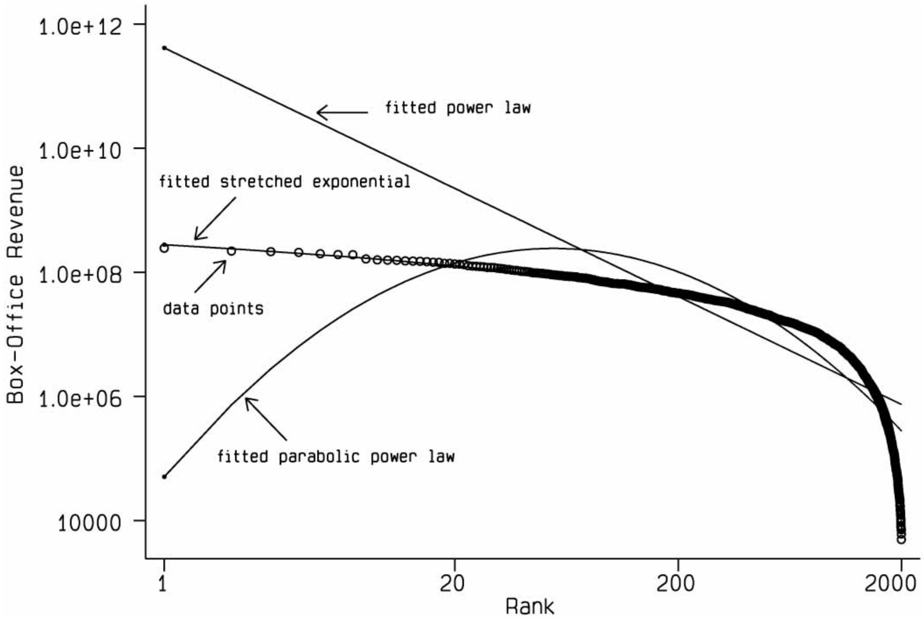

Fig. 1. Data and fitted distributions on a logarithmic scale

Table 1. Parameter estimates: power law

	Full sample	Hit movies
Constant	26.749 (0.2080)	19.861 (0.0257)
log *Rank*	−1.741 (0.0311)	−0.401 (0.0058)
Observations	2015	206
R^2 in logs	0.609	0.959
R^2 in levels	0.307	0.947

Notes: Estimated standard errors in parentheses.
Hit movies defined as revenue > US$45 million.

Table 2. Parameter estimates: parabolic power law

	Full sample	Hit movies
Constant	10.829 (0.4108)	19.256 (0.0109)
log *Rank*	4.224 (0.1448)	−0.013 (0.0062)
(log *Rank*)^2	−0.526 (0.0126)	−0.055 (0.0008)
Observations	2015	206
R^2 in logs	0.790	0.998
R^2 in levels	0.791	0.998

Notes: Estimated standard errors in parentheses.
Hit movies defined as revenue > US$45 million.

The graphical comparisons illustrated in Fig. 1 are formalized, and now tested statistically for deviations from the power law distribution. Table 1 shows the estimates of a fitted linear equation to the log–log plot: $\log Revenue = \alpha + \beta \log Rank + \mu$. In terms of log Revenue the model explains about 60% of the variation, but in terms of the *level* of revenue, the model explains only 30% of the variation.[7] The Pareto distribution model was also estimated to the upper tail of the revenue distribution, estimating the log linear regression only on those 'hit' movies earning box-office revenue in excess of US$45 million. Confining the attention to the upper tail, the Pareto model provides a much better fit to the data with about 95% of the variation in the level of box-office revenue being explained.

The parabolic power law specification, was also estimated, a model that is a parabola in the log–log size-rank plot: $\log Revenue = \alpha + \beta \log Rank + \gamma(\log Rank)^2 + \mu$. This model permits deviation from the linear log–log size-rank plot which can be interpreted as returns to information. Table 2 reports the estimates of the parabolic power law. The estimates show a statistically superior fit to the entire distribution of box-office revenue as compared to the linear power law estimates. The model explains about 79% of the variation in terms of both the logarithms and levels of revenue. When confined to the portion of the revenue distribution in excess of US$45 million – the 'hit' movies – the estimated parabolic power law model explains 0.998 of the variation in box-office revenue. It is apparent from the

[7] In terms of the level of revenue, the R^2 is the squared correlation between fitted revenue and actual revenue.

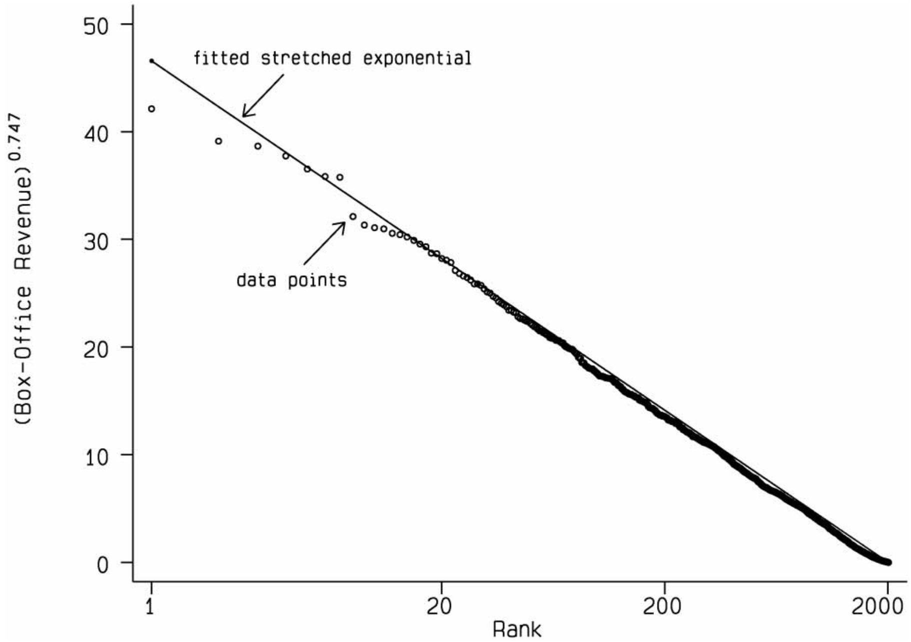

Fig. 2. Diagnostic plot for fitted stretched exponential distribution

Table 3. Parameter estimates: stretched exponential

	Full sample	Hit movies
α	46.562 (0.4890)	137.732 (13.4221)
β	−6.119 (0.0685)	−20.555 (2.1811)
γ	0.747 (0.0023)	0.886 (0.0183)
Observations	2015	206
R^2 in levels	0.908	0.999

Notes: Estimated standard errors in parentheses.
Hit movies defined as revenue > US$45 million.

regression results reported in Tables 1 and 2 that there is a clear departure from linearity in the log–log *rank*-size plot, even in the upper tail of the distribution.

As an alternative to the power law type distributions, the stretched exponential distribution is now investigated. A stretched exponential will be linear when plotting $Revenue^{\gamma}$ as a function of $\log Rank$: $Revenue^{\gamma} = \alpha + \beta \log Rank + \mu$. The parameters of the stretched exponential distribution is estimated by non-linear least squares and these estimates are displayed in Table 3. The estimate of the exponent $\gamma = 0.747$ is statistically less than 1 at the 1% marginal significance level, permitting the exponential model to be formally rejected in favour of the stretched exponential model. The overall fit of the stretched exponential model is quite good, with an R^2 of about 0.91 as compared with the R^2 of 0.31 for the power law model and 0.78 for the parabolic power law model.[8]

Figure 1 plots the estimated size-rank distributions for the power law and parabolic power law models against the empirical size-rank distribution. It is clear that these models which can predict well for asymptotic tail behaviour predict poorly when they are fit to the entire distribution. Figure 2 plots the box-office revenue data against the fitted stretched exponential distribution. The fit is quite good over the range of data, although there is slightly more variation for the top-ranked movies.

IV. Interpreting the Stretched Exponential Estimates

From the parameter estimates of the stretched exponential the theoretical distribution can be used to calculate some useful quantities. The number of levels in the multiplicative cascade model is $1/\gamma$ and the estimate of this is 1.339. This number is similar to the exponent in the Pareto distribution in that it is the amount by which the distribution's argument is powered up before it is exponentiated. The number of levels in the information transmission cascade is consistent with limited transmissibility.

[8] These R^2 values are all calculated in terms of the *level* of box-office revenue.

The mean of the stretched exponential distribution is $x_0\Gamma(1/\gamma)/\gamma$ where $x_0 = a^{1/\gamma}$ is a reference scale from which one can calculate the other moments of the distribution. Evaluating this expression using the point estimates yields a value of 204.186 million for the mean film earnings. Other quantiles of the fitted stretched exponential distribution can also be calculated. For example, the 95th percentile of the distribution is $3^{1/\gamma}x_0$ which when evaluated at the point estimates yields 743.879 million.

One of the attractions of the power law distributions in explaining the movie business is that they allow for the heavy tails and skewness that are characteristic of box-office outcomes. Frisch and Sornette (1997) are concerned with extreme deviations, and this exists when the probability density function extends to arbitrarily large values. They are also interested in the tail behaviour of the distribution. In statistics, probabilities in the extreme 5% or 1% of the tails are typically considered as extreme, but here the concern is with much smaller tail probabilities. The stretched exponential distribution also has this property because this distribution is not truncating the upper tail in its estimates of the probability of a movie earning a larger amount than previous movies. The distribution accounts for the deviation from the strict Pareto power law in a way that does not place artificial restrictions on the possibility that a movie can earn far more than experience suggests. Also, the distribution more closely resembles the entire range of movie outcomes than the alternatives.

V. Conclusion

There is strong evidence that the distribution of motion picture outcomes does not follow a power law or a even a parabolic power law. Empirical evidence from North American box-office revenue shows that the stretched exponential distribution provides a statistically superior fit. This is consistent with a model of consumer demand where shocks are multiplicative and there is decay in information transmission that leads to local audience saturation. Future research will seek to relate the models of demand stochastics to the information cascade literature with an eye toward generating empirically testable hypotheses.

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